Representation Theory of Groups - Blatt 8

11:30-12:15, Seminarraum 9, Oskar-Morgenstern-Platz 1, 2.Stock

http://www.mat.univie.ac.at/~gagt/rep_theory2017

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Question 1. A locally compact group G with left Haar measure μ is of *subexponential growth* if

$$\lim_n \mu(U^n)^{\frac{1}{n}} = 1$$

for all compact neighbourhoods of the identity $U \subset G$. Show that any group of subexponential growth is unimodular.

Question 2. Let $G = N \rtimes H$ be the subgroup of $SL_4(\mathbb{C})$, where H and N are defined as follows:

$$H := \{ \begin{pmatrix} A & 0 \\ 0 & (A^t)^{-1} \end{pmatrix} \mid A \in GL_2(\mathbb{C}) \}$$

and

$$N := \{ \begin{pmatrix} I_2 & A \\ 0 & I_2 \end{pmatrix} \mid A \in M_2(\mathbb{C}), A^t = A \}.$$

Show that, for any representation $\pi : G \to U(\mathcal{H})$, if $\xi \in \mathcal{H}$ is invariant under H, then it is also invariant under G.

For the following question, we will consider, for every regular Borel measure μ on G and for every unitary representation $\pi : G \to U(\mathcal{H})$, the operator $\pi(\mu) \in B((H))$ defined by the formula:

$$\langle \pi(\mu)\xi, \eta \rangle := \int_G \langle \pi(x)\xi, \eta \rangle d\mu(x)$$

for every $\xi, \eta \in \mathcal{H}$. It is known that weak containment of the trivial representation in π is related to the spectrum of $\pi(\mu)$. We will consider an example that shows negative behaviour in this context.

Question 3. Fix an irrational real number θ, and consider the dense subgroup of S^1 generated by $e^{2\pi i\theta}$. Let μ be the probability measure on S^1 defined by:

$$\mu = \sum_{n \geq 0} 2^{-n} \delta_{e^{2\pi i n\theta}},$$

and let Φ be the direct sum of all the non-trivial unitary characters of S_1.

1. Show that 1 belongs to the spectrum of the operator $\Phi(\mu)$;
2. Show that 1_{S^1} is not weakly contained in Φ.