Errata
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Errata

Changes appear in yellow. Line $k+$ (resp., line $k-$) denotes the $k$th line from the top (resp., the bottom) of a page. My thanks go to the following individuals who have contributed to this list: Theresa Dvorak, Gudrun Szewieczek, William Jagy, Jonathan Eckhardt, Sebastian Woblistin, Yu Jiang, Annemarie Luger, Johannes Wächtler, Daniel Scherl, Kristoffer Varholm, Constantino Santos, Minjae Park, Teck-Cheong Lim, Peter Elbau, Walter Schachermayer, Sebas Pedersen.

Page 6. 7+: Furthermore, $K^{j+1} \in C^k(U_j, M)$ for any

Page 19. Problem 1.26, second item: $y(x_0 + x) = y(x_0 - x)$

Page 45. 10+: $\Delta(t) = \left[\phi(t, t_0, y_0) - \phi(t, s_0, y_0)\right]$ and use . . .

Page 57. 4+: Taking $m \to \infty$ we finally obtain

Page 73. Problem 3.13: It should read $\text{deg}(q(t)) \leq \text{deg}(p(t)) + s$ where $s$ is the size of the largest Jordan block corresponding to the eigenvalue $\beta$. Moreover, here is an extended hint:

(Hint: Investigate (3.48) using the following fact: $\int_t^t (t-s)^m p(s)e^{\beta s} ds = q(t)e^{\beta t}$, where $q(t)$ is a polynomial of degree $\text{deg}(q) = \text{deg}(p)$ if $\beta \neq 0$ and $\text{deg}(q) = \text{deg}(p) + m + 1$ if $\beta = 0$. First show $m = 0$ using integration by parts and then use induction and again integration by parts.)
Example:

\[(e^t \dot{c}(t) + 4te^t \ddot{c}(t) + (2 + 4t^2)e^t c(t)) - 2t(e^t \dot{c}(t) + 2te^t c(t)) - 2e^t c(t)\]

Problem 3.34: Consider the equation \(\ddot{x} + q_0(t)x = 0\).

Problem 3.37:

\[y^{(n)} + \sum_{k=0}^{n-2} \sum_{j=k}^{n} \binom{j}{k} q_j(t) Q^{(j-k)}(t) y^{(k)} = 0.\]

Problem 3.38:

\[\dot{y} + e^{-Q(t)}y^2 + e^{Q(t)}q_0(t) = 0.\]

Theorem 3.26:

\[|x(t)| \leq Ce^{-(\alpha - b_0 C)t}|x_0|, \quad |x_0| < \frac{\delta}{C}, \quad t \geq 0. \quad (3.168)\]

Proof of Theorem 4.5: An argument that \(h_2(z)\) has the same radius of convergence is missing:

If there is a second solution of the form \(u_2(z) = z^{\alpha_2}h_2(z)\) the same argument can be used for \(h_2(z)\). Otherwise one has to use (4.56) below in place of (4.39). Note that the present theorem will also follow as a special case of Theorem 4.13.

Problem 4.5: \(\Gamma(z) = \frac{(-1)^n}{n!(z+n)} + O(1)\).

Problem 4.11 (iii): \(J_{\nu+1}(z) - J_{\nu-1}(z) = 2J_\nu(z)\)

\[y(x) = y(x_0)c(z, x, x_0) + p(x_0)y'(x_0)s(z, x, x_0) + \int_{x_0}^{x} s(z, x, t)g(t)r(t)dt. \quad (5.50)\]

Problem 5.13:

\[Q(y) = \frac{q(x(y))}{r(x(y))} - \frac{(p(x(y)))^{1/4}}{r(x(y))}(p(x(y))(p(x(y))r(x(y)))^{-1/4})'.\]

\[\min_{x \in [a, b]} \frac{q(x)}{r(x)} \leq E_0. \quad (5.77)\]

In fact, \(\theta_0(\lambda, x)\) as defined in (5.89) is the Prüfer angle for \(-u_0(\lambda, x)\), but this will be of no importance for our purpose.
Page 169.

\[ \#_{(-\infty, \lambda)}(L) = \left\lceil \frac{\theta_a(\lambda, b) - \beta}{\pi} \right\rceil - 1, \quad (5.91) \]

Moreover, note that we also have:

\[ \#_{(-\infty, \lambda]}(L) = \left\lfloor \frac{\theta_a(\lambda, b) - \beta}{\pi} \right\rfloor + 1 = \left\lfloor \frac{\alpha - \theta_b(\lambda, a)}{\pi} \right\rfloor, \]

Page 172. Replace “As in the case of Theorem 5.18 one proves” by ”As an immediate consequence of Lemma 5.16 we obtain”

Page 207. Problem 6.25: The requirement that \( U'(x) \) never vanishes is missing.

Page 213. Proof of Theorem 7.4: Exchange the definitions of \( Q_3 \) and \( Q_4 \).

Page 222. The proof of Lemma 7.13 only shows that \( \omega_\sigma(x) \) contains a regular periodic orbit. However, the claim follows from Lemma 7.13 if \( \omega_\sigma(x) \) is connected. Moreover, connectedness follows from compactness as in the proof of Lemma 6.6. In particular, the connectedness assumption in Theorem 7.16 is superfluous.

Page 225. First equation in the example:

\[ f(x, y) = \left( -\eta E(x, y)^2 y - U'(x) \right), \]

Page 259. Theorem 9.3: there are neighborhoods \( U(x_0) \) of \( x_0 \) and \( U \) of 0 and a function \( h^{+, \alpha} \in C^k(E^{+, \alpha} \cap U, E^{-, \alpha}) \) such that

Page 261. Theorem 9.4: there are neighborhoods \( U(x_0) \) of \( x_0 \) and \( U \) of 0 and functions \( h^{\pm} \in C^k(E^{\pm} \cap U, E^\pm \oplus E^\mp) \) such that

Page 266. Proof of Lemma 9.7: The equation \( A^{-1} \circ \varphi \circ \vartheta = \varphi \circ \vartheta \circ A^{-1} \) should read \( l \circ \varphi \circ \vartheta = \varphi \circ \vartheta \circ l \). This last equation implies \( \varphi \circ \vartheta = I + l \), where \( l \) is a solution of \( Ll(x) = g(x) - g(x + l(x)) \). Using the estimates for the inverse of \( L \) and for \( g \) one obtains \( l \equiv 0 \) and thus \( \varphi \) is a homeomorphism.

Page 278. The very last equation on the bottom of the page is only true if \( \Phi_t \) is linear. Set \( \Phi_t = e^{tA} + G_t \), where \( G_t \) is bounded, and replace this equation by

\[ h_t = \Phi_t \circ \varphi \circ e^{-tA} - I = e^{tA} \circ h \circ e^{-tA} + G_t \circ \varphi \circ e^{-tA}, \]

where both terms are bounded.

Page 289.

\[ \varphi(x) = (x_1 + x_2^2, x_2). \quad (9.38) \]

Page 296. Second sentence after (11.10): Change \( W^s \) to \( W^+ \).

Page 300. Problem 11.3: The assumption that the set has no isolated points needs to be added.

Page 333. Change \( W^s \) to \( W^+ \) and \( W^u \) to \( W^- \) in the picture and the text before the picture.