Unitarizability of holomorphically induced representations of a split solvable Lie group

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Let G be a Lie group and $\mathfrak{g}$ its Lie algebra. A totally complex positive polarization induces a G-invariant Kähler structure on the coadjoint orbit $\mathcal{O}^*$ in $\mathfrak{g}^*$. If the orbit is integral, one has a G-invariant holomorphic line bundle L, called *quantization bundle*, on $\mathcal{O}^*$. Then a unitary representation of G is naturally defined on the Hilbert space $\mathcal{H}^2(L)$ of the square integrable holomorphic sections of L if $\mathcal{H}^2(L)$ is non-zero. Even if $\mathcal{H}^2(L) = \{0\}$, there may exist a reproducing kernel Hilbert space $\mathcal{H}(L)$ of holomorphic sections on L such that one defines a unitary representations of G on $\mathcal{H}(L)$.

We determine the condition of the existence of such $\mathcal{H}(L)$ for a certain 5-dimensional split solvable Lie group.