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Time-dependent scattering of generalized plane waves by a wedge

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We obtain explicit formulas for the scattering of plane waves with arbitrary profile by a wedge under Dirichlet, Neumann and Dirichlet-Neumann boundary conditions. The diffracted wave is given by a convolution of the profile function with a suitable kernel corresponding to the boundary conditions. We prove the existence and uniqueness of solutions in appropriate classes of distributions and establish the Sommerfeld type representation for the diffracted wave.
As an application, we establish (i) stability of long-time asymptotic local perturbations of the profile functions and (ii) the limiting amplitude principle in the case of a harmonic incident wave. **Copyright © 2015 John Wiley & Sons, Ltd.**

Keywords: scattering; plane wave

1. Introduction

In this paper, we extend our results [1–6] on scattering of harmonic plane waves by the two-dimensional wedge

$$W := \{y = (y_1, y_2) \in \mathbb{R}^2 : y_1 = \rho\cos\theta,\ \ y_2 = \rho\sin\theta,\ \rho \geq 0,\ 0 \leq \theta \leq \phi\},$$

with angle $\phi \in (0, \pi)$. In those papers, scattering was studied for harmonic incident waves

$$u_{\rm in}(y,t) = e^{-i\omega_0(t-n_0\cdot y)} f(t - n_0 \cdot y) \text{ for } t \in \mathbb{R} \text{ and } y \in Q, \tag{1.1}$$

where $n_0 = (\cos\alpha, \sin\alpha)$ and $Q := \mathbb{R}^2 \setminus W$ is the angle of magnitude $\Phi := 2\pi - \phi$, $\Phi \in (\pi, 2\pi)$. The boundary is $\partial Q = \Gamma_1 \cup \Gamma_2 \cup 0$, where $\Gamma_1 := \{(y_1, 0) : y_1 > 0\}$ and $\Gamma_2 := \{(\rho\cos\phi, \rho\sin\phi) : \rho > 0\}$. Further, the profile function f is a Heaviside-type smooth function:

$$f \in C^\infty(\mathbb{R}),\quad \operatorname{supp} f \subset [0, \infty),\quad \text{and}\quad f(s) = 1 \text{ for } s \geq s_0 \tag{1.2}$$

where $s_0 > 0$. The diffraction is described by the mixed problem

$$\begin{cases} \Box u(y,t) = 0,\ y \in Q;\quad Bu(y,t)|_{\Gamma_1\cup\Gamma_2} = 0,\ t \in \mathbb{R} \\ u(y,t) = u_{\rm in}(y,t),\ y \in Q,\ t < 0. \end{cases} \tag{1.3}$$

Here $\Box = \partial_t^2 - \triangle$, $B = (B_1, B_2)$ and $Bu|_{\Gamma_1\cup\Gamma_2} = (B_1 u|_{\Gamma_1}, B_2 u|_{\Gamma_2})$, where $B_{1,2}$ are equal to either the identity operator I or to $\partial/\partial n$, where n is the outward normal to Q. The DD problem corresponds to $B_1 = B_2 = I$, the NN problem corresponds to $B_1 = B_2 = \partial/\partial n$, and the DN problem corresponds to $B_2 = I, B_1 = \partial/\partial n$. We gave an explicit formula for the solution to (1.3) and proved the uniqueness, existence and the limiting amplitude principle in [1–6]. Now we generalize those results to the case of nonsmooth and nonperiodic incident wave

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$$u_{\rm in}(y,t) = F(t - n_0 \cdot y), \quad y \in \mathbb{R}^2, \quad t \in \mathbb{R}, \tag{1.4}$$

where F is a tempered distribution with support in $\overline{\mathbb{R}^+}$. Our main results are formulas for the solutions to nonstationary problems (1.3)

$$u = u_{\rm in} + u_s; \quad u_s = F_\delta * Js, (y;t) \in Q \times \mathbb{R}, \tag{1.5}$$

where Js is a suitable distribution corresponding to the type of boundary conditions either Dirichlet (DD) or Neumann (NN) or Dirichlet in a side of the angle and Neumann on the other side of the angle (DN). Here $F_\delta(y,t) := F(t)\delta(y)$, and the convolution is well defined in the sense of distributions (see Theorem 3.4 for the DD case).

Moreover, we give an explicit formula for the solution when $F(s) = \delta(s)$. We study also the case when F is a locally summable function such that

$$F(s) = 0, \;\; s < 0, \;\; \sup(1 + |s|)^p |F(s)| < \infty, \;\; s \in \mathbb{R} \tag{1.6}$$

for some $p \in \mathbb{R}$. We analyze the stabilization of solutions as $t \to \infty$. Namely, we prove that the solution locally tends to a limit as $t \to \infty$, if and only if $F(s) \to C$ as $s \to \infty$.

We also generalize the limiting amplitude principle that was proved for smooth Heaviside-type incident waves: in [2, 3] for the DD case, in [4, 5] for the DN case, and in [6] for the NN case. Namely, we consider the incident waves with $F(s) - a^0 e^{-i\omega_0 s} \to 0$, as $s \to \infty$, and write the corresponding nonstationary solution in the form

$$u(y,t) = A(y,t)e^{-i\omega_0 t}.$$

We prove that $A(y,t) \to A_\infty(y)$ as $t \to \infty$, where $A_\infty(y)$ is a solution to the corresponding stationary Helmholtz equation.

The key role in this asymptotic analysis plays the Sommerfeld–Maluzhinetz type representation for the diffracted wave

$$u_d(\rho,\theta,t) := \frac{i}{4\Phi} \int_{\mathbb{R}} Z(\beta + i\theta) F(t - \rho\cosh\beta) d\beta, \;\; \theta \in \Theta := [\phi, 2\pi] \setminus \{\theta_1, \theta_2\} \tag{1.7}$$

in the case of a locally summable incident wave. We will use it for analysis of long-time asymptotic behavior of the diffracted wave.

The representation was justified first in [2–6] for the Heaviside-type smooth incident wave (1.1) using the method of complex characteristics [7–9]. Here we extend this representation to locally summable incident waves. The representation was used in [6] and [10] to find convergence rate to the limiting amplitude.

Let us comment on previous works. Nonstationary scattering of the incident wave (1.4) by wedge was considered for the first time in the case $F(s) = h(s)$ by Sobolev [11–13] in 1934, by Keller and Blank [14] in 1951, by Kay [15] in 1953, by Oberhettinger [16] in 1958, by Borovikov [17] in 1966, by Bernard [18–20] in 1991–1993, and by Rottbrand [21, 22] in 1998. For the step function, Sobolev constructed in [11] a particular solution in the form

$$u(y,t) = g(\zeta(y,t)). \tag{1.8}$$

Here $\zeta(y,t)$ is an 'algebraic' function defined by the equation

$$bt - m(\zeta)y_1 - n(\zeta)y_2 - \chi(\zeta) = 0,$$

where $m(\zeta), n(\zeta)$ and $\chi(\zeta)$ are suitable complex analytic functions related by $m^2(\zeta) + n^2(\zeta) = 1$. Sobolev refers to formula (1.8) as the Sobolev–Smirnov representation [23] and relates it to dilation invariance of the problem. The problem is solved explicitly using conformal mappings onto unit circle and Schwarz's reflection principle: antisymmetric reflections in the DD case and symmetric reflections in the NN case. The resulting formulas read as (1.5), and we will show it in a subsequent paper.

In the next paper [12] (mainly included in [13]), Sobolev relates this process of reflections with the wave propagation on logarithmic Riemann surface served as in a spirit of Sommerfeld's ideas cited in [13]. In these papers, Sobolev introduced his famous discontinuous 'weak solutions' to the wave equations served as the cornerstone for the Theory of Distributions developed later by L. Schwartz.

Keller and Blank [14] also considered the diffraction of Heaviside incident wave by a wedge developing Busemann's 'conical flow method', which is similar to Sobolev's approach: the dilation invariance of the wave equation allows to reduce the problem to Laplace equation on a circle with piecewise constant boundary values. The obtained solution [14] coincides with the Sobolev formula (and with our solution) as we have shown in [24].

Kay's approach [15] relies on a separation of variables. Any solution of the wave equation is represented in a form of Whittaker functions series [25, p. 279]. The author proves that the series coincide with Keller–Blank solution in the case of Heaviside incident wave (see p. 434 of [15]).

In [16], Oberhettinger has considered time-dependent problems in wedges with the DD and NN boundary conditions and a general profile function F. The problem is reduced to $F(t) = h(t)e^{i\omega t}$ by the Laplace transform. The particular solution to the corresponding stationary problem is constructed in an integral form using modified Hankel function [16, (11)]. The final formula for the time-dependent problem is a convolution with the corresponding kenel (formula (108) of [16])

In [18–20], Bernard developed Oberhettiger's results in the Sommerfeld type representation for impedance wedges using the Malyuzhinetz method.

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A solution for incident wave with $F(s) = s_+^{-1/2}$ has been constructed by Borovikov [17], who used the obtained formula to reproduce Sobolev's solution.

Rottbrand [21, 22] considered the diffraction of the plane wave (1.4) with $F(s) = \int_0^s g(\tau)d\tau$ where $g \in L^1(\mathbb{R})$, $\operatorname{supp} g \subset (0, \infty)$. The problem is reduced by a conformal map to the Rawlins's mixed problem, and the solution is represented by an infinite series of Bessel functions [21, Section 3].

The formulas obtained in [11–21] appear quite different. All these works concern a solution of some particular form: Sommerfeld type representation in [17–20] or some algebraic form in [11–15]. The uniqueness of the time-dependent solution in an appropriate functional class was not established up to now. Moreover, it is well known that the solution is not unique if its singularity is not specified. A systematic mathematical analysis of the nonstationary scattering of plane harmonic waves in the cases of the DD, DN, and NN problems was developed in [1–6, 24]. The case of general nonperiodic profile function is analyzed in the present paper.

In our paper, we construct the solution in a suitable space of distributions for general incident wave (1.4) with any tempered distribution F with the support in $\overline{\mathbb{R}^+}$. Moreover, we prove that the solution is unique in this class and is given by the convolution (1.5). Let us stress that we deduce the existence and uniqueness of solutions from our previous results [2, 3].

We plan to obtain in the future the Sobolev formula for theta-function incident wave under the DD and NN boundary conditions [12]. This justification of the Sobolev diffraction formula was one of our main motivations in writing this paper.

Let us outline the plan of our paper. In Section 2, we reduce the problem (1.3) using the Fourier–Laplace transform. In Section 3, we prove the existence and uniqueness of solutions in suitable classes of distributions and establish the convolution formula and the Sommerfeld type representation for the diffracted wave. In Sections 4 and 5, we apply our results to establish (i) the stability of solution long-time asymptotics under local perturbations of the profile functions and (ii) the limiting amplitude principle in the case of a harmonic incident wave. In Appendix A, we give some facts from the Paley–Wiener theory and calculate some Fourier transforms.

2. Formulation of the scattering problem

The front of incident wave $u_{\text{in}}(y,t)$ at any moment of time $t \le 0$ is a straight line $\{y : t - n_0 \cdot y = 0\}$ in $\mathbb{R}^2$. For $n_0 \cdot y > t$, we have $u_{\text{in}}(y,t) = 0$ by (1.6). We impose the following conditions on vector n_0. First, we suppose that $\phi - \pi/2 < \alpha < \pi/2$. Then the front of $u_{\text{in}}(y,t)$ lies in Q for $t < 0$.

Second, we suppose that the incident wave is reflected by both sides of the wedge. This is equivalent to the condition $0 < \alpha < \phi$. These two conditions on vector n_0 are expressed by the following inequalities:

$$\max(0, \phi - \pi/2) < \alpha < \min(\pi/2, \phi) \tag{2.1}$$

(see Figure 1). The extension of our results to other angles ϕ and α does not pose any new conceptual difficulties. In particular, formulas (1.7)–(A4) remain valid for all angles Φ and α.

Let us denote by $u(y,t)$ a solution of problem (1.3) and by $u_s(y,t) := u(y,t) - u_{\text{in}}(y,t)$ the scattered wave. Then u_s is a solution to the following mixed problem:

$$\begin{cases} \Box u_s(y,t) = 0, \ y \in Q, \ \ Bu_s(y,t)|_{\Gamma_1 \cup \Gamma_2} = -Bu_{\text{in}}(y,t)|_{\Gamma_1 \cup \Gamma_2}, \ t \in \mathbb{R}, \\ u_s(y,t) = 0, \ y \in Q, \ t < 0. \end{cases} \tag{2.2}$$

Let us define the meaning of this mixed problem. First, let us introduce the space of solutions to (2.2). By $S'(\overline{Q} \times \overline{\mathbb{R}^+})$, we denote the space of tempered distributions in $\mathbb{R}^3$ with supports in $\overline{Q} \times \overline{\mathbb{R}^+}$.

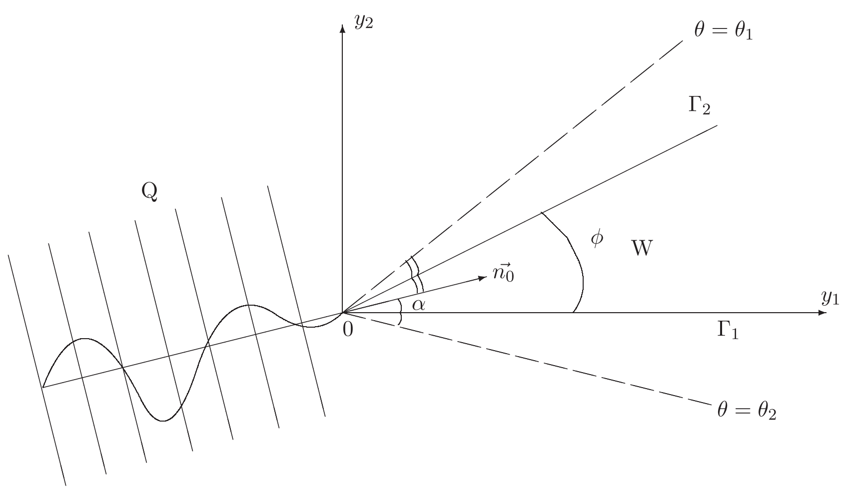

Figure 1. An incident plane wave.

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Definition 2.1 (see Def 2.1 [2])
(i) E_ε is the Banach space of functions $u(y) \in C(\overline{Q}) \cap C^1(\dot{\overline{Q}})$ with finite norm $\|u\|_\varepsilon = \sup_{y\in Q} |u(y)| + \sup_{y\in\dot{\overline{Q}}} \{y\}^\varepsilon |\nabla u(y)| < \infty$, where $\{y\} := \frac{|y|}{1+|y|}$ and $\dot{\overline{Q}} := \overline{Q} \setminus 0$.
(ii) $\mathcal{M}_\varepsilon$ is the space of tempered distributions $u(y,t) \in S'(\overline{Q} \times \overline{\mathbb{R}^+})$ such that its Fourier–Laplace transform $\hat{u}(y,\omega)$ is a holomorphic function of $\omega \in \mathbb{C}^+$ with the values in E_ε.

Roughly speaking, $\mathcal{M}_\varepsilon$ is the space of functions with the asymptotics $|\nabla u(y,t)| \sim |y|^{-\varepsilon}$ at the vertex, that is, as $|y| \to 0$.

For $u \in \mathcal{M}_\varepsilon$, the Fourier transform of the system (2.2) is

$$\left.\begin{cases} (\Delta + \omega^2)\hat{u}_s(y,\omega) = 0 \;\; y \in Q \\ \hat{u}_s(y,\omega) = -\hat{F}(\omega) e^{i\omega y_1 \cos\alpha}, \;\; y \in \Gamma_1 \\ \hat{u}_s(y,\omega) = -\hat{F}(\omega) e^{-i\omega y_2 [\cos(\alpha+\Phi)/\sin\Phi]}, \;\; y \in \Gamma_2 \end{cases}\right| \omega \in \mathbb{C}^+ \tag{2.3}$$

in the case of the DD problem, and similar equations hold for the NN and DN problems (see Appendix A1).

Let us note that the boundary conditions in (2.3) are well defined for $\hat{u}_s(y,\omega) \in E_\varepsilon$ in contrast to the boundary conditions in (2.2), which are not well defined for tempered distributions $u_s(y,t)$. This suggests the following definition.

Definition 2.2
We call $u_s(y,t) \in \mathcal{M}_\varepsilon$ a solution to (2.2) if $\hat{u}_s(y,\omega)$ is a solution to (2.3).

3. Existence and uniqueness

In this section, we prove the uniqueness and existence of solution to the scattering problem (1.3) in the class $\mathcal{M}_\varepsilon$, using methods and results of [2–6]. We will assume that

$$F \in S'(\mathbb{R}), \;\; \text{supp}\, F \subset \overline{\mathbb{R}^+}. \tag{3.1}$$

We will prove the existence and uniqueness of a solution to problem (2.2) with any fixed boundary operators B_1 and B_2.

3.1. *Uniqueness*

Theorem 3.1
A solution to problem (2.2) is unique in the class $\mathcal{M}_\varepsilon$ for any $\varepsilon \in (0,1)$.

Proof
Let $u_s(y,t) \in \mathcal{M}_\varepsilon$ satisfy (2.2). By Definition 2.2, it suffices to prove the uniqueness of the solution $\hat{u}(y,\omega)$ to problems (2.3) for any $\omega \in \mathbb{C}^+$ in the space E_ε. This uniqueness is proved in Sections 7 and 8 of [2] for the DD problem, in [4,5] for the DN problem, and [6] for the NN problem. □

3.2. *Existence*

Let us recall the functions $\mathcal{S}_s(y,\omega), \mathcal{S}_d(y,\omega), \mathcal{S}_r(y,\omega)$ introduced in [3,4] and in [6] for the DD, DN, and NN problems, respectively, which are the densities of the scattered, diffracted, and reflected waves, respectively, and

$$\left.\begin{aligned} \mathcal{S}_r(\rho,\theta,\omega) &:= \begin{cases} -e^{i\omega\rho\cos(\theta-\theta_1)}, & \phi < \theta < \theta_1 \\ 0, & \theta_1 < \theta < \theta_2 \\ -e^{i\omega\rho\cos(\theta-\theta_2)}, & \theta_2 < \theta < 2\pi \end{cases} \\ \mathcal{S}_d(\rho,\theta,\omega) &:= \frac{i}{4\Phi}\int_{\mathbb{R}} e^{i\omega\rho\cosh\beta} Z(\beta + i\theta)\, d\beta, \;\; \theta \neq \theta_{1,2} \\ \mathcal{S}_s(\rho,\theta,\omega) &:= \mathcal{S}_r(\rho,\theta,\omega) + \mathcal{S}_d(\rho,\theta,\omega), \;\; \theta \neq \theta_{1,2} \end{aligned}\right| \rho > 0, \;\; \omega \in \overline{\mathbb{C}^+}. \tag{3.2}$$

Here

$$\theta_1 := 2\phi - \alpha, \;\; \theta_2 := 2\pi - \alpha \tag{3.3}$$

are the 'critical' directions (see [3, Def. 7.1]), and

$$Z(\beta) = -H\left(-\frac{i\pi}{2} + \beta\right) + H\left(-\frac{5i\pi}{2} + \beta\right), \;\; \beta \in \mathbb{C}, \tag{3.4}$$

where H is the Malyuzhinetz type kernel for the DD, NN, and DN problems (see Appendix A2). The formulas for $\mathcal{S}_r$ for the DN and NN problems are given in Appendix A3.

By [3, Thm 8.1], the function $\mathcal{S}_s(y,\omega) \in C_b(\overline{Q} \times \mathbb{C}^+)$, and it is analytic in $\omega \in \mathbb{C}^+$. This implies that

$$\mathcal{S}_s \in HP(\mathbb{C}^+, S'(\overline{Q})) \tag{3.5}$$

by Definition A.2 from Appendix A0.

Let us define

$$\mathcal{Z}(\beta) := Z(\beta) + Z(-\beta), \;\; l(\lambda) = \begin{cases} \ln(\lambda + \sqrt{\lambda^2 - 1}), & \lambda \geq 1 \\ 0 & , \;\; \lambda \in (0,1). \end{cases} \tag{3.6}$$

In Appendix A4, we calculate the inverse Fourier transforms $F_{\omega \to t}$ of $\mathcal{S}_r$ and $\mathcal{S}_d$ that we denote by $\mathcal{J}_r(\rho,\theta,t)$ and $\mathcal{J}_d(\rho,\theta,t)$, respectively,

$$\left.\begin{aligned} \mathcal{J}_r(\rho,\theta,t) &= \begin{cases} \delta(t - \rho\cos(\theta - \theta_1)), & \phi < \theta < \theta_1 \\ 0, & \theta_1 < \theta < \theta_2 \\ \delta(t - \rho\cos(\theta - \theta_2)), & \theta_2 < \theta < 2\pi \end{cases} \\ \mathcal{J}_d(\rho,\theta,t) &= \frac{i}{4\Phi}\frac{\mathcal{Z}(l(t/\rho) + i\theta)}{\sqrt{t^2 - \rho^2}} h(t - \rho) \\ \mathcal{J}_s(\rho,\theta,t) &= \mathcal{J}_r(\rho,\theta,t) + \mathcal{J}_d(\rho,\theta,t), \;\; \theta \neq \theta_{1,2} \end{aligned}\right| \;\; \rho > 0, \;\; t \in \mathbb{R}, \tag{3.7}$$

where $h(\cdot)$ denotes the Heaviside function. Let us note that $\mathcal{J}_r(\rho,\theta,t) = \mathcal{J}_d(\rho,\theta,t) = 0$ for $t < 0$. For the NN and DN problems, the functions $\mathcal{J}_r$ and $\mathcal{J}_d$ are calculated in Appendix A5.

For our application, it is crucially important that $\mathcal{J}_d, \; \mathcal{J}_r \; \mathcal{J}_s \in S'(\overline{Q} \times \overline{\mathbb{R}^+})$. This follows immediately from Lemma A.1, (3.5) and the fact that $\mathcal{J}_s$ is the inverse Fourier transform of $\mathcal{S}_s$.

To prove the main theorem, we need the following Lemma collecting the basic results of [2–6].

Lemma 3.2
Let F be a smooth profile function (1.2). Then the unique solution $\hat{u}_s(y,\omega) \in E_\varepsilon$ to (2.3) is given by

$$\hat{u}_s(\rho,\theta,\omega) = \hat{F}(\omega)\mathcal{S}_s(\rho,\theta,\omega), \quad \omega \in \mathbb{C}^+, \tag{3.8}$$

where $\hat{F}(\omega) = \hat{f}(\omega - \omega_0)$, and the parameter ε is given by

$$\varepsilon = \begin{cases} 1 - \dfrac{\pi}{\Phi} & \text{for the DD and NN cases} \\ 1 - \dfrac{\pi}{2\Phi} & \text{for the DN case.} \end{cases} \tag{3.9}$$

Formula (3.8) is proved in [3, (3.15)], while (3.9) was given in Section 10 of [3] for the DD problem, in Section 6 of [6] for the NN problem, and in Section 16 of [4] for the DN problem.

This lemma implies, in particular, that

$$\mathcal{S}_s(\cdot,\omega) \in E_\varepsilon, \; \omega \in \mathbb{C}^+, \tag{3.10}$$

as for any $\omega \in \mathbb{C}^+$ we can choose a smooth profile function (1.2) such that $\hat{f}(\omega - \omega_0) \neq 0$.

Corollary 3.3
The function $\mathcal{S}_s(y,\omega)$ is a solution to problem (2.3) with $\hat{F} \equiv 1$.

Our main result is the following theorem.

Theorem 3.4
Let F satisfy (3.1). Then
(i) There exists a generalized solution $u_s(y,t) \in \mathcal{M}_\varepsilon$ to problem (2.2) with ε given by (3.9).
(ii) The solution is given by the convolution

$$u_s = F_\delta * \mathcal{J}_s, \;\; (y,t) \in \overline{Q} \times \mathbb{R}, \tag{3.11}$$

where $F_\delta(y,t) := F(t)\delta(y)$, and the convolution is well defined in the sense of distributions.

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Proof

(i) By Definition 2.2, the problem (2.2) is equivalent to (2.3). For any distribution (3.1), it is natural to define the solution to (2.3) again by (3.8). Indeed, $\hat{u}_s(\cdot,\omega) \in E_\varepsilon$ for $\omega \in \mathbb{C}^+$ as $\mathcal{S}_s(\cdot,\omega) \in E_\varepsilon$ by (3.10). Moreover, $\hat{u}_s$ is a solution to (2.3) by Corollary 3.3. It remains to prove that

$$u_s(y,t) := F^{-1}_{\omega\to t}\hat{u}_s(y,\omega) \in \mathcal{M}_\varepsilon.$$

It suffices to check that $u_s \in S'(\overline{Q}\times\overline{\mathbb{R}^+})$, or equivalently, $\hat{u}_s \in HP(\mathbb{C}^+, S'(\overline{Q}))$ by Lemma A.1 of Appendix A0. First, $\hat{F}$ satisfies (A1) by (1.6). Second, $\mathcal{S}_s \in HP(\mathbb{C}^+, S'(\overline{Q}))$ by (3.5). Hence, the product (3.8) also belongs to $HP(\mathbb{C}^+, S'(\overline{Q}))$ by Lemma A.1 and bound (A2).

(ii) The convolution representation (3.11) follows from (3.8). The convolution is well defined as the intersection of the supports of $F_\delta(y',t')$ and $\mathcal{J}_s(y-y',t-t')$ is a bounded set for any fixed $y \in \overline{Q}$ and $t \in \mathbb{R}$. □

3.3. Sommerfeld type representation of the diffracted wave

Let us substitute the splitting from the last line of (3.7) into (3.11). Then we obtain the corresponding splitting $u_s = u_r + u_d$. By (3.8), we obtain for the DD case

$$u_r = F^{-1}_{\omega\to t}[\hat{F}\mathcal{S}_r] = F_\delta * \mathcal{J}_r,\;\; u_d = F^{-1}_{\omega\to t}[\hat{F}\mathcal{S}_d] = F_\delta * \mathcal{J}_d.$$

Similar formulas hold for the NN and DN cases. The explicit expressions of u_r for all types of boundary conditions are given in Appendix A6.

Lemma 3.5
Suppose that

$$F \in L^1_{loc}(\mathbb{R}) \tag{3.12}$$

and (1.6) holds. Then the diffracted wave u_d admits the representation (1.7) with a suitable kernel Z for any boundary conditions of the DD, NN, or DN types.

Proof
It suffices to prove that

$$\begin{aligned}&\hat{u}_d(\rho,\theta,\omega)\\ &= \hat{F}(\omega)\mathcal{S}_d(\rho,\theta,\omega) = \frac{i}{4\Phi}\int_{\mathbb{R}} e^{i\omega t}\left(\int_{\mathbb{R}} Z(\beta+i\theta)F(t-\rho\cosh\beta)d\beta\right)dt,\;\; \omega\in\mathbb{C}^+.\end{aligned} \tag{3.13}$$

Let us denote

$$q = \frac{\pi}{2\Phi}.$$

From (3.4), (A3), and (A4), we obtain the decay

$$|Z(\beta+i\theta)| \le C(\theta)e^{-2q|\beta|}\;,\quad \theta\in\Theta \tag{3.14}$$

for the DD and NN problems, and

$$|Z(\beta+i\theta)| \le C(\theta)e^{-q|\beta|}\;,\quad \theta\in\Theta \tag{3.15}$$

for the DN problem. Hence, (1.6) and the Fubini Theorem imply that

$$\begin{aligned}&\frac{i}{4\Phi}\int_{\mathbb{R}} e^{i\omega t}\left(\int_{\mathbb{R}} Z(\beta+i\theta)F(t-\rho\cosh\beta)d\beta\right)dt\\ &= \frac{i}{4\Phi}\int_{\mathbb{R}} Z(\beta+i\theta)\left(\int_{\mathbb{R}} e^{i\omega t}F(t-\rho\cosh\beta)dt\right)d\beta = \hat{F}(\omega)\mathcal{S}_d(\rho,\theta,\omega),\;\; \omega\in\mathbb{C}^+\end{aligned}$$

by formula (3.2) for $\mathcal{S}_d$. □

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4. Stabilization of the diffracted wave

Let l_k be the critical rays $l_k := \{(\rho, \theta_k) : \ \rho > 0\}$, where θ_k are given by (3.3), $k = 1, 2$.

Lemma 4.1
Let (2.1), (3.12), and (1.6) hold. Then for any type of the boundary conditions (DD, NN, and DN) and $t \in \mathbb{R}$, there exist the limits

$$u_d(\rho, \theta_k \pm 0, t) := \lim_{\varepsilon \to 0+} u_d(\rho, \theta_k \pm \varepsilon, t), \ \ \rho > 0, \ \ k = 1, 2$$

in the sense of distribution of $\rho > 0$, and the jumps of u_d on the critical rays are given by

$$[u_d]_k(\rho, t) := u_d(\rho, \theta_k + 0, t) - u_d(\rho, \theta_k - 0, t) = (-1)^{k+1} F(t - \rho), \quad \rho > 0, \ k = 1, 2. \tag{4.1}$$

Proof
We will use representation (1.7) and consider the DD case for concreteness. The cases of the NN and DN problems are analyzed similarly (see Appendix A6). Formulas (3.4) and (A3) imply the following representation:

$$\begin{aligned} & Z(\beta + i\theta) \\ & = -\coth(q\beta + ic_0) \mp \coth(q\beta + ic_1) \pm \coth(q\beta + ic_2) + \coth(q\beta + ic_3) \end{aligned} \tag{4.2}$$

for the DD and NN cases, respectively, where

$$c_k := q(\theta - p_k); \ \ p_0 = \alpha, \ \ p_1 = \theta_1, \ \ p_2 = \theta_2, \ \ p_3 = 2\pi + \alpha.$$

First, let us consider the case when $F(s)$ is a Hölder function of $s \geq 0$ satisfying (1.6). Then the Sokhotski–Plemelj formulas imply

$$\begin{aligned} & [u_d]_k(\rho, t) \\ & = \int_{-1}^{1} F(t - \rho \cosh \beta) \left[\coth(q\beta + i0) - \coth(q\beta - i0)\right] d\beta = (-1)^{k+1} F(t - \rho), \ \rho < t, \end{aligned}$$

because $\coth(q\beta + ic_k)$ with $k = 0$ and $k = 3$ are continuous on the critical rays for α satisfying (2.1). For F satisfying (1.6), (4.1) holds in the sense of distributions. □

Theorem 4.2
Let the incident wave profile F satisfy (1.6), and

$$F(s) \to C, \qquad s \to \infty. \tag{4.3}$$

Then
(i) The diffracted wave converges in the long-time limit:

$$u_d(\rho, \theta, t) \underset{t \to \infty}{\longrightarrow} u_d(\theta, \infty) := \frac{iC}{4\Phi} \int_{\mathbb{R}} Z(\beta + i\theta) d\beta, \ \ \rho > 0, \ \ \theta \in \Theta, \tag{4.4}$$

and in particular,

$$[u_d]_k(\rho, t) \underset{t \to \infty}{\longrightarrow} C, \quad \text{for} \ \ \rho > 0, \ \ k = 1, 2. \tag{4.5}$$

(ii) Conversely, (4.5) implies (4.3).

Proof
We can use (1.7) by Lemma 3.5.

(i) Conditions (1.6) and (1.7) imply that

$$u_d(\rho, \theta, t) = \frac{i}{4\Phi} \int_{-l(t/\rho)}^{l(t/\rho)} Z(\beta + i\theta) F(t - \rho \cosh \beta) d\beta, \ \ \theta \in \Theta,$$

where $l(\cdot)$ is defined by (3.6). Then (4.4) follows from (3.14) by the Lebesgue Dominate Convergence Theorem. Convergence (4.5) follows from (4.1) and (4.3).
(ii) Equation (4.3) follows from (4.1) and (4.5).

□

Corollary 4.3
For any type of boundary conditions (DD, NN, or DN), the function $u_d(\theta, \infty)$ is a piecewise constant function of $\theta \in \Theta$ with the jumps at $\theta = \theta_1$ and $\theta = \theta_2$.

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Proof
For the DD and NN cases, formula (4.2) implies that $Z(\beta)$ is a holomorphic function on $\mathbb{C}\setminus P$, where $P = \cup_{l=0,1,2,3}\{ip_l + 2ik\Phi :\ k \in \mathbb{Z}\}$. Moreover, $p_0, p_3 \notin \Theta$ by (2.1). Hence, $Z(\beta + i\theta)$ may have a pole $\beta \in \mathbb{R}$ only at $\beta = 0$, and it holds only for $\theta = \theta_1$ or $\theta = \theta_2$. Therefore, the corollary follows from decay (3.14) and the Cauchy Theorem.

For the DN case, the proof is similar, relying on decay (3.15). □

5. Limiting amplitude principle

Consider the incident wave

$$F^0(t) := a^0 e^{-i\omega_0 t} h(t),\ \ t \in \mathbb{R},$$

where $\omega_0 \neq 0$ (the case of $\omega_0 = 0$ is covered by Theorem (4.2)). By (1.7), the corresponding diffracted wave is given by

$$u_d^0(\rho,\theta,t) = i\frac{e^{-i\omega_0 t}}{4\Phi}\int_{-l(t/\rho)}^{l(t/\rho)} e^{i\omega\rho\cosh\beta} Z(\beta + i\theta)d\beta,$$

where Z is given by (4.2) for the DD and NN problems (and by (A4) for the DN problem) and $l(\cdot)$ is defined by (3.6). The limiting amplitude of this wave is

$$A^0(\rho,\theta) = \frac{i}{4\Phi}\int_{\mathbb{R}} a^0 e^{i\omega_0\rho\cosh\beta} Z(\beta + i\theta)d\beta.\ \ \forall\theta \in \Theta \tag{5.1}$$

because $l(t/\rho) \to \infty$, as $t \to \infty$, while Z satisfies (3.14) for the DD and NN problems and (3.15) for the DN problem.

For general incident wave (1.4), the diffracted wave (1.7) can be written as $u_d(\rho,\theta,t) = e^{-i\omega_0 t}A^0(\rho,\theta)$ with amplitude

$$A_d(\rho,\theta,t) := e^{i\omega_0 t}\frac{i}{4\Phi}\int_{-l(t/\rho)}^{l(t/\rho)} Z(\beta + i\theta)F(t - \rho\cosh\beta)d\beta,\ \ \rho > 0,\ \ \theta \in \Theta,\ \ t > 0. \tag{5.2}$$

In the following theorem, we prove that the amplitude is asymptotically close to the amplitude (5.1) if F is asymptotically close to F^0.

Theorem 5.1 (Limiting amplitude principle)
Suppose that

$$R(t) := F(t) - F^0(t) \to 0,\ \ t \to \infty. \tag{5.3}$$

Then for any $\delta > 0$,

$$u_d(\rho,\theta,t) - e^{-i\omega_0 t}A_0(\rho,\theta) \to 0,\ \ t \to \infty,$$

uniformly in bounded $\rho > 0$ and $\theta - \theta_k \geq \delta$.

Proof
By definitions (5.1) and (5.2),

$$A_d(\rho,\theta,t) - A^0(\rho,\theta)$$
$$= -\frac{i}{4\Phi}\int_{|\beta|\geq l(t/\rho)} F^0(-\rho\cosh\beta)Z(\beta + i\theta)d\beta + \frac{i}{4\Phi}\int_{-l(t/\rho)}^{l(t/\rho)} e^{i\omega_0 t}Z(\beta + i\theta)R(t - \rho\cosh\beta)d\beta.$$

Estimates (3.14) and (3.15) imply that

$$\frac{i}{4\Phi}\int_{|\beta|\geq l(t/\rho)} F^0(-\rho\cosh\beta)Z(\beta + i\theta)d\beta = \frac{i}{4\Phi}\int_{|\beta|\geq l(t/\rho)} e^{i\omega_0\rho\cosh\beta}Z(\beta + i\theta)d\beta \to 0,\ \ t \to \infty$$

uniformly in $\rho > 0$ and $\theta \in \Theta$. It remains to prove that

$$R_1(\rho,\theta,t) := \int_{-l(t/\rho)}^{l(t/\rho)} e^{i\omega_0 t}Z(\beta + i\theta)R(t - \rho\cosh\beta)d\beta \to 0,\ \ t \to \infty$$

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uniformly in bounded $\rho > 0$ and $\theta - \theta_k \geq \delta > 0$. First, (3.14), (3.15), and (5.3) imply that for any $\varepsilon > 0$ there exists $\beta(\varepsilon)$ s.t.

$$\int_{|\beta|\geq\beta(\varepsilon)} |Z(\beta + i\theta)R(t - \rho\cosh\beta)|d\beta < \varepsilon/2 \tag{5.4}$$

uniformly in $\rho > 0$ and $\theta \in \Theta$. Second, (5.3) implies that for $0 < \rho \leq b < \infty$ and $\theta - \theta_k \geq \delta > 0$ there exists $t(\varepsilon, \delta, b)$ such that

$$2\beta(\varepsilon)|Z(\beta + i\theta)R(t - \rho\cosh\beta)| < \varepsilon/2, \;\; |\beta| < \beta(\varepsilon), \;\; \theta \in \Theta, \quad t > t(\varepsilon, \delta, b). \tag{5.5}$$

Then for $0 < \rho \leq b < \infty$ and $t > t(\varepsilon, \delta, b)$, we have

$$|R_1(\rho, \theta, t)| \leq \int_{|\beta|\leq\beta(\varepsilon)} |Z(\beta + i\theta)R(t - \rho\cosh\beta)|d\beta + \int_{|\beta|\geq\beta(\varepsilon)} |Z(\beta + i\theta)R(t - \rho\cosh\beta)|d\beta < \varepsilon$$

by (5.5) and (5.4). □

Appendix

A.1. The Paley–Wiener theory

For a function $u(t) \in S(\mathbb{R})$, we denote its Fourier transform as

$$\hat{u}(\omega) := F_{t\to\omega}u(\omega) := \int_{\mathbb{R}} e^{i\omega t}u(t)dt, \;\; \omega \in \mathbb{R}.$$

This transform is extended by continuity to tempered distributions $u \in S'(\mathbb{R})$. When $\operatorname{supp} u \subset \overline{\mathbb{R}^+}$, the distribution $\hat{u}(\omega)$ admits an analytic extension to the upper half plane $\mathbb{C}^+ := \{z \in \mathbb{C} : \operatorname{Im} z > 0\}$ and

$$|\hat{f}(\omega)| \leq C(1 + |\omega|)^m |\operatorname{Im}\omega|^{-N}, \;\; \omega \in \mathbb{C}^+ \tag{A1}$$

for some $m, N \in \mathbb{N}$ by the Paley–Wiener theorem. We will call this analytic continuation the Fourier–Laplace transform of f. Conversely, if an analytic function $G(\omega)$ in $\mathbb{C}^+$ satisfies (A1), then there exists its boundary value as $\operatorname{Im}\omega \to 0+$ in the sense of $S'(\mathbb{R})$; see [26, Thm I.5.2].

Let us introduce functional space of distributions and its Fourier–Laplace 'transform'. First we define the seminorms

$$\|\varphi\|_{m,N} := \sup_{x\in\mathbb{R}^n, |\alpha|\leq m} (1 + |x|)^N |\partial_x^\alpha \varphi(x)| < \infty$$

for functions from the Schwartz space $S(\mathbb{R}^n)$. Let us recall that $S'(\overline{Q} \times \overline{\mathbb{R}^+})$ denotes the space of tempered distributions in $\mathbb{R}^3$ with supports in $\overline{Q} \times \overline{\mathbb{R}^+}$. This space is a subspace of the dual space to a countable-normed space $S(\mathbb{R}^n)$, so for each $u \in S'(\overline{Q} \times \overline{\mathbb{R}^+})$, there exist $m, N \in \mathbb{N}$ such that

$$|\langle u(y,t), \varphi(y,t)\rangle| \leq C\|\varphi\|_{m,N}, \;\; \varphi \in S(\mathbb{R}^3).$$

In fact, this follows from the definition of topology on countable-normed spaces [27, Ch. I, §4, p.34].

From this estimate, we obtain Paley–Wiener type theorem for distributions, which is a straightforward generalization of [26, Thm 5.2, Ch. II].

Lemma A.1

(i) Let $u \in S'(\overline{Q} \times \overline{\mathbb{R}^+})$. Then its partial Fourier transform $\hat{u}(y,\omega)$ extends to an analytic function on $\mathbb{C}^+$ with values in $S'(\overline{Q})$, and (cf. (A1)) there exist $m, N \in \mathbb{N}$ s.t.

$$|\langle \hat{u}(y,\omega), \varphi(y)\rangle| \leq C\|\varphi\|_{m,N}(1 + |\omega|)^m |\operatorname{Im}\omega|^{-N}, \;\; \varphi \in S(\overline{Q}). \tag{A2}$$

(ii) Conversely, let $\hat{u}(y,\omega)$ be an analytic function of $\omega \in \mathbb{C}^+$ with values in $S'(\overline{Q})$, and the bound (A2) holds for some $m, N \in \mathbb{N}$. Then $\hat{u}(y,\omega)$ is the Fourier–Laplace transform of a distribution $u \in S'(\overline{Q} \times \overline{\mathbb{R}^+})$.

Definition A.2

We denote by $HP(\mathbb{C}^+, S'(\overline{Q}))$ the space of holomorphic functions in $\mathbb{C}^+$ with values in $S'(\overline{Q})$ satisfying bound (A2) for some $m, N \in \mathbb{N}$.

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A1. The 'stationary' scattering problem takes the forms

$$\begin{cases} (\Delta+\omega^2)\,\hat{u}_s(y,\omega) \;=\; 0, & y\in Q\\ \frac{\partial}{\partial y_2}\hat{u}_s(y,\omega) \;=\; -i\omega\hat{F}(\omega)\sin\alpha\, e^{i\omega y_1\cos\alpha}, & y\in\Gamma_1\\ \frac{\partial}{\partial \mathbf{n_2}}\hat{u}_s(y,\omega) \;=\; i\omega\hat{F}(\omega)\sin(\Phi+\alpha)e^{-i\omega y_2\frac{\cos(\Phi+\alpha)}{\sin\Phi}}, & y\in\Gamma_2\end{cases}$$

for the NN problem and

$$\left.\begin{cases} (\Delta+\omega^2)\hat{u}_s(y,\omega)=0, & y\in Q\\ \partial_{y_2}\hat{u}_s(y,\omega)=-i\omega g(\omega)\sin\alpha e^{i\omega y_1\cos\alpha}, & y\in\Gamma_1\\ \hat{u}_s(y,\omega)=-\hat{F}(\omega)e^{-i\omega y_2\frac{\cos(\alpha+\Phi)}{\sin\Phi}}, & y\in\Gamma_2\end{cases}\right| \quad \omega\in\mathbb{C}^+.$$

for the DN problem.

A2. The Malyuzhinetz integral kernels are given by

$$H(\beta)=\coth\left(q\left(\beta+\frac{i\pi}{2}-i\alpha\right)\right)\mp\coth\left(q\left(\beta-\frac{3i\pi}{2}+i\alpha\right)\right) \tag{A3}$$

for the DD and NN problems, respectively, and by

$$H(\beta)=\frac{1}{\sinh\left[q\left(\beta+\frac{i\pi}{2}-i\alpha\right)\right]}+\frac{1}{\sinh\left[q\left(\beta-\frac{3i\pi}{2}+i\alpha\right)\right]} \tag{A4}$$

for the DN problem (see also [5,6] where the DN and NN problems were considered in details). Here $q=\frac{\pi}{2\Phi}$.

A3. The densities of 'stationary' reflected waves S_r are given by

$$\mathcal{S}_r(\rho,\theta,\omega)=\left.\begin{cases}-e^{i\omega\rho\cos(\theta-\theta_1)} & \phi\le\theta\le\theta_1\\ 0 & \theta_1<\theta<\theta_2\\ e^{i\omega\rho\cos(\theta-\theta_2)}, & \theta_2\le\theta\le 2\pi\end{cases}\right|\omega\in\overline{\mathbb{C}^+}$$

for the DN problem and by

$$\mathcal{S}_r(\rho,\theta,\omega)=\left.\begin{cases}e^{i\omega\rho\cos(\theta-\theta_1)} & \phi\le\theta\le\theta_1\\ 0 & \theta_1<\theta<\theta_2\\ e^{i\omega\rho\cos(\theta-\theta_2)}, & \theta_2\le\theta\le 2\pi\end{cases}\right|\omega\in\overline{\mathbb{C}^+}$$

for the NN problem.

A4. The inverse Fourier transform.

Lemma A.3

(i) The inverse Fourier transforms of the functions $\mathcal{S}_d$ and $\mathcal{S}_r$ given by (3.2) are the functions $\mathcal{J}_d(y,\omega)$ and $\mathcal{J}_r(y,\omega)$, given by (3.7).

Proof

(i) We need to prove that for $\theta\in\Theta$

$$\mathcal{J}_d(\rho,\theta,t)=F^{-1}_{\omega\to t}\mathcal{S}_d(\rho,\theta,\omega)=F^{-1}_{\omega\to t}\left[\frac{i}{4\Phi}\int_R e^{i\omega\rho\cosh\beta}Z(\beta+i\theta)d\beta\right]. \tag{A5}$$

First, we note that

$$\frac{i}{4\Phi}\int_{\mathbb{R}} e^{i\omega\rho\cosh\beta}Z(\beta+i\theta)d\beta=\int_0^\infty e^{i\omega\rho\cosh\beta}\mathcal{Z}(\beta+i\theta)d\beta,$$

where $\mathcal{Z}$ is defined by (3.6). The integrals converge by (3.14).

Changing the variables $t:=\rho\cosh\beta,\ d\beta=\frac{dt}{\sqrt{t^2-\rho^2}}$, we obtain

$$\frac{i}{4\Phi}\int_{\mathbb{R}} e^{i\omega\rho\cosh\beta}Z(\beta+i\theta)d\beta=F_{t\to\omega}\left[\mathcal{J}_d(\rho,\theta,t)\right],$$

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where $\mathcal{J}_d$ is given by (3.7). Hence, (A5) follows.
Representation (3.7) for $\mathcal{J}_r$ follows from (3.2) directly. □

For the DN problem, $\mathcal{J}_r$ is

$$\mathcal{J}_r(\rho,\theta,t) := \left\{\begin{array}{ll} \delta(t-\rho\cos(\theta-\theta_1)), & \phi<\theta<\theta_1 \\ 0, & \theta_1<\theta<\theta_2 \\ -\delta(t-\rho\cos(\theta-\theta_2)), & \theta_2<\theta<2\pi \end{array}\right| t\geq 0,\ \ \mathcal{J}_r(\rho,\theta,t)=0,\ t<0.$$

For the NN problem,

$$\mathcal{J}_r(\rho,\theta,t) := \left\{\begin{array}{ll} -\delta(t-\rho\cos(\theta-\theta_1)), & \phi<\theta<\theta_1 \\ 0, & \theta_1<\theta<\theta_2 \\ -\delta(t-\rho\cos(\theta-\theta_2)), & \theta_2<\theta<2\pi \end{array}\right| t\geq 0,\ \ \mathcal{J}_r(\rho,\theta,t)=0,\ t<0.$$

A5. Expressions for the reflected waves are

$$u_r(y,t) := \begin{cases} \mp F(t-n_1\cdot y), & \phi\leq\theta\leq\theta_1 \\ 0 & \theta_1<\theta<\theta_2 \\ \mp F(t-n_2\cdot y), & \theta_2\leq\theta\leq 2\pi \end{cases}$$

for the DD and NN problems, respectively, and

$$u_r(y,t) := \begin{cases} F(t-n_1\cdot y), & \phi\leq\theta\leq\theta_1 \\ 0 & \theta_1<\theta<\theta_2 \\ -F(t-n_2\cdot y), & \theta_2\leq\theta\leq 2\pi \end{cases}$$

for the DN problem.

A6. Jumps of the diffracted wave in the cases of the NN and DN problems.
For the DN problem, the function Z from (4.2) takes the form

$$Z(\beta+i\theta) = -\frac{1}{\sinh(q\beta+ic_0)} - \frac{1}{\sinh(q\beta+ic_1)} - \frac{1}{\sinh(q\beta+ic_2)} + \frac{1}{\sinh(q\beta+ic_3)}$$

The jumps of the diffracted wave u_d on the critical rays for the NN and DN problems are given by

$$[u_d]_k(\rho,t) = F(t-\rho), \quad \rho>0,\ k=1,2$$

(cf. with (4.1)).

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