PDE WITH LOSS OF DERIVATIVES - Abstract

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Let $X_1, \dots, X_p$ be complex valued vectorfields in a neighborhood of the origin in $\mathbb{R}^n$ that satisfy the bracket condition at the origin. That is: the Lie algebra generated by the $X_1, \dots, X_p$ evaluated at the origin spans the tangent space at the origin. According to a theorem of Hörmander if the vectorfields are real and satisfy the bracket condition at the origin then the operator

$$E = \sum X_i^* X_i,$$

where the X_i^* are L_2 adjoints of the X_i, is locally hypoelliptic in a neighborhood U of the origin.

The above theorem is proved by establishing the following subelliptic estimate.

$$(\bullet) \qquad \|u\|_\varepsilon^2 \leq C \sum \|X_i u\|^2,$$

for all $C_0^\infty(W)$.

Here we consider the case when the $X_1, \dots, X_p$ are complex (i.e. have complex coefficients) and satisfy the bracket condition. In that case, if the bracket condition involves only one bracket then $(\bullet)$ is satisfied with $\varepsilon = \frac{1}{2}$. However, in general when the bracket condition involves more than one bracket, the subelliptic estimate $(\bullet)$ no longer holds. We will present examples in which subellipticity does not hold and for which the operator E has local existence and is hypoelliptic with a loss of derivatives. We will discuss general theorems of this type for certain classes of vectorfields.