The Differentiable Sphere Theorem

Speaker: Josef Zehetbauer

University of Vienna, Faculty of Mathematics

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ABSTRACT

The topological sphere theorem states that any compact, simply connected Riemannian manifold of dimension n, whose sectional curvatures lie in the interval $(1/4, 1]$, is homeomorphic to the standard sphere S^n. It was proved around 1960 by Marcel Berger and Wilhelm Klingenberg. The main step of the proof is the establishment of a proper lower bound for the injectivity radius, which is accomplished by using methods from comparison geometry. In 1956, John Milnor proved that there exists a smooth manifold which is homeomorphic, but not diffeomorphic to S^7. In light of this result one can ask whether a n-dimensional Riemannian manifold, satisfying the assumptions of the topological sphere theorem, is even diffeomorphic to S^n. This statement is known as the differentiable sphere theorem, and it was proved in 2007 by Simon Brendle and Richard Schoen. The argument is based on the Ricci flow, an evolution equation for the Riemannian metric, and in this talk I will try to give an idea of the proof.