

DMT PAR-reduction by weighted cancellation waveforms

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Abstract

DMT (discrete multitone) signals have a high peak-to-average power ratio (PAR). In the transmitters, the PAR governs the necessary resolution of the digital-to-analog converter (DAC) and is an important factor for the power consumption of the line-driver.

In a recent paper, we presented a low complexity PAR-reduction method for DMT-transmitters. Here we improve a central part of this method. Computer simulations show a small but clear clip rate and clip power performance improvement with almost no increase in complexity.

1 Introduction

The samples of a discrete multitone (DMT) or orthogonal frequency division multiplexing (OFDM) signal have a nearly Gaussian amplitude distribution. Thus, it has a high *peak-to-average power ratio*¹ (PAR).

Figure 1 shows the parts of the transmitter that are of importance for PAR-reduction. The high PAR is mainly a problem in the digital-to-analog converter (DAC) and in the line-driver.

The DAC clips all samples that exceed a certain maximum amplitude, the *clip level*. If the PAR is reduced, a lower clip level is needed to reach a desired clip rate. Consequently, the preferred quantization of the IFFT output can be obtained with a lower precision, and cheaper, DAC.

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¹The peak-to-average power ratio is defined as the ratio $\|\mathbf{x}\|_{\infty}^2/\sigma^2$ of the maximum absolute value squared of a DMT frame $\mathbf{x} = (x_1, x_2, \dots, x_{2N})$ to its expected mean power σ^2 .

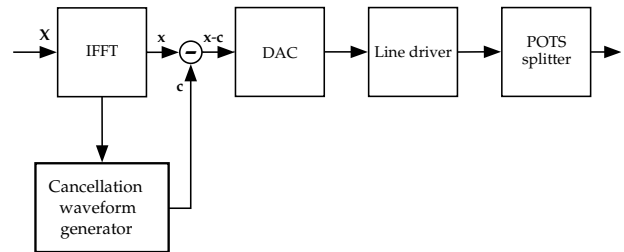


Figure 1: A DMT transmitter.

The *line-driver* amplifies the DAC output and applies it to the line. In our target system, it must be a class A amplifier. Therefore, the power consumption is determined by the range over which the line driver must provide linear amplification, that is, by the amplitude of the highest peaks. Reducing the PAR reduces this range, and thus also the power consumption. This further reduces the cost of the modem.

The results reported in this paper are off-spring of the Telia Research and ST Microelectronics (former SGS-Thomson) joint development of a DMT-VDSL (very high bit-rate digital subscriber loop) system using the Zipper [1, 2, 3] duplex method. In [4] we introduced a low-complexity PAR-reduction method, which is developed for use in this system. The PAR is reduced by adding a waveform that is bandlimited to certain *peak reduction tones*, which consequently cannot be used for data transmission. A special circuit, the *cancellation waveform generator*, constructs this waveform so that it has sharp and unique peak(s) in counterphase to the largest peak(s) of the DMT frame. Our method is based on an iterative method [5, 6, 7, 8, 9], but with additional restrictions and refinements in order to adapt it to our target system.

The main characteristics of the approach in [4] are

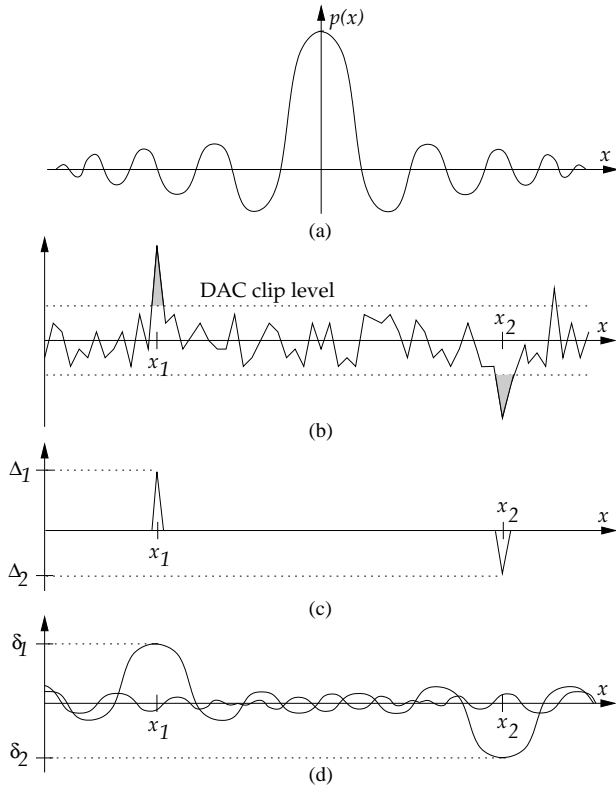


Figure 2: Weighted peak reduction: (a) The pre-calculated and stored peak-cancellation kernel; (b) the DMT signal; (c) identified potential clips; (d) the translated and rescaled peak-cancellation kernels.

that we *avoid iterations*, due to the low latency budget in the VDSL requirements. We also *replace multiplications with bit-shifts* in the necessary computations. This reduces the complexity considerably, with no significant change in performance. We use *only the first 60 tones* for peak reduction, thus maintaining the data rate. This frequency band cannot be used for data transmission, since it is occupied by POTS (plain old telephone service) and therefore filtered out in the POTS splitter. It can, however, be used to reduce the PAR in the DAC and the line driver. In [4], we also give special attention to when several peaks are reduced simultaneously. This was done by reducing the peaks down to a certain peak reduction level, slightly below the clip level. In the present paper, we introduce a more sophisticated solution, *weighted peak reduction*.

In Section 2 we present the new algorithm, the simulation results are given in Section 3. An interesting result is that weighting almost completely removes the (clip rate) performance loss that comes from replacing multiplications with bit-shifts.

For further references to PAR-related papers the reader may consult [4], which also contains a more detailed explanation of the target system and the implementation.

2 Weighted peak reduction

Our method tries to reduce each peak to a fixed *peak reduction level*, slightly below or equal to the clip level. Figure 2 shows the steps needed to reduce two clips per DMT frame. A pre-calculated waveform, the *peak-cancellation kernel*, is stored in a memory. It is bandlimited to the first 60 subcarriers. A simple programmable processor, or a dedicated circuit, identifies potential clips in the IFFT output, as illustrated in Figure 2(b) and (c). Note that Δ_i is the distance down to the peak reduction level. For each peak, the cancellation waveform generator (parallelly) scales and subtracts one copy of the stored waveform from the DMT-signal. With notation as in the figure, the amplitudes δ_i are computed such that the constructed waveform has amplitude Δ_i at x_i . This is done in equation (3) below.

Our new algorithm for reducing up to M peaks per frame is:

1. Locate the (at most) M highest amplitude samples that exceed the clip level. (This could be replaced by finding the at most M *first* such peaks.)
2. Find the position x_i of each such peak and the amplitude Δ_i by which it exceeds the peak reduction level.
3. Compute the reduction amplitudes δ_i . In the case of bit-shift scaling, each δ_i is then rounded upwards to an even power of two.
4. Compute the sum of scaled copies of the peak-cancellation kernel p .
5. Subtract the generated waveform from the IFFT output.

In this algorithm, we want to choose δ_j such that

$$\Delta_i = \sum_{j=1}^M \delta_j \cdot p[x_i - x_j], \quad (1)$$

where p is read with circular addressing from the memory. With the notation $\mathbf{\Delta} \stackrel{\text{def}}{=} (\Delta_1, \dots, \Delta_M)^T$ and $\mathbf{\delta} \stackrel{\text{def}}{=} (\delta_1, \dots, \delta_M)^T$ we write the linear system (1) in matrix form

$$\mathbf{\Delta} = W\mathbf{\delta},$$

where

$$(W)_{i,j} = p[x_i - x_j]. \quad (2)$$

Thus a solution is given by

$$\mathbf{\delta} = W^+ \mathbf{\Delta}, \quad (3)$$

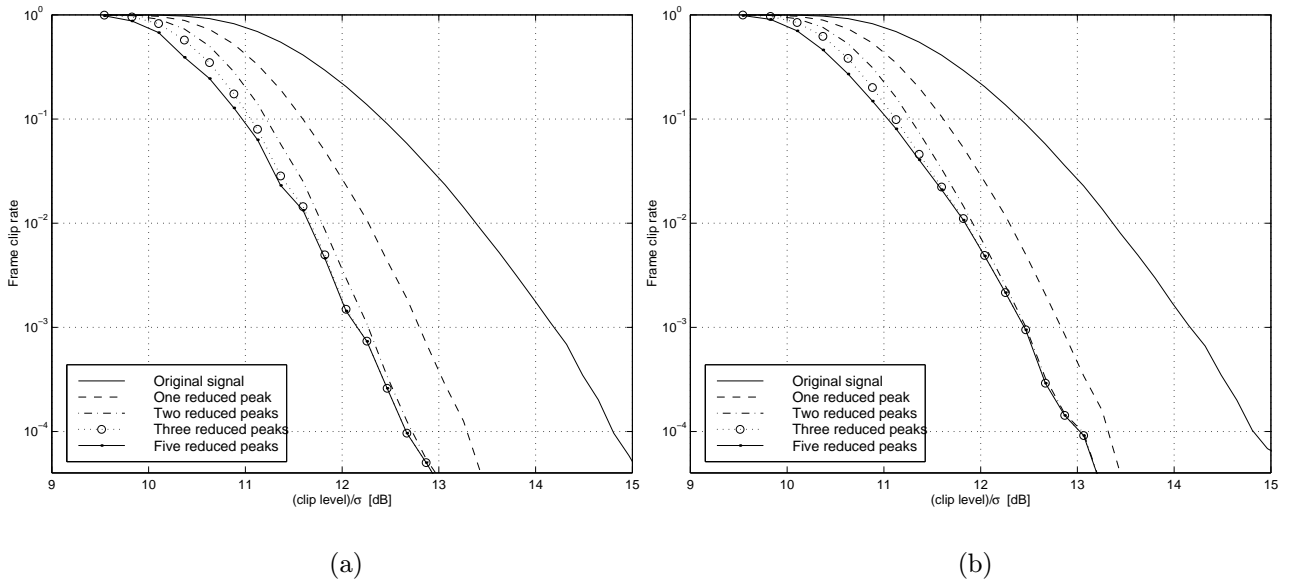


Figure 3: Frame clip rate plotted against the clip level (divided by the signal's standard deviation σ): (a) Weighting; (b) Weighting and bit-shift scaling.

where W^+ denotes the pseudoinverse [10] of W . If, for example, $M = 2$ and p is symmetric with maximum amplitude $a = |p[0]|$, then $W = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$ with $|b| < |a|$. Thus, W is invertible and $W^+ = W^{-1}$. This is the case (when $M = 2$) for the sinc-function used in our simulations.

The peak reduction level was introduced in [4] as a safety margin against the interaction between translated waveforms. There, we concluded that a good choice of this level is 98% of the clip level, and this gave a clear performance improvement.

However, this is not the best safety margin for all frames. On the one hand, when a frame has only one potential clip, then no safety margin is needed. Instead, the higher waveform amplitude increases the probability of creating new clips. On the other hand, sometimes this safety margin can be too small to get all clips reduced. The weighting is a way around this problem of a fixed safety margin. The reduction amplitudes are always chosen so that the peaks are reduced exactly to the peak reduction level, which then can be set equal to the DAC clip level. Moreover, when there are many ways to do this, then the pseudoinverse always gives the minimum 2-norm solution [10], that is, the vector δ that has minimum length.

	Multiplicative	Bit-shift
No weighting	2.0 dB	1.7 dB
Weighting	2.2 dB	1.9 dB

Table 1: Clip level reduction when reducing (at most) two peaks with a fixed frame clip rate of 10^{-4} .

3 Computer simulation results

We have examined the performance of our method with MATLAB simulations. The first 60 DMT subcarriers (of 2048) are used for peak reduction. All the remaining subcarriers are bit-loaded [11] for an 800 m long TP2 cable [12]. The signal constellations used are QAM for an even number of bits, and n-cross, 2-PAM or 8-square for an odd number of bits². Round-off errors in the DAC are not simulated.

The used peak approximation waveform is a sinc-function. Other choices of this waveform, for example as proposed in [6], might give some further performance improvement.

The simulated peak cancellation generator tries to reduce the largest peak(s) that exceed the clip level.

The number of simulated frames (ordered as in Figure 4) is (a) 175 400, (b) 197 400, (c) 238 800 and (d) 175 200.

3.1 Clip rate

Given a fixed frame clip rate of 10^{-4} , Table 1 summarizes the clip rate performance shown in Figure 3 and the corresponding figures in [4]. It shows the clip level improvement when reducing up to two peaks per frame. Reducing (at most) one peak gives 1.5 dB reduction in all cases.

The peak reduction level is set to 98% of the clip level, except when weighting is combined with multiplicative scaling. Then setting the levels equal gave

²Thus the simulations take into consideration any effects that might arise because the tail of the distribution is not exactly Gaussian.

a slightly *lower* clip level (see Table 1), with the additional benefits described in Section 1. Comparisons at a fixed clip level show that reducing several peaks reduces the clip power up to 10 dB extra, compared to reducing (at most) one peak.

At higher clip rates (not interesting for our target system), weighting increases clip power with no large change of the clip level. This suggests that in this case³ weighting sometimes produces higher amplitude waveforms when the peak reduction fails. Therefore, a possible subject for further investigation is to introduce a threshold to determine when to use the peak reduced signal. For example, if peak reduction fails with a high amplitude waveform, then it could be better to use the original IFFT output. This could improve the clip power performance with no change in clip rate performance, but such an investigation is out of the scope of this paper.

3.3 Complexity

The presented results suggest choosing $M = 2$, since larger M only gives a very small further improvement. For such small M , the complexity of the weighting-computations (2) and (3) is negligible, and the total complexity is roughly $2N$ comparisons, $2MN$ additions, and $2MN$ bit-shifts, as computed in [4].

4 Summary and conclusions

We have showed that our weighted PAR-reduction method gives a small but clear performance improvement compared to the method presented in [4], and with almost no increase in complexity.

One interesting result is that the combination of weighting and bit-shift scaling gives almost the same clip rate performance as the much more complex case of no weighting and multiplicative scaling.

References

- [1] Mikael Isaksson, Petra Deutgen, Frank Sjöberg, Sarah Kate Wilson, Per Ödling, and Per Ola Börjesson. Zipper – a flexible duplex method for VDSL. In *Proceedings of the International Workshop on Copper Wire Access Systems (CWAS)*, pages 95–99, Budapest, Hungary, October 1997.
- [2] Frank Sjöberg, Mikael Isaksson, Petra Deutgen, Rickard Nilsson, Per Ödling, and Per Ola Börjesson. Performance evaluation of the Zipper duplex method. In *Proceedings of the International Conference on Communication*, pages 1035–1039, Atlanta, GA, June 1998.
- [3] Frank Sjöberg, Mikael Isaksson, Rickard Nilsson, Per Ödling, Sarah Kate Wilson, and Per Ola Börjesson. Zipper – a duplex method for VDSL based on DMT. To appear in *IEEE Transactions on Communication*, 1999.
- [4] Per Ola Börjesson, Hans G. Feichtinger, Niklas Grip, Mikael Isaksson, Norbert Kaiblinger, Per Ödling, and Lars-Erik Persson. A low-complexity PAR-reduction method for DMT-VDSL. In *Proceedings of the 5th International Symposium on Digital Signal Processing for Communication Systems (DSPCS'99)*, pages 164–169, Perth, Australia, February 1999.
- [5] Alan Gatherer and Michael Polley. Clip mitigation techniques for T1.413 Issue 3. ANSI document T1E1.4/97-397, Sacramento, CA, December 1998.
- [6] José Tellado and John M. Cioffi. PAR reduction in multicarrier transmission systems. ANSI document T1E1.4/97-367, Sacramento, CA, December 1997.
- [7] José Tellado and John M. Cioffi. Revisiting DMT's PAR. ANSI document T1E1.4/98-083, Austin, TX, March 1998.
- [8] José Tellado and John M. Cioffi. Revisiting DMT's Peak-to-Average Ratio. ETSI document TM6:TD08, Antwerp, Belgium, April 1998.
- [9] José Tellado and John M. Cioffi. Peak power reduction for multicarrier transmission. In *Communications Theory Mini-Conference, Globecom '98*, Sydney, Australia, November 1998.
- [10] Gilbert Strang. *Linear Algebra and its applications*. Harcourt Brace Jovanovich College publishers, third edition, 1988. Library of Congress Catalog Card Number: 87-083019.
- [11] Jerry Tu and John M. Cioffi. A loading algorithm for the concatenation of coset codes with multi-channel modulation methods. In *Proceedings of the IEEE Global Telecommunications Conference (Globecom '90)*, pages 1183–1187, San Diego, CA, November 1990.
- [12] ANSI T1E1.4. *Very-high-speed Digital Subscriber Lines. System Requirements*. VDSL SR: 98-043R5, draft technical document, revision 16, September 1998.

³The plots show this behaviour when the frame clip rate without peak reduction is of the order 10^{-1} or higher.