Representation Theory of Groups - Blatt 7

11:30-12:15, Seminarraum 9, Oskar-Morgenstern-Platz 1, 2.Stock

http://www.mat.univie.ac.at/~gagt/rep_theory2017

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On this exercise sheet we will consider the (left) Haar measure μ on a locally compact group G. This exists, and has the property that it is left invariant and unique up to a scalar function (we will not prove existence or uniqueness, but you can use them).

Question 1. (Constructing the modular function) Use the properties of the left Haar measure μ on G to show that there is a function $c : G \to \mathbb{R}$ such that $\mu(Eg) = c(g)\mu(E)$ for every measurable subset $E \subset G$.

Remark: the modular function is continuous - look a proof of this up.

A group G is called *unimodular* if the function c is identically equal to 1 (i.e $c(g) = 1$ for all $g \in G$).

Question 2. Let G be a compact group and let μ be the left Haar measure. Show that G is unimodular and that $\mu(G) < \infty$.

Question 3. (A non-unimodular group) Let G be the group generated by matrices of the form:

$$\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$$

where $a \in \mathbb{R}_{>0}$ and $b \in \mathbb{R}$. Show that $a^{-2}dadb$ is a left Haar measure and $a^{-1}dadb$ is a right Haar measure for G.

Question 4. Outline an argument that might show that $SL(2, \mathbb{R})$ is unimodular.