

Errata

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Ordinary Differential Equations and Dynamical Systems

Graduate Studies in Mathematics, Vol. 140
American Mathematical Society, Providence, Rhode Island, 2012

The official web page of the book:
<http://www.mat.univie.ac.at/~gerald/ftp/book-ode/>

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Updated as of February 28, 2018

Errata

Changes appear in **yellow**. Line $k+$ (resp., line $k-$) denotes the k th line from the top (resp., the bottom) of a page. My thanks go to the following individuals who have contributed to this list: Theresa Dvorak, Gudrun Szewieczek, William Jagy, Jonathan Eckhardt, Sebastian Woblistin, Yu Jiang, Annemarie Luger, Johannes Wächtler, Daniel Scherl, Kristoffer Varholm, Constantino Santos, Minjae Park, Teck-Cheong Lim, Peter Elbau, Walter Schachermayer, Sebas Pedersen, Raphael Stuhlmeier, Eric Wahlén, Lukas Peham, Jakob Holböck, Guillaume Berger.

Page 6. 7+: Furthermore, $K^{j+1} \in C^k(U_j, M)$ for any

Page 19. Problem 1.26, second item: $y(x_0+x) = y(x_0 - x)$

Page 45. 10+: $\Delta(t) = \|\phi(t, t_0, y_0) - \phi(t, s_0, y_0)\|$ and use ...

Page 57. 4+: Taking $m \rightarrow \infty$ we finally obtain

Page 62.

$$\exp(tJ) = \exp(\alpha t \mathbb{1}) \exp(tN) = e^{\alpha t} \sum_{j=0}^{k-1} \frac{t^j}{j!} N^j. \quad (3.18)$$

Page 73. Problem 3.13: It should read $\deg(q(t)) \leq \deg(p(t)) + \mathbf{s}$ where s is the size of the largest Jordan block corresponding to the eigenvalue β . Moreover, here is an extended hint:

(Hint: Investigate (3.48) using the following fact: $\int^t (t-s)^m p(s) e^{\beta s} ds = q(t) e^{\beta t}$, where $q(t)$ is a polynomial of degree $\deg(q) = \deg(p) + m$ if $\beta \neq 0$ and $\deg(q) = \deg(p) + m + 1$ if $\beta = 0$. To see this differentiate $\int^t s^k e^{\beta s} ds$ with respect to β .)

Page 84. Line before (3.98): Let $X(t)$ be the identity matrix with the first column replaced by $\phi_1(t)$,

Page 89. Example:

$$(e^{t^2} \ddot{c}(t) + 4te^{t^2} \dot{c}(t) + (2 + 4t^2)e^{t^2} c(t)) - 2t(e^{t^2} \dot{c}(t) + 2te^{t^2} c(t)) - 2e^{t^2} c(t)$$

Page 90. Problem 3.34: Consider the equation $\ddot{x} + q_0(t)x = 0$.

Page 90. Problem 3.37:

$$y(t) = Q(t)^{-1} x(t), \quad Q(t) = e^{-\frac{1}{n} \int^t q_{n-1}(s) ds}.$$

$$y^{(n)} + Q(t)^{-1} \sum_{k=0}^{n-2} \sum_{j=k}^n \binom{j}{k} q_j(t) Q^{(j-k)}(t) y^{(k)} = 0,$$

Page 90. Problem 3.38:

$$\dot{y} + e^{-Q(t)} y^2 + e^{Q(t)} q_0(t) = 0.$$

Page 100. Paragraph after the proof of Theorem 3.23: ... is constant. To this end recall that Corollary 3.5 tells us when the system corresponding to $B(t) = 0$ is stable. Moreover, ...

Page 101. Theorem 3.26:

$$|x(t)| \leq C e^{-(\alpha - b_0 C)t} |x_0|, \quad |x_0| < \frac{\delta}{C}, \quad t \geq 0. \quad (3.168)$$

Page 120. Proof of Theorem 4.5: An argument that $h_2(z)$ has the same radius of convergence is missing:

If there is a second solution of the form $u_2(z) = z^{\alpha_2} h_2(z)$ the same argument can be used for $h_2(z)$. Otherwise one has to use (4.56) below in place of (4.39). Note that the present theorem will also follow as a special case of Theorem 4.13.

Page 126. Problem 4.5: $\Gamma(z) = \frac{(-1)^n}{n!(z+n)} + O(1)$.

Page 126. Problem 4.8:

$$h_j = \frac{1}{j} \sum_{k=0}^{j-1} p_{j-k} h_k,$$

Page 126. Problem 4.10: For (i) you can use that $\Gamma(z)$ has no zeros and hence $\Gamma(z)^{-1}$ is an entire function.

Page 126. Problem 4.11 (iii): $J_{\nu+1}(z) - J_{\nu-1}(z) = 2J'_\nu(z)$

Page 145. Problem 5.3: We have $x \in [0, 1]$ and there is only one boundary condition $u(t, 1) = 0$.

Page 145. Problem 5.5:

$$m \frac{d^2}{dt^2} u(t, n) = k(u(t, n+1) - u(t, n)) - k(u(t, n-1) - u(t, n)),$$

Page 154.

$$y(x) = y(x_0)c(z, x, x_0) + p(x_0)y'(x_0)s(z, x, x_0) - \int_{x_0}^x s(z, x, t)g(t)r(t)dt. \quad (5.50)$$

$$s(z, x, x_0) = \frac{-u(x)v(x_0) + u(x_0)v(x)}{W(u, v)}. \quad (5.51)$$

Page 155. Problem 5.13:

$$Q(y) = \frac{q(x(y))}{r(x(y))} - \frac{(p(x(y))r(x(y)))^{1/4}}{r(x(y))} (p(x(y))(p(x(y))r(x(y)))^{-1/4})'.$$

Page 162.

$$\min_{x \in [a, b]} \frac{q(x)}{r(x)} \leq E_0. \quad (5.77)$$

Page 166. Problem 5.22: with $f^{(2j)}(0) = f^{(2j)}(1) = 0$ for $0 \leq j \leq k$.

Page 168. In fact, $\theta_b(\lambda, x)$ as defined in (5.89) is the Prüfer angle for $-u_b(\lambda, x)$, but this will be of no importance for our purpose.

Page 169.

$$\#_{(-\infty, \lambda)}(L) = \left\lfloor \frac{\theta_a(\lambda, b) - \beta}{\pi} \right\rfloor = \left\lfloor \frac{\alpha - \theta_b(\lambda, a)}{\pi} \right\rfloor - 1, \quad (5.91)$$

Moreover, note that we also have:

$$\#_{(-\infty, \lambda]}(L) = \left\lfloor \frac{\theta_a(\lambda, b) - \beta}{\pi} \right\rfloor + 1 = \left\lfloor \frac{\alpha - \theta_b(\lambda, a)}{\pi} \right\rfloor,$$

Page 172. Replace “As in the case of Theorem 5.18 one proves” by ”As an immediate consequence of Lemma 5.16 we obtain”

Page 174. Problem 5.29: suppose $0 \leq \alpha_2 < \alpha_1 < \pi$ and show

Page 175. Problem 5.30:

$$W'(u_0, u_1) = (q_1 - \lambda_1 r_1 - q_0 + \lambda_0 r_0)u_0 u_1 + \left(\frac{1}{p_0} - \frac{1}{p_1} \right) p_0 u'_0 p_1 u'_1. \quad (5.111)$$

Page 175. Problem 5.31:

$$u'(x) = a \left(\frac{\cos(x+b)}{x^{1/2}} + \frac{(-3/4 - \nu^2) \sin(x+b)}{2x^{3/2}} + O(x^{-5/2}) \right).$$

Page 183. Problem 5.33:

$$G_{\pm}(z, x, y) = \frac{W(z)^{-1}}{1 \mp \rho_{\pm}(z)} \begin{cases} u_{+}(z, x)u_{-}(z, y) \pm \rho_{+}(z)u_{-}(z, x)u_{+}(z, y), & y < x, \\ u_{-}(z, y)u_{+}(z, x) \pm \rho_{+}(z)u_{+}(z, y)u_{-}(z, x), & y > x, \end{cases}$$

with $W(z) = W(u_{+}(z), u_{-}(z))$.

Page 184. Problem 5.35: (iii) $c(z, \ell) = s'(z, \ell)$.

Page 207. Problem 6.25: The requirement that $U'(x)$ never vanishes is missing.

Page 213. Proof of Theorem 7.4: Exchange the definitions of Q_3 and Q_4 .

Page 214. Problem 7.4: The trajectory enters Q_3 and satisfies $x(t) < x_0$ in Q_3 since ... where $y(t)$ decreases, implying $x(t) \geq x_1 = \frac{1-y_1}{\lambda}$ when ... If $y_2 \geq y_0$, that is, if

$$\lambda \mu^2 ((\mu \lambda)^2 - 1) \left(x_0 - \frac{1 + \mu}{1 + \mu \lambda} \right) > 0, \quad (1)$$

Page 222. The proof of Lemma 7.13 only shows that $\omega_{\sigma}(x)$ contains a regular periodic orbit. However, the claim follows from Lemma 7.14 if $\omega_{\sigma}(x)$ is connected. Moreover, connectedness follows from compactness as in the proof of Lemma 6.6. In particular, the connectedness assumption in Theorem 7.16 is superfluous.

Page 225. First equation in the example:

$$f(x, y) = \left(-\eta E(x, y)^2 y - U'(x) \right),$$

Page 231. Sentence before (8.9): Moreover, for any positively invariant neighborhood $U \subseteq W^+(\Lambda)$ we have

Page 233. Problem 8.2: Let $V_R = \{x \in M \mid L(x) \leq R\}$ be a relatively compact set

Page 235. Replace the last sentence by: Using the Routh–Hurwitz criterion one can show that the two new fixed points are asymptotically stable for $1 < r < \frac{\sigma(3+b+\sigma)}{\sigma-b-1}$ if $1+b < \sigma$ and $1 < r$ if $1+b \geq \sigma$. For the classical values $\sigma = 10$, $b = 8/3$ this gives $1 < r < 470/19 = 24.74$.

Page 240.

$$\frac{d}{dt} \begin{pmatrix} p \\ q \end{pmatrix} = \text{grad}_s H(p, q), \quad (8.46)$$

Page 261. Theorem 9.4: there are neighborhoods $U(x_0)$ of x_0 and U of 0 and functions $h^\pm \in C^k(E^\pm \cap U, E^0 \oplus E^\mp)$ such that

Page 266. Proof of Lemma 9.7: The equation $A^{-1} \circ \varphi \circ \vartheta = \varphi \circ \vartheta \circ A^{-1}$ should read $f \circ \varphi \circ \vartheta = \varphi \circ \vartheta \circ f$. This last equation implies $\varphi \circ \vartheta = \mathbb{I} + l$, where l is a solution of $Ll(x) = g(x) - g(x + l(x))$. Using the estimates for the inverse of L and for g one obtains $l \equiv 0$ and thus φ is a homeomorphism.

Page 268. The very last equation on the bottom of the page is only true if Φ_t is linear. Set $\Phi_t = e^{tA} + G_t$, where G_t is bounded, and replace this equation by

$$h_t = \Phi_t \circ \varphi \circ e^{-tA} - \mathbb{I} = e^{tA} \circ h \circ e^{-tA} + G_t \circ \varphi \circ e^{-tA},$$

where both terms are bounded.

Page 268.

$$\varphi(x) = (x_1 + x_2^2, x_2). \quad (9.38)$$

Page 286. Line before (10.19) which is zero at p , positive for $x \neq p$, whose level sets $S_\delta = \{x \in U(p) | L(x) \leq \delta\}$ are connected for sufficiently small δ , and such that $x \in U(p)$ implies $f^n(x) \in U(p)$ and

Page 289.

$$x(m) = \Pi(m, m_0)x_0 + \sum_{j=m_0}^{m-1} \Pi(m, j+1)g(j), \quad (10.28)$$

$$x(m) = \Pi(m, m_0)x_0 - \sum_{j=m-1}^{m_0} \Pi(m, j+1)g(j), \quad m < m_0. \quad (10.29)$$

Page 296. Second sentence after (11.10): Change W^s to W^+ .

Page 300. Problem 11.3: The assumption that the set has no isolated points needs to be added.

Page 306. Lemma 11.10: the number of periodic points of period at most l is equal to $\text{tr}(A^l)$.

Page 308. Problem 11.10: the number of periodic orbits of period at most n .

Page 316. Problem 11.13: It seems too difficult to give a counterexample. The problem should be ignored.

Page 333. Change W^s to W^+ and W^u to W^- in the picture and the text before the picture.

Page 337. Problem 13.3:

$$\dot{q} = p, \quad \dot{p} = -\sin(q) + \varepsilon \sin(t).$$

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$$\begin{aligned} M(t) &= \int_{-\infty}^{\infty} p_0(s) (\delta p_0(s) + \gamma \cos(\omega(s-t))) ds \\ &= \frac{4\delta}{3} - \sqrt{2}\pi\gamma\omega \operatorname{sech}\left(\frac{\pi\omega}{2}\right) \sin(\omega t). \end{aligned} \quad (13.26)$$

Thus the perturbed Duffing equation is chaotic for ε sufficiently small provided