

Algebraic Approach to Non-Standard Analysis and Colombeau Theory of Generalized Functions

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Abstract

We construct a chain (of infinitely many) algebras of generalized functions and show they are spacial Colombeau algebras in the sense that for each of these algebras there exists a Colombeau type of embedding of the space of Schwartz distributions (the proof is based on convolution, but also on Zorn lemma). One of these algebras is a full Colombeau algebra (with an explicit embedding). What is different from most of the similar constructions is that the scalars of these algebras form algebraically closed Cantor complete non-Archimedean fields. Our framework is the so called *distributional non-standard model* of the real numbers especially designed for the purpose of non-linear theory of generalized functions. No background on non-standard analysis is required by the audience; the language will be essentially algebraic: ordered fields, algebraically closed fields, maximal ideals, etc.

For supporting material we refer to Todor's webpage <http://web.me.com/ttodorov> (under "Current Research Projects"), where you will find **links to**:

1. Todorov & Vernaeve, *Asymptotic Fields and Power Series* (under preparation). The reading requires a background in non-standard analysis.
2. For a "**gentle**" **introduction to non-standard analysis** we refer to the Master Thesis of Ray Cavalcante, where the non-standard analysis is presented in terms of **nets of complex numbers** $\mathbb{C}^{\mathbb{R}^+}$ (notationally similar to Colombeau theory).
3. For the **more advanced introduction to distributional non-standard model** of analysis we refer to the Master Thesis of Guy Berger, where the non-standard analysis is presented in terms of nets of complex numbers $\mathbb{C}^{\mathcal{D}}$.
4. For an **up-to-date version** of the **distributional non-standard model** we refer to Section 6 in Todorov & Vernaeve, *Full Algebra of Generalized Functions and Non-Standard Asymptotic Analysis*, Logic and Analysis, Vol. 1, Issue 3, 2008.
5. For the terminology on ordered fields we refer to any textbook on abstract algebra; in particular to S. Lang (Chapter XI).