Errata

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Errata

Changes appear in yellow. Line $k+$ (resp., line $k-$) denotes the kth line from the top (resp., the bottom) of a page. My thanks go to the following individuals who have contributed to this list: Theresa Dvorak, Gudrun Szewieczek, William Jagy, Jonathan Eckhardt, Sebastian Woblistin, Yu Jiang, Annemarie Luger, Johannes Wächtler, Daniel Scherl, Kristoffer Varholm, Constantino Santos, Minjae Park, Teck-Cheong Lim, Peter Elbau, Walter Schachermayer, Sebas Pedersen, Raphael Stuhlmeier, Eric Wahlén, Lukas Peham, Jakob Holböck.

Page 6. 7+: Furthermore, $K^{j+1} \in C^k(U_j, M)$ for any

Page 19. Problem 1.26, second item: $y(x_0+x) = y(x_0 - x)$

Page 45. 10+: $\Delta(t) = |\phi(t, t_0, y_0) - \phi(t, s_0, y_0)|$ and use ...

Page 57. 4+: Taking $m \to \infty$ we finally obtain

Page 62.

$$\exp(tJ) = \exp(\alpha t\mathbb{I}) \exp(tN) = \mathrm{e}^{\alpha t} \sum_{j=0}^{k-1} \frac{t^j}{j!} N^j. \tag{3.18}$$

Page 73. Problem 3.13: It should read $\deg(q(t)) \le \deg(p(t)) + s$ where s is the size of the largest Jordan block corresponding to the eigenvalue β. Moreover, here is an extended hint:

(Hint: Investigate (3.48) using the following fact: $\int^t (t-s)^m p(s) \mathrm{e}^{\beta s} ds = q(t)\mathrm{e}^{\beta t}$, where $q(t)$ is a polynomial of degree $\deg(q) = \deg(p) + m$ if $\beta \neq 0$ and $\deg(q) = \deg(p) + m + 1$ if $\beta = 0$. To see this differentiate $\int^t s^k \mathrm{e}^{\beta s} ds$ with respect to β.)

Page 84. Line before (3.98): Let $X(t)$ be the identity matrix with the first column replaced by $\phi_1(t)$,

Page 89. Example:

$$\big(\mathrm{e}^{t^2}\ddot{c}(t) + 4t\mathrm{e}^{t^2}\dot{c}(t) + (2+4t^2)\mathrm{e}^{t^2}c(t)\big) - 2t\big(\mathrm{e}^{t^2}\dot{c}(t) + 2t\mathrm{e}^{t^2}c(t)\big) - 2\mathrm{e}^{t^2}c(t)$$

Page 90. Problem 3.34: Consider the equation $\ddot{x} + q_0(t)x = 0$.

Page 90. Problem 3.37:

$$y(t) = Q(t)^{-1}x(t), \qquad Q(t) = \mathrm{e}^{-\frac{1}{n}\int^t q_{n-1}(s)ds}.$$

$$y^{(n)} + Q(t)^{-1}\sum_{k=0}^{n-2}\sum_{j=k}^{n}\binom{j}{k} q_j(t) Q^{(j-k)}(t) y^{(k)} = 0,$$

Page 90. Problem 3.38:

$$\dot{y} + \mathrm{e}^{-Q(t)}y^2 + \mathrm{e}^{Q(t)}q_0(t) = 0.$$

Page 100. Paragraph after the proof of Theorem 3.23: ... is constant. To this end recall that Corollary 3.5 tells us when the system corresponding to $B(t) = 0$ is stable. Moreover, ...

Page 101. Theorem 3.26:

$$|x(t)| \le C\mathrm{e}^{-(\alpha - b_0 C)t}|x_0|, \qquad |x_0| < \frac{\delta}{C},\ t \ge 0. \tag{3.168}$$

Page 120. Proof of Theorem 4.5: An argument that $h_2(z)$ has the same radius of convergence is missing:

If there is a second solution of the form $u_2(z) = z^{\alpha_2}h_2(z)$ the same argument can be used for $h_2(z)$. Otherwise one has to use (4.56) below in place of (4.39). Note that the present theorem will also follow as a special case of Theorem 4.13.

Page 126. Problem 4.5: $\Gamma(z) = \frac{(-1)^n}{n!(z+n)} + O(1)$.

Page 126. Problem 4.8:

$$h_j = -\frac{1}{j}\sum_{k=0}^{j-1} p_{j-k}h_k,$$

Page 126. Problem 4.10: For (i) you can use that $\Gamma(z)$ has no zeros and hence $\Gamma(z)^{-1}$ is an entire function.

Page 126. Problem 4.11 (iii): $J_{\nu+1}(z) - J_{\nu-1}(z) = -2J'_\nu(z)$

Page 145. Problem 5.3: We have $x \in [0,1]$ and there is only one boundary condition $u(t,1) = 0$.

Page 145. Problem 5.5:

$$m\frac{d^2}{dt^2}u(t,n) = -k(u(t,n+1) - u(t,n)) - k(u(t,n-1) - u(t,n)),$$

Page 154.

$$y(x) = y(x_0)c(z,x,x_0) + p(x_0)y'(x_0)s(z,x,x_0) - \int_{x_0}^{x} s(z,x,t)g(t)r(t)dt. \quad (5.50)$$

$$s(z,x,x_0) = \frac{-u(x)v(x_0) + u(x_0)v(x)}{W(u,v)}. \quad (5.51)$$

Page 155. Problem 5.13:

$$Q(y) = \frac{q(x(y))}{r(x(y))} - \frac{(p(x(y))r(x(y)))^{1/4}}{r(x(y))}\big(p(x(y))((p(x((y))r(x(y)))^{-1/4})'\big)'.$$

Page 162.

$$\min_{x\in[a,b]} \frac{q(x)}{r(x)} \le E_0. \quad (5.77)$$

Page 166. Problem 5.22: with $f^{(2j)}(0) = f^{(2j)}(1) = 0$ for $0 \le j \le k$.

Page 168. In fact, $\theta_b(\lambda,x)$ as defined in (5.89) is the Prüfer angle for $-u_b(\lambda,x)$, but this will be of no importance for our purpose.

Page 169.

$$\#_{(-\infty,\lambda)}(L) = \left\lceil \frac{\theta_a(\lambda,b) - \beta}{\pi} \right\rceil = \left\lceil \frac{\alpha - \theta_b(\lambda,a)}{\pi} \right\rceil - 1, \quad (5.91)$$

Moreover, note that we also have:

$$\#_{(-\infty,\lambda]}(L) = \left\lfloor \frac{\theta_a(\lambda,b) - \beta}{\pi} \right\rfloor + 1 = \left\lfloor \frac{\alpha - \theta_b(\lambda,a)}{\pi} \right\rfloor,$$

Page 172. Replace "As in the case of Theorem 5.18 one proves" by "As an immediate consequence of Lemma 5.16 we obtain"

Page 174. Problem 5.29: suppose $0 \le \alpha_2 < \alpha_1 < \pi$ and show

Page 175. Problem 5.30:

$$W'(u_0,u_1) = (q_1 - \lambda_1 r_1 - q_0 + \lambda_0 r_0)u_0u_1 + \left(\frac{1}{p_0} - \frac{1}{p_1}\right)p_0u_0'p_1u_1'. \quad (5.111)$$

Page 175. Problem 5.31:

$$u'(x) = a\left(\frac{\cos(x+b)}{x^{1/2}} + \frac{(-3/4-\nu^2)\sin(x+b)}{2x^{3/2}} + O(x^{-5/2})\right).$$

Page 183. Problem 5.33:

$$G_\pm(z,x,y) = \frac{W(z)^{-1}}{1\mp\rho_+(z)}\begin{cases} u_+(z,x)u_-(z,y)\pm\rho_+(z)u_-(z,x)u_+(z,y), & y<x,\\ u_-(z,y)u_+(z,x)\pm\rho_+(z)u_+(z,y)u_-(z,x), & y>x,\end{cases}$$

with $W(z) = W(u_+(z), u_-(z))$.

Page 184. Problem 5.35: (iii) $c(z,\ell) = s'(z,\ell)$.

Page 207. Problem 6.25: The requirement that $U'(x)$ never vanishes is missing.

Page 213. Proof of Theorem 7.4: Exchange the definitions of Q_3 and Q_4.

Page 214. Problem 7.4: The trajectory enters Q_3 and satisfies $x(t) < x_0$ in Q_3 since ... where $y(t)$ decreases, implying $x(t) > x_1 = \frac{1-y_1}{\lambda}$ when .... If $y_2 \ge y_0$, that is, if

$$\lambda\mu^2((\mu\lambda)^2-1)(x_0 - \frac{1+\mu}{1+\mu\lambda}) > 0, \tag{1}$$

Page 222. The proof of Lemma 7.13 only shows that $\omega_\sigma(x)$ contains a regular periodic orbit. However, the claim follows from Lemma 7.14 if $\omega_\sigma(x)$ is connected. Moreover, connectedness follows from compactness as in the proof of Lemma 6.6. In particular, the connectedness assumption in Theorem 7.16 is superfluous.

Page 225. First equation in the example:

$$f(x,y) = \begin{pmatrix} y \\ -\eta E(x,y)^2 y - U'(x)\end{pmatrix},$$

Page 233. Problem 8.2: Let $V_R = \{x \in M | L(x) < R\}$ be a relatively compact set

Page 235. Replace the last sentence by: Using the Routh–Hurwitz criterion one can show that the two new fixed points are asymptotically stable for $1 < r < \frac{\sigma(3+b+\sigma)}{\sigma-b-1}$ if $1+b<\sigma$ and $1<r$ if $1+b\ge\sigma$. For the classical values $\sigma=10$, $b=8/3$ this gives $1<r<470/19=24.74$.

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$$\frac{d}{dt}\begin{pmatrix} p \\ q\end{pmatrix} = +\mathrm{grad}_s H(p,q), \tag{8.46}$$

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$$H(p,q) = \frac{1}{2}(pM^{-1}p + qWq) \tag{8.77}$$

Page 246. Line before equation (8.78). Then the symplectic transform $(P, Q) = (V^T M^{-1/2} p, V^T M^{+1/2} q)$ (Problem 8.15) gives the decoupled system

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$$H(p,q) = \sum_{j=1}^{n} \frac{p_j}{2m} + \sum_{j=0}^{n} U_0(q_{j+1} - q_j), \qquad q_0 = q_{n+1} = 0, \tag{8.81}$$

Page 246. In order to better explain what Problem 8.18 is about the text between "If we assume that the particles ... of the Jacobian matrix of the potential." should be replaced by:

If we assume that the particles are coupled by springs, the potential would be $U_0(x) = \frac{k}{2}x^2$, where $k > 0$ is the so called spring constant, and we have a harmonic oscillator with

$$M = m\mathbb{I}, \qquad W = k \begin{pmatrix} 2 & -1 & & & \\ -1 & 2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & 2 & -1 \\ & & & -1 & 2 \end{pmatrix}.$$

The motion is decomposed into n modes corresponding to the eigenvectors of W, which are given by

$$v^j = \sqrt{\frac{2}{n+1}} \begin{pmatrix} \sin(\eta_j) \\ \sin(2\eta_j) \\ \vdots \\ \sin(n\eta_j) \end{pmatrix}, \qquad \eta_j = \frac{\pi j}{n+1}.$$

The corresponding eigenvalues are $m\omega_j^2$, where $\omega_j^2 = \frac{2k}{m}(1-\cos(\eta_j)) = \frac{4k}{m}\sin(\frac{\eta_j}{2})^2$. Consequently the j'th mode corresponds to the initial condition $(p(0), q(0)) = (0, v^j)$ and is given by

$$q^j(t) = \cos(\omega_j t) v^j, \qquad p^j(t) = -m\omega_j \sin(\omega_j t) v^j.$$

The energy of the j'th mode is $H(p^j, q^j) = \frac{m\omega_j^2}{2}$.

Page 247. Problem 8.16: Ignore the hint.

Page 259. Theorem 9.3: there are neighborhoods $U(x_0)$ of x_0 and U of 0 and a function $h^{+,\alpha} \in C^k(E^{+,\alpha} \cap U, E^{-,-\alpha})$ such that

Page 261. Theorem 9.4: there are neighborhoods $U(x_0)$ of x_0 and U of 0 and functions $h^{\pm} \in C^k(E^{\pm} \cap U, E^0 \oplus E^{\mp})$ such that

Page 266. Proof of Lemma 9.7: The equation $A^{-1} \circ \varphi \circ \vartheta = \varphi \circ \vartheta \circ A^{-1}$ should read $f \circ \varphi \circ \vartheta = \varphi \circ \vartheta \circ f$. This last equation implies $\varphi \circ \vartheta = \mathbb{I} + l$, where l is a solution of $Ll(x) = g(x) - g(x + l(x))$. Using the estimates for the inverse of L and for g one obtains $l \equiv 0$ and thus φ is a homeomorphism.

Page 268. The very last equation on the bottom of the page is only true if Φ_t is linear. Set $\Phi_t = \mathrm{e}^{tA} + G_t$, where G_t is bounded, and replace this equation by

$$h_t = \Phi_t \circ \varphi \circ \mathrm{e}^{-tA} - \mathbb{I} = \mathrm{e}^{tA} \circ h \circ \mathrm{e}^{-tA} + G_t \circ \varphi \circ \mathrm{e}^{-tA},$$

where both terms are bounded.

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$$\varphi(x) = (x_1 + x_2^2, x_2). \tag{9.38}$$

Page 286. Line before (10.19) which is zero at p, positive for $x \neq p$, whose level sets $S_\delta = \{x \in U(p) | L(x) \leq \delta\}$ are connected for sufficiently small δ, and such that $x \in U(p)$ implies $f^n(x) \in U(p)$ and

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$$x(m) = \Pi(m, m_0)x_0 + \sum_{j=m_0}^{m-1} \Pi(m, j+1)g(j), \tag{10.28}$$

$$x(m) = \Pi(m, m_0)x_0 - \sum_{j=m-1}^{m_0} \Pi(m, j+1)g(j), \quad m < m_0. \tag{10.29}$$

Page 296. Second sentence after (11.10): Change W^s to W^+.

Page 300. Problem 11.3: The assumption that the set has no isolated points needs to be added.

Page 306. Lemma 11.10: the number of periodic points of period at most l is equal to $\mathrm{tr}(A^l)$.

Page 308. Problem 11.10: the number of periodic orbits of period at most n.

Page 316. Problem 11.13: It seems too difficult to give a counterexample. The problem should be ignored.

Page 333. Change W^s to W^+ and W^u to W^- in the picture and the text before the picture.

Page 337. Problem 13.3:

$$\dot{q} = p, \qquad \dot{p} = -\sin(q) + \varepsilon \sin(t).$$

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$$\begin{aligned} M(t) &= \int_{-\infty}^{\infty} p_0(s) \left(\delta p_0(s) + \gamma \cos(\omega(s-t))\right) ds \\ &= \frac{4\delta}{3} - \sqrt{2}\pi\gamma\omega \,\mathrm{sech}(\frac{\pi\omega}{2}) \sin(\omega t). \end{aligned} \tag{13.26}$$

Thus the perturbed Duffing equation is chaotic for ε sufficiently small provided