# **Representation Theory of Groups** - Blatt 6

11:30-12:15, Seminarraum 9, Oskar-Morgenstern-Platz 1, 2.Stock

http://www.mat.univie.ac.at/~gagt/rep_theory2017


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**Question 1.** Give an example of a unitary representation of some group $G$ on a Hilbert space $\mathcal{H}$ of that has at least one proper subspace $V \subset \mathcal{H}$ that is invariant but not closed.

**Question 2.** Consider $\mathbb{R}$ as a topological group under addition.

a) As a topological group with the discrete topology show that:

   i) Given any subgroup $H \leq \mathbb{R}$ and any one dimensional representation $\chi : H \to \mathbb{C}^\times$ that there is an extension of $\chi$ to all of $\mathbb{R}$ (Hint: find a partially ordered set here and try and use Zorn's lemma).

   ii) Using part i), conclude that given any finite set $F \subset \mathbb{R}$ and a tuple $\{z_i\}_{i \in F}$ where each $z_i \in \mathbb{C}^\times$, there exists a one dimensional representation of $\mathbb{R}$ that assigns $i \in F$ to $z_i$ in $\mathbb{C}^\times$.

b) As a topological group with the metric topology, every one dimensional representation $\chi : \mathbb{R} \to \mathbb{C}^\times$ is of the form:

$$\chi(x) = e^{sx}$$

where $s$ is some complex number (Hint: for an analytic proof, one might want to suppose that the representation is not only continuous, but differentiable).

The contents of the above question indicate that unnatural topologies will give “too many" representations, but natural ones give us “just the right number".

**Question 3.** Consider $G := \frac{\mathbb{R}}{\mathbb{Z}}$ with its natural quotient topology (from the metric topology on $\mathbb{R}$. Let $E$ be the Banach space $C(G; \mathbb{C})$ with the supremum norm. Show:

a) the "left regular" map $\rho : G \to BGL(E)$, given (pointwise) by $(\rho([r])(f))(x) = f(x + [r])$ is continuous in the supremum norm.

b) Show that $\rho$ is not continuous in the operator norm on $BGL(E)$.