HYP

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This is a MATHEMATICA package for handling hypergeometric series. It provides quite a few tools for

(A) manipulating factorial expressions
(B) transforming binomial sums into hypergeometric notation
(C) summing hypergeometric series
(D) transforming hypergeometric series
(E) applying contiguous relations
(F) doing formal limits of hypergeometric expressions
(G) transforming hypergeometric MATHEMATICA expressions into TEX-code.
(H) applying Gosper's [3] and Zeilberger's [10, 11, 12] algorithms

The tools for items (A), (B), (F), (G) are contained in the file `hyp.m`, the basic package. This file must be loaded at the very beginning of your MATHEMATICA session. (Ignore error messages occuring when loading `hyp.m`.) The file `hyp.m` defines the basic objects, the rules and functions for items (A), (B), (F), and (G), and predefines all the remaining ones. The tools for (C) are the contents of the file `summatio.m`, those for (D) are the contents of the files `transfor.m` and `transfor.mli`, and those for (E) are the contents of the file `contig.m`. You also have access to summation and transformation formulas in form of equations. This is the contents of the files `summatio.mgl` and `transfor.mgl`, respectively. The file `output` defines some nice MATHEMATICA output features for `SUM`, `Product`, `Integrate`, `Abs`, `Floor`, `Ceiling`, `Pi`, and `Infinity`. The tools for item (H) rely on Peter Paule and Markus Schorn's *Mathematica* implementation of Gosper's and Zeilberger's algorithms. The current version 1.1 or updates can be received via e-mail request to `peter.paule@risc.uni-linz.ac.at`.

However, the philosophy of this package is:

Do it by yourself!

The idea is that you should be able to control each step in a series of manipulations by yourself. So, for instance, this package does not make any attempt to sum or transform a series automatically. So, it is you who has to tell the package which command, summation, or transformation has to be applied next. Therefore a basic knowledge of hypergeometric series is required (cf. [1, sec. 1.1, 2.1; 7, sec. 1.1; 2, pp. 1–6]). This handbook provides you with a list of the rules, functions, summations, transformations that are available. The main sources for identities that are included in this package have been the books [2, 7, 1]. They contain a fairly comprehensive collection of known summation and transformation formulas for hypergeometric series. In particular, the (almost) complete Appendices of [7] and [2] (for $q \uparrow 1$) are included in this package.

Finally you should be warned that there is no guarantee that a formula that has been obtained using this package is actually valid. Many formulas or operations are only valid under certain restrictions for the parameters. This package only helps you to do calculations fast. It is up to you to check that the manipulations you are doing are actually being allowed.

For a brief summary of the main features of this package the user is referred to [4] which is the contents of the AMS-TEX file `hyp_hypq.tex`.

Hypergeometric notation

All the notation and terminology is adopted from [2, pp. 1–6]. The *(generalized) hypergeometric series* is defined by

$$ {}_rF_s\left[\begin{matrix} a_1,\dots,a_r \\ b_1,\dots,b_s \end{matrix};z\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_r)_n}{n!\,(b_1)_n \cdots (b_s)_n} z^n \ , $$

where the rising factorial $(a)_n$ is given by $(a)_n := a(a+1)\cdots(a+n-1)$, $n \ge 1$, $(a)_0 := 1$. A hypergeometric series ${}_{r+1}F_r$ is called *very well-poised* if $a_i + b_i = 1 + a_0$ for $i = 1, 2, \dots, r$, and among the parameters a_i occurs $1 + a_0/2$. We use the standard abbreviation for very well-poised hypergeometric series,

$$ {}_{r+1}V_r(a_0; a_2, a_3, \dots, a_r; z) := {}_{r+1}F_r\left[\begin{matrix} a_0, 1+a_0/2, a_2, a_3, \dots, a_r \\ a_0/2, 1+a_0-a_2, 1+a_0-a_3, \dots, 1+a_0-a_r \end{matrix};z\right]. $$

The *bilateral hypergeometric series* is defined by

$$ {}_rH_s\left[\begin{matrix} a_1,\dots,a_r \\ b_1,\dots,b_s \end{matrix};z\right] = \sum_{n=-\infty}^{\infty} \frac{(a_1)_n \cdots (a_r)_n}{(b_1)_n \cdots (b_s)_n} z^n \ . $$

We also use the compact Gasper-Rahman notation

$$ (a_1, a_2, \dots, a_r)_n := (a_1)_n\,(a_2)_n \cdots (a_r)_n $$

and

$$ \Gamma\left[\begin{matrix} a_1, a_2, \dots, a_r \\ b_1, b_2, \dots, b_s \end{matrix}\right] := \frac{\Gamma(a_1)\,\Gamma(a_2)\,\cdots\,\Gamma(a_r)}{\Gamma(b_1)\,\Gamma(b_2)\,\cdots\,\Gamma(b_s)} \ . $$

The file `hyp.m`

The objects which are defined in the file hyp.m are

`AbsGreater`, `AbsSmaller`, `AbsUndetermined`, `Add`, `AmSLaTeX`, `AmSTeX`, `baszerl`, `baszus`, `Binomialp`, `Div`, `Drucke`, `Ers`, `erw1`, `erw2`, `Expandq`, `F`, `FH`, `Factorialp`, `FCancel`, `FEinf`, `FFormat`, `FOrdne`, `FPerm`, `FSUM`, `FTausche`, `GAMMA`, `Gleichung`, `GlTausche`, `GOSPER`, `Gzerl`, `Gzus`, `H`, `HEinf`, `HF`, `Hoch`, `HOrdne`, `HPerm`, `HShift`, `HSUM`, `hypAttributes`, `inv`, `LaTeX`, `Limes`, `lina1`, `lina2`, `linz`, `LS`, `Mal`, `ManipulationsListe`, `MinusOne`, `Multinomialp`, `neg1`, `neg2`, `p`, `P`, `paufl`, `PosListe`, `PSort`, `pzerl`, `pzus`, `RS`, `SchreibeZahl`, `SimplifyP`, `Sub`, `Subst`, `SUM`, `SUMErw1`, `SUMErw2`, `SUMExpand`, `SUMF`, `SUMH`, `SUMInfinity`, `SUMRegeln`, `SUMSammle`, `SUMShift`, `SUMTausche`, `SUMUmkehr`, `SUMZerl`, `TeX`, `TeXFV`, `TeXMat`, `trans`, `V`, `ZB`, `zerl`, `zus1`, `zus2`, `zus3`.

These objects can be divided into 10 groups: There are the basic objects,

`Binomialp`, `F`, `Factorialp`, `GAMMA`, `H`, `Multinomialp`, `p`, `SUM`, `V`,

the rules for manipulating factorial expressions

`baszerl`, `baszus`, `erw1`, `erw2`, `Expandq`, `Gzerl`, `Gzus`, `inv`, `lina1`, `lina2`, `linz`, `MinusOne`, `neg1`, `neg2`, `paufl`, `pzerl`, `pzus`, `trans`, `zerl`, `zus1`, `zus2`, `zus3`,

the rules for manipulating sums and hypergeometric series,

`FCancel`, `FEinf`, `FFormat`, `FH`, `FOrdne`, `FSUM`, `FPerm`, `FTausche`, `HEinf`, `HF`, `HOrdne`, `HPerm`, `HShift`, `HSUM`, `SUMErw1`, `SUMErw2`, `SUMExpand`, `SUMF`, `SUMH`, `SUMInfinity`, `SUMRegeln`, `SUMSammle`, `SUMShift`, `SUMTausche`, `SUMUmkehr`, `SUMZerl`,

two functions for controlled use of rules,

`Ers`, `Posliste`,

one function for substitution of an expression instead of another expression, `Subst`,

some objects for doing limits of hypergeometric expressions,

`AbsGreater`, `AbsSmaller`, `AbsUndetermined`, `Limes`,

one object for simplifying arguments in hypergeometric expressions,

`SimplifyP`,

some objects for converting expressions into TEX-code,

`AmSLaTeX`, `AmSTeX`, `LaTeX`, `TeX`, `TeXFV`, `TeXMat`,

two objects for on-line help,

`hypAttributes`, `ManipulationsListe`,

and the function

`Drucke`,

which enables you to directly send an expression to the printer. Also there are

`Add`, `Div`, `Gleichung`, `GlTausche`, `Hoch`, `LS`, `Mal`, `PSort`, `RS`, `Sub`,

for manipulating equations and writing expressions in a "normalized" form (`PSort`) in order to be able to quickly check if two expressions agree. These objects are particularly important when using objects from `summatio.mgl` and `transfor.mgl`. Finally there are

`GOSPER`, `ZB`,

for applying Gosper's and Zeilberger's algorithms. Peter Paule's and Markus Schorn's *Mathematica* implementation of these algorithms which is needed for using `GOSPER` and `ZB` also provides

`Zb`, `Gosper`, `RunMode`, `FileName`, `SolAmount`, `Fnk`, `GoRat`, `GoSol`, `Cert`, `DegBound`, `System`, `SystemDimension`.

Regarding these objects the user is referred to the documentation and the description [5] of this implementation.

Most of the tools for manipulating expressions that are provided by this package are rules. This has the advantage that very often you do not have to specify to which part of an expression you want to apply a rule, since there is just one subexpression to which the rule applies. However, if there are more subexpressions to which a rule applies, you will sometimes want to apply the rule only to some of the subexpressions. To handle this conveniently, there are the functions `Ers` and `PosListe`.

The file `summatio.m`

This file basically contains the Appendix II of [2] for $q \uparrow 1$ in form of rules. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `summatio.m` are

`S1001`, `S2101`, `S2103`, `S2104`, `S2105`, `S2106`, `S2131`, `S2132`, `S2210`, `S2240`, `S3201`, `S3202`, `S3204`, `S3231`, `S3232`, `S3233`, `S3234`, `S3235`, `S3261`, `S3291`, `S3340`, `S4306`, `S4307`, `S4331`, `S4332`, `S5431`, `S5432`, `S5540`, `S6531`, `S6532`, `S7631`, `S7632`, `S7691`, `SListe`, `SumListe`.

The numbering of each rule is S⟨d1⟩⟨d2⟩⟨n1⟩ following the following system: The number ⟨d1⟩ is the number of the upper parameters, the number ⟨d2⟩ is the number of the lower parameters of the hypergeometric series to which the rule applies. The number ⟨n1⟩ allows to distinguish the rules applying to hypergeometric series with equal numbers of upper and lower parameters. ⟨n1⟩ is within the range 01–30 if the summation is a one-term summation and its q-analogue (in the package HYPQ) has the same numbering, it is within the range 31–60 if the summation is a one-term summation and its q-analogue (in the package HYPQ) has a different numbering, it is within the range 61–90 if the summation is a two- or more-term summation and its q-analogue (in the package HYPQ) has the same numbering, it is within the range 90–120 if the summation is a two- or more-term summation and its q-analogue (in the package HYPQ) has a different numbering.

For terminating series there is a check if one of the parameters is of the form $(-n)$ where n is a nonnegative integer. Depending on your input you might be asked if some expression is a nonnegative integer (see the examples for `S3201`). Be sure to give an affirmative answer only for *one* of several expressions, otherwise the package will try to find the minimum of all of these, which might cause problems. This remark also applies to other rules which put this question, e.g. `SUMUmkehr`, `FSUM`, or in case that automatic evaluating is active (cf. `P`). The rule `SListe` enables you to quickly check if one of the summation rules `S1001`–`S7691` can be directly applied.

The file `transfor.m`

This file basically contains the Appendix III of [2] for $q \uparrow 1$ in form of rules. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `transfor.m` are

`T2103`, `T2104`, `T2106`, `T2107`, `T2110`, `T2112`, `T2131`, `T2132`, `T2133`, `T2134`, `T2135`, `T2136`, `T2137`, `T2138`, `T2139`, `T2139`, `T2140`, `T2141`, `T2163`, `T2191`, `T2192`, `T3204`, `T3205`, `T3206`, `T3207`, `T3217`, `T3231`, `T3232`, `T3233`, `T3234`, `T3235`, `T3236`, `T3237`, `T3238`, `T3239`, `T3240`, `T3261`, `T3262`, `T3263`, `T3264`, `T4301`, `T4302`, `T4303`, `T4304`, `T4306`, `T4309`, `T4310`, `T4312`, `T4313`, `T4331`, `T4332`, `T4362`, `T4391`, `T5401`, `T5402`, `T5403`, `T5468`, `T6501`, `T6531`, `T6532`, `T6533`, `T6534`, `T7631`, `T7632`, `T7633`, `T7634`, `T7635`, `T7636`, `T7637`, `T7691`, `T7692`, `T7693`, `T7694`, `T7740`, `T8731`, `T8732`, `T9831`, `T9832`, `T9833`, `T9834`, `T9835`, `T9836`, `T9837`, `T9838`, `T9891`, `T9892`, `T9893`, `T9894`, `T9940`, `T111031`, `T111032`, `TListe`, `TransListe`.

The comments for the file `summatio.m` regarding the numbering of the rules and optional questions for input also apply here. The rule `TListe` enables you to quickly check if one of the transformation rules `T2103`–`T9891` can be directly applied.

The file `transfor.mli`

Each of the objects of this file corresponds to a transformation rule of the file `transfor.m`. Each object gives a list of all the outcomes under application of a particular transformation after before having permuted the upper and lower parameters of the involved basic hypergeometric series. All the objects in this file are rules. These rules help to prove conjectured transformation formulas quickly. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `transfor.mli` are

`Tli2103`, `Tli2104`, `Tli2106`, `Tli2107`, `Tli2110`, `Tli2112`, `Tli2131`, `Tli2132`, `Tli2133`, `Tli2134`, `Tli2135`, `Tli2136`, `Tli2137`, `Tli2138`, `Tli2139`, `Tli2139`, `Tli2140`, `Tli2141`, `Tli2163`, `Tli2191`, `Tli2192`, `Tli3204`, `Tli3205`, `Tli3206`, `Tli3207`, `Tli3217`, `Tli3231`, `Tli3232`, `Tli3233`, `Tli3234`, `Tli3235`, `Tli3236`, `Tli3237`, `Tli3238`, `Tli3239`, `Tli3240`, `Tli3261`, `Tli3262`, `Tli3263`, `Tli3264`, `Tli4301`, `Tli4302`, `Tli4303`, `Tli4304`, `Tli4306`, `Tli4309`, `Tli4310`, `Tli4312`, `Tli4313`, `Tli4331`, `Tli4332`, `Tli4362`, `Tli4391`, `Tli5401`, `Tli5402`, `Tli5403`, `Tli5468`, `Tli6501`, `Tli6531`, `Tli6532`, `Tli6533`, `Tli6534`, `Tli7631`, `Tli7632`, `Tli7633`, `Tli7634`, `Tli7635`, `Tli7636`, `Tli7637`, `Tli7691`, `Tli7692`, `Tli7693`, `Tli7694`, `Tli7740`, `Tli8731`, `Tli8732`, `Tli9831`, `Tli9832`, `Tli9833`, `Tli9834`, `Tli9835`, `Tli9836`, `Tli9837`, `Tli9838`, `Tli9891`, `Tli9892`, `Tli9893`, `Tli9894`, `Tli9940`, `Tli111031`, `Tli111032`.

The files `summatio.mgl` and `transfor.mgl`

These files contain the same summations, respectively transformations, as `summatio.m`, respectively `transfor.m`, but in form of equations. You do not have to load these files by hand since they are loaded automatically once an object of this file is called. The respective objects are

`Sgl1001`, `Sgl2101`, `Sgl2103`, `Sgl2104`, `Sgl2105`, `Sgl2106`, `Sgl2131`, `Sgl2132`, `Sgl2210`, `Sgl2240`, `Sgl3201`, `Sgl3202`, `Sgl3204`, `Sgl3231`, `Sgl3232`, `Sgl3233`, `Sgl3234`, `Sgl3235`, `Sgl3261`, `Sgl3291`, `Sgl3340`, `Sgl4306`, `Sgl4307`, `Sgl4331`, `Sgl4332`, `Sgl5431`, `Sgl5432`, `Sgl5540`, `Sgl6531`, `Sgl6532`, `Sgl7631`, `Sgl7632`, `Sgl7691`, `SumListe$gl`,

and

`Tgl2103`, `Tgl2104`, `Tgl2106`, `Tgl2107`, `Tgl2110`, `Tgl2112`, `Tgl2131`, `Tgl2132`, `Tgl2133`, `Tgl2134`, `Tgl2135`, `Tgl2136`, `Tgl2137`, `Tgl2138`, `Tgl2139`, `Tgl2139`, `Tgl2140`, `Tgl2141`, `Tgl2163`, `Tgl2191`, `Tgl2192`, `Tgl3204`, `Tgl3205`, `Tgl3206`, `Tgl3207`, `Tgl3217`, `Tgl3231`, `Tgl3232`, `Tgl3233`, `Tgl3234`, `Tgl3235`, `Tgl3236`, `Tgl3237`, `Tgl3238`, `Tgl3239`, `Tgl3240`, `Tgl3261`, `Tgl3262`, `Tgl3263`, `Tgl3264`, `Tgl4301`, `Tgl4302`, `Tgl4303`, `Tgl4304`, `Tgl4306`, `Tgl4309`, `Tgl4310`, `Tgl4312`, `Tgl4313`, `Tgl4331`, `Tgl4332`, `Tgl4362`, `Tgl4391`, `Tgl5401`, `Tgl5402`, `Tgl5403`, `Tgl5468`, `Tgl6501`, `Tgl6531`, `Tgl6532`, `Tgl6533`, `Tgl6534`, `Tgl7631`, `Tgl7632`, `Tgl7633`, `Tgl7634`, `Tgl7635`, `Tgl7636`, `Tgl7637`, `Tgl7691`, `Tgl7692`, `Tgl7693`, `Tgl7694`, `Tgl7740`, `Tgl8731`, `Tgl8732`, `Tgl9831`, `Tgl9832`, `Tgl9833`, `Tgl9834`, `Tgl9835`, `Tgl9836`, `Tgl9837`, `Tgl9838`, `Tgl9891`, `Tgl9892`, `Tgl9893`, `Tgl9894`, `Tgl9940`, `Tgl111031`, `Tgl111032`, `TransListe$gl`.

When calling one of these objects you will be put a question. If the variables of the called summation or transformation are undefined, the question is

```
Do you want to set values for the equation?  [y|n]:
```

Enter `y` if you want to set values, even only for some of them, if you do not need to set values enter `n`. If some of the variables of the called summation or transformation are already defined, you will be asked the question

```
Some variables have a value.  Should the variables
{[V,a,r,i,a,b,l,e,s]} be cleared?  Do you want to set
values for the equation (v)?  [y|n|yv|nv]:
```

Now you have four options depending on if you want to set values or not and if you want to clear the already defined variables or not. For example, if you want to set values but do not want to clear the defined variables, enter `nv`. (Cf. the examples in `Sgl2101`.)

In addition there are the functions and variables

`Add, Div, Gleichung, GlTausche, Hoch, LS, Mal, RS, Sub,`

for manipulating equations. In fact, once you have called one of the objects `Sgl*` or `Tgl*`, the right-hand side of the displayed equation will have been assigned to `RS`, the left-hand side to `LS`, and thus the equation itself to `Gleichung` (cf. the example in `Gleichung`). The functions `Add, Div, GlTausche, Hoch, Mal, Sub`, and also `Ers`, allow you to manipulate the equation.

The file `contig.m`

This file contains a vast number of contiguous relations in form of rules. You do not have to load this file by hand since it is loaded automatically once an object of this file is called. The objects that are defined by `contig.m` are

`C01`, `C02`, `C14`, `C15`, `C16`, `C17`, `C18`, `C19`, `C20`, `C21`, `C22`, `C23`, `C24`, `C25`, `C26`, `C27`, `C30`, `C31`, `C32`, `C33`, `C34`, `C35`, `C36`, `C40`, `C41`, `C42`, `C43`, `C44`, `C45`, `C46`, `C49`, `C50`, `C51`, `C52`, `C53`, `C54`, `C55`, `C56`, `C57`, `C58`, `C59`, `C60`, `C61`, `C62`, `C63`, `C64`, `C65`, `C66`, `C67`, `C68`, `C69`, `C70`, `C71`, `C72`, `C73`, `C74`, `C75`, `C76`, `C77`, `C78`, `C79`, `C80`, `C81`, `C82`, `C83`, `C84`, `C85`, `C86`, `C87`, `C88`, `C89`, `C90`, `C91`, `C92`, `C93`, `C94`, `C95`, `C96`, `C97`, `C98`, `C99`, `C100`, `C101`, `C102`, `C103`, `C104`, `C105`, `C106`, `C107`, `C108`, `C109`, `C110`, `C111`, `C112`, `C113`, `C114`, `C115`, `C116`, `C117`, `C118`, `C119`, `C120`, `C121`, `ContigListe`.

Simultaneous use of HYP and HYPQ

It is possible to load both packages, HYP and HYPQ. In this case, the objects of the package that is loaded last will override those objects of the other package which have identical names. However, you can use the overrided objects by calling them by their *full* names. To determine the full name of an object the following rule applies:

If the object `Object` is defined in the file `File.ext`, then the full name of `Object` is `File‘ext‘Object`.

For instance, if you load `hyp.m` first and then `hyp.q` and want to use `Limes` with an ordinary (i.e. non-basic) hypergeometric expression, then you have to type `Hyp‘m‘Limes` instead of `Limes`. (Calling `Limes` would invoke the *basic* hypergeometric `Limes`.) For information on contexts in MATHEMATICA confer [9].

On-line help

For each object of this package on-line help is supported in the usual way. For instance, quick information about `Limes` (not having the manual at hand) is available in the following way.

```
In[1]:= ?Limes
Description:  Function for doing formal limits of hypergeometric
  expressions.  If required for taking the limit, you will be asked whether
  or not the absolute value of some variable or expression is smaller than
  1.  Your decision is stored for the rest of your MATHEMATICA session.  If
  you want to change your decision later, use "AbsGreater", "AbsSmaller",
  or "AbsUndetermined", respectively.
Warning:  This function uses primitive algebraic techniques to do the
  limit.  There is no check if taking the limit is actually allowed.  So it
  is left to you to check the validity of a result of "Limes".
Usage:  Limes[Expr, x->x0].
See also:  AbsGreater, AbsSmaller, AbsUndetermined, MinusOne.
```

The screen output

The screen output of the examples in this manual imitates the output under usage of code tables 437, 860, 863, or 865 (cf. the `read.me`). The output under other code tables is a little bit less attractive, but similar. For instance, the examples for `GAMMA` then would read as follows.

```
In[1]:= GAMMA[2*a+1]

Out[1]= Ga(1 + 2 a)

In[2]:= GAMMA[{a,b},{c,d}]

          [      ]
          | a, b |
Out[2]= Ga|      |
          | c, d |
          [      ]
```

A brief dictionary

Most of the names of the objects are obviously German influenced. To help those who are not so familiar with German, brief German–English and English–German vocabularies are provided.

A German–English vocabulary

German	English	*Mathematica* objects in HYP containing the word
abspalten	split	`lina1, lina2`
auflösen	dissolve	`paufl`
drucken	print	`Drucke`
einfügen	insert	`FEinf`
ersetzen	replace	`Ers`
erweitern	extend	`erw1, erw2, SUMErw1, SUMErw2`
Gleichung	equation	`Gleichung`
"2 hoch 3"	"2 to the 3"	`Hoch`
"2 mal 3"	"2 times 3"	`Mal`
ordnen	order	`FOrdne`
Regel	rule	`SUMRegeln`
sammeln	collect	`SUMSammle`
schreiben	write	`SchreibeZahl`
tauschen	exchange, interchange	`SUMTausche, GlTausche`
umkehren	reverse	`SUMUmkehr`
Zahl	number	`SchreibeZahl`
zerlegen	split	`zerl1, pzerl, Gzerl, SUMZerl`
zusammen	together	`zus1, zus2, zus3`

An English–German vocabulary

English	German	*Mathematica* objects in HYP containing the word
collect	sammeln	`SUMSammle`
dissolve	auflösen	`paufl`
equation	Gleichung	`Gleichung`
exchange	tauschen	`SUMTausche, GlTausche`
extend	erweitern	`erw1, erw2, SUMErw1, SUMErw2`
insert	einfügen	`FEinf`
interchange	tauschen	`SUMTausche, GlTausche`
number	Zahl	`SchreibeZahl`
order	ordnen	`FOrdne`
print	drucken	`Drucke`
replace	ersetzen	`Ers`
reverse	umkehren	`SUMUmkehr`
rule	Regel	`SUMRegeln`
split	abspalten	`lina1, lina2`
split	zerlegen	`zerl1, pzerl, Gzerl, SUMZerl`
”2 times 3”	”2 mal 3”	`Mal`
”2 to the 3”	”2 hoch 3”	`Hoch`
together	zusammen	`zus1, zus2, zus3`
write	schreiben	`SchreibeZahl`

Alphabetic List of the objects with desriptions

AbsGreater

Description: Function for declaring the absolute value of a variable or expression to be greater than 1. This declaration is used by `Limes`.

Usage: `AbsGreater[Expr].`

See also: `AbsSmaller, AbsUndetermined, Limes.`

AbsSmaller

Description: Function for declaring the absolute value of a variable or expression to be smaller than 1. This declaration is used by `Limes`.

Usage: `AbsSmaller[Expr].`

See also: `AbsGreater, AbsUndetermined, Limes.`

AbsUndetermined

Description: Function for declaring the absolute value of a variable or expression to be neither smaller nor greater than 1. This declaration is used by `Limes`.

Usage: `AbsUndetermined[Expr].`

See also: `AbsGreater, AbsSmaller, Limes.`

Add

Description: Function that adds `Expr` to `Gleichung`.

Usage: `Add[Expr].`

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n

                               (-a + c)
               [ a, -n        ]        n
Out[1]=  F     [       ;  1   ] == ---------
        2 1    [   c          ]      (c)
                                        n

In[2]:= Add[p[c-a,n]]

                                                      (-a + c)
               [ a, -n        ]                               n
Out[2]=  F     [       ;  1   ] + (-a + c)  == (-a + c)  + ---------
        2 1    [   c          ]           n            n     (c)
                                                                n
```

```
In[3]:= Gleichung

                                                 (-a + c)
              [ a, -n        ]                            n
Out[3]=  F    [        ; 1   ] + (-a + c)  == (-a + c)  + ---------
        2 1   [   c          ]           n           n     (c)
                                                              n
```

See also: `Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Div, Sub, Hoch, GlTausche, Ers.`

AmSLaTeX

Description: Switch that changes the output of TeXForm to be usable with $\mathcal{A}\mathcal{M}\mathcal{S}$-LaTeX. By default the output of TeXForm is usable with $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX.

Usage: `AmSLaTeX.`

Example(s):

```
In[1]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[2]:= TeXForm[F[{a,b},{c},z]]

Out[2]//TeXForm=
  {} _{2} F _{1} \!\left [ \matrix { a, b}\\ { c}\endmatrix ; {\displaystyle
   z}\right ]

In[3]:= AmSLaTeX

In[4]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-LaTeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[5]:= TeXForm[F[{a,b},{c},z]]

Out[5]//TeXForm=
  {} _{2} F _{1} \!\left [ \begin{matrix} { a, b}\\ { c}\end{matrix} ;
   {\displaystyle z}\right ]
```

See also: `AmSTeX, LaTeX, TeX, TeXMat, TeXFV.`

AmSTeX

Description: Switch that changes the output of TeXForm to be usable with $\mathcal{AMS}$-TEX. By default the output of TeXForm is usable with $\mathcal{AMS}$-TEX.

Usage: `AmSTeX.`

Example(s):

```
In[1]:= TeX

In[2]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with Plain-TeX and LaTeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[3]:= TeXForm[F[{a,b},{c},z]]

Out[3]//TeXForm=
  {} _{2} F _{1} \!\left [ \matrix { a, b\cr c} ; {\displaystyle z} \right ]

In[4]:= AmSTeX

In[5]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[6]:= TeXForm[F[{a,b},{c},z]]

Out[6]//TeXForm=
  {} _{2} F _{1} \!\left [ \matrix { a, b}\\ { c}\endmatrix ; {\displaystyle
   z}\right ]
```

See also: `AmSLaTeX, LaTeX, TeX, TeXMat, TeXFV.`

---

baszerl

Description: $(a)_n \to m^n \prod_{k=0}^{m-1}((a+k)/m)_{n/m}$,
$\Gamma(a) \to m^{a-1/2}(2\pi)^{(1-m)/2} \prod_{k=0}^{m-1}\Gamma((a+k)/m)$.

The parameter `m` has to be entered on request.

Usage: `Expr/.baszerl.`

Example(s):

```
In[1]:= p[a,n]
```

```
Out[1]= (a)
           n

In[2]:= %/.baszerl
split into ?  terms:  2

         n   a      1 + a
Out[2]= 2  (-)    (-----)
            2 n/2    2   n/2

In[3]:= p[b,4*m]

Out[3]= (b)
           4 m

In[4]:= %/.baszerl
split into ?  terms:  4

         4 m   b     1 + b    2 + b    3 + b
Out[4]= 4    (-)  (-----)  (-----)  (-----)
              4 m    4   m    4   m    4   m

In[5]:= GAMMA[2*c]

Out[5]= Γ(2 c)

In[6]:= %/.baszerl
split into ?  terms:  4

         -(1/2) + 2 c    c    1 + 2 c    2 + 2 c    3 + 2 c
        4             Γ(-) Γ(-------) Γ(-------) Γ(-------)
                        2       4          4          4
Out[6]= ---------------------------------------------------
                                  3/2  3/2
                                 2    π
```

See also: `baszerl`, `baszus`, `Ers`, `PosListe`, `ManipulationsListe`.

---

baszus

Description: $(a)_n \to m^{-mn}(am)_{mn}/\prod_{k=1}^{m-1}(a+k/m)_n$,
$\Gamma(a) \to m^{1/2-am}(2\pi)^{(m-1)/2}\Gamma(am)/\prod_{k=1}^{m-1}\Gamma(a+k/m)$.

The parameter `m` has to be entered on request. This operation is basically the inverse of `baszerl`.

Usage: `Expr/.baszus.`

Example(s):

```
In[1]:= p[a/2,n]*p[(a+1)/2,n]
```

```
         a     1 + a
Out[1]= (-)   (-----)
         2 n     2   n

In[2]:= Ers[%,baszus,{1}]
put together ?  terms:  2

                1 + a
        (a)    (-----)
           2 n    2   n
Out[2]= ---------------
          2 n  1   a
         2    (- + -)
               2   2 n

In[3]:= ExpandAll[%]

        (a)
           2 n
Out[3]= ------
          2 n
         2

In[4]:= GAMMA[(a-1)/2]*GAMMA[a/2]*p[b/3,m]*p[(b+1)/3,m]*p[(b+2)/3,m]

          -1 + a     a    b     1 + b     2 + b
Out[4]= Γ(------) Γ(-)  (-)   (-----)   (-----)
            2        2   3 m     3   m     3   m

In[5]:= Ers[%,baszus,{3}]
put together ?  terms:  3

          -1 + a     a           1 + b     2 + b
        Γ(------) Γ(-)  (b)     (-----)   (-----)
            2        2     3 m     3   m     3   m
Out[5]= ---------------------------------------
                 3 m  1   b     2   b
                3    (- + -)   (- + -)
                      3   3 m  3   3 m

In[6]:= ExpandAll[%]

            1    a    a
        Γ(-(-) + -) Γ(-)  (b)
            2    2    2     3 m
Out[6]= -----------------------
                  3 m
                 3
```

```
In[7]:= PosListe[%]

           -3 m                  1    a               a
Out[7]= {{3     , {{1}}}, {Γ(-(-) + -), {{2}}}, {Γ(-), {{3}}}, {(b)    , {{4}}}}
                                 2    2               2               3 m

In[8]:= Ers[%%,baszus,{2}]
put together ?  terms:  2

         1 - 2 (-(1/2) + a/2)                1    a
        2                     Sqrt[π] Γ(2 (-(-) + -)) (b)
                                             2    2      3 m
Out[8]= ----------------------------------------------------
                                 3 m
                                3

In[9]:= ExpandAll[%]

        4 Sqrt[π] Γ(-1 + a) (b)
                               3 m
Out[9]= --------------------------
                  a  3 m
                 2  3
```

See also: `Ers`, `PosListe`, `ManipulationsListe`.

Binomialp

Description: `Binomialp[n,k]` is the binomial coefficient, written in terms of factorial symbols (Pochhammer symbols) `p`.

Usage: `Binomialp[n,k]`.

Example(s):

```
In[1]:= Binomialp[n,k]

        (1 - k + n)
                   k
Out[1]= ------------
            (1)
               k

In[2]:= Binomialp[6,3]
```

```
         (4)
            3
Out[2]= ----
         (1)
            3
```

See also: `Factorialp, Multinomialp.`

C01

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A)\\(B)\end{matrix};z\right] \longrightarrow 1+z\frac{\prod\limits_{i=1}^{r}A_i}{\prod\limits_{i=1}^{s}B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix}1,(A+1)\\2,(B+1)\end{matrix};z\right] $$

Usage: `Expr/.C01.`

See also: `C64, ContigListe, Ers, PosListe.`

C02

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A),1\\(B)\end{matrix};z\right] \longrightarrow \frac{1}{z}\frac{\prod\limits_{i=1}^{s}(B_i-1)}{\prod\limits_{i=1}^{r}(A_i-1)}\,{}_rF_s\left[\begin{matrix}(A-1),1\\(B-1)\end{matrix};z\right]-\frac{1}{z}\frac{\prod\limits_{i=1}^{s}(B_i-1)}{\prod\limits_{i=1}^{r}(A_i-1)} $$

Usage: `Expr/.C02.`

See also: `C64, ContigListe, Ers, PosListe.`

C14

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}a,(A)\\(B)\end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix}a-1,(A)\\(B)\end{matrix};z\right]+z\frac{\prod\limits_{i=1}^{r-1}A_i}{\prod\limits_{i=1}^{s}B_i}\,{}_rF_s\left[\begin{matrix}a,(A+1)\\(B+1)\end{matrix};z\right] $$

Usage: `Expr/.C14[m1].`
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C15

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1,(A) \\ (B) \end{matrix};z\right] - z\frac{\prod_{i=1}^{r-1} A_i}{\prod_{i=1}^{s} B_i}{}_rF_s\left[\begin{matrix} a+1,(A+1) \\ (B+1) \end{matrix};z\right] $$

Usage: `Expr/.C15[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C16

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};z\right] \longrightarrow \frac{1}{z}\frac{\prod_{i=1}^{s}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}{}_rF_s\left[\begin{matrix} a,(A-1) \\ (B-1) \end{matrix};z\right] - \frac{1}{z}\frac{\prod_{i=1}^{s}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}{}_rF_s\left[\begin{matrix} a-1,(A-1) \\ (B-1) \end{matrix};z\right] $$

Usage: `Expr/.C16[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C17

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a-1,(A) \\ (B) \end{matrix};z\right] + z\frac{\prod_{i=1}^{r-1} A_i}{\prod_{i=1}^{s} B_i}{}_rF_s\left[\begin{matrix} a,(A+1) \\ (B+1) \end{matrix};z\right] $$

Usage: `Expr/.C17[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C18

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1,(A) \\ (B) \end{matrix};z\right] - z\frac{\prod_{i=1}^{r-1} A_i}{\prod_{i=1}^{s} B_i}{}_rF_s\left[\begin{matrix} a+1,(A+1) \\ (B+1) \end{matrix};z\right] $$

Usage: `Expr/.C18[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C19

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a,(A) \\ (B) \end{matrix};z\right] \longrightarrow \frac{1}{z}\frac{\prod_{i=1}^{s}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}\,{}_rF_s\left[\begin{matrix} a,(A-1) \\ (B-1) \end{matrix};z\right] - \frac{1}{z}\frac{\prod_{i=1}^{s}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}\,{}_rF_s\left[\begin{matrix} a-1,(A-1) \\ (B-1) \end{matrix};z\right]
$$

Usage: `Expr/.C19[m1]`.
`m1` is the position of the special upper parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C20

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} (A) \\ b,(B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} (A) \\ b+1,(B) \end{matrix};z\right] + \frac{z}{b\,(1+b)}\frac{\prod_{i=1}^{r}A_i}{\prod_{i=1}^{s-1}B_i}\,{}_rF_s\left[\begin{matrix} (A+1) \\ b+2,(B+1) \end{matrix};z\right]
$$

Usage: `Expr/.C20[n1]`.
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C21

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} (A) \\ b,(B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} (A) \\ b-1,(B) \end{matrix};z\right] - \frac{z}{(b-1)\,b}\frac{\prod_{i=1}^{r}A_i}{\prod_{i=1}^{s-1}B_i}\,{}_rF_s\left[\begin{matrix} (A+1) \\ b+1,(B+1) \end{matrix};z\right]
$$

Usage: `Expr/.C21[n1]`.
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C22

Description: Contiguous relation in form of a rule.

$$
\begin{aligned}
{}_rF_s\left[\begin{matrix} (A) \\ b,(B) \end{matrix};z\right] \longrightarrow {} & \frac{(b-2)\,(b-1)}{z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r}(A_i-1)}\,{}_rF_s\left[\begin{matrix} (A-1) \\ b-2,(B-1) \end{matrix};z\right] \\
& - \frac{(b-2)\,(b-1)}{z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r}(A_i-1)}\,{}_rF_s\left[\begin{matrix} (A-1) \\ b-1,(B-1) \end{matrix};z\right]
\end{aligned}
$$

Usage: `Expr/.C22[n1]`.
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C23

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A)\\ b,(B)\end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix}(A)\\ b+1,(B)\end{matrix};z\right] + \frac{z}{b\,(1+b)}\frac{\prod_{i=1}^{r} A_i}{\prod_{i=1}^{s-1} B_i}\,{}_rF_s\left[\begin{matrix}(A+1)\\ b+2,(B+1)\end{matrix};z\right] $$

Usage: `Expr/.C23[n1]`.
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C24

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A)\\ b,(B)\end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix}(A)\\ b-1,(B)\end{matrix};z\right] - \frac{z}{(b-1)\,b}\frac{\prod_{i=1}^{r} A_i}{\prod_{i=1}^{s-1} B_i}\,{}_rF_s\left[\begin{matrix}(A+1)\\ b+1,(B+1)\end{matrix};z\right] $$

Usage: `Expr/.C24[n1]`.
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C25

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A)\\ b,(B)\end{matrix};z\right] \longrightarrow \frac{(b-2)\,(b-1)}{z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r}(A_i-1)}\,{}_rF_s\left[\begin{matrix}(A-1)\\ b-2,(B-1)\end{matrix};z\right] - \frac{(b-2)\,(b-1)}{z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r}(A_i-1)}\,{}_rF_s\left[\begin{matrix}(A-1)\\ b-1,(B-1)\end{matrix};z\right] $$

Usage: `Expr/.C25[n1]`.
`n1` is the position of the special lower parameter.

See also: `C64, ContigListe, Ers, PosListe.`

C26

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}a,b,(A)\\ (B)\end{matrix};z\right] \longrightarrow \frac{b}{b-a}\,{}_rF_s\left[\begin{matrix}a,b+1,(A)\\ (B)\end{matrix};z\right] + \frac{a}{a-b}\,{}_rF_s\left[\begin{matrix}a+1,b,(A)\\ (B)\end{matrix};z\right] $$

Usage: `Expr/.C26[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C27

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow \frac{(a-b-1)}{a-1}\,{}_rF_s\left[\begin{matrix} a-1, b, (A) \\ (B) \end{matrix}; z\right] + \frac{b}{a-1}\,{}_rF_s\left[\begin{matrix} a-1, b+1, (A) \\ (B) \end{matrix}; z\right] $$

Usage: `Expr/.C27[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C30

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a-1, b+1, (A) \\ (B) \end{matrix}; z\right] + (1-a+b)\, z\, \frac{\prod_{i=1}^{r-2} A_i}{\prod_{i=1}^{s} B_i}\,{}_rF_s\left[\begin{matrix} a, b+1, (A+1) \\ (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C30[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C31

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow \frac{1}{(b-a)\, z}\, \frac{\prod_{i=1}^{s} (B_i-1)}{\prod_{i=1}^{r-2} (A_i-1)}\,{}_rF_s\left[\begin{matrix} a, b-1, (A-1) \\ (B-1) \end{matrix}; z\right] - \frac{1}{(b-a)\, z}\, \frac{\prod_{i=1}^{s} (B_i-1)}{\prod_{i=1}^{r-2} (A_i-1)}\,{}_rF_s\left[\begin{matrix} a-1, b, (A-1) \\ (B-1) \end{matrix}; z\right] $$

Usage: `Expr/.C31[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C32

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a-1, b-1, (A) \\ (B) \end{matrix}; z\right] + (a+b-1)\, z\, \frac{\prod_{i=1}^{r-2} A_i}{\prod_{i=1}^{s} B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix} a, b, a+b, (A+1) \\ -1+a+b, (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C32[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C33

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1, b+1, (A) \\ (B) \end{matrix}; z\right] - (1+a+b)\, z \frac{\prod_{i=1}^{r-2} A_i}{\prod_{i=1}^{s} B_i} {}_{r+1}F_{s+1}\left[\begin{matrix} a+1, b+1, a+b+2, (A+1) \\ a+b+1, (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C33[m1,m2]`.
`m1`, `m2` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C34

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ b, (B) \end{matrix}; z\right] \longrightarrow \frac{(b-1)}{b-1-a} {}_rF_s\left[\begin{matrix} a, (A) \\ b-1, (B) \end{matrix}; z\right] + \frac{a}{1+a-b} {}_rF_s\left[\begin{matrix} a+1, (A) \\ b, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C34[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C35

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ b, (B) \end{matrix}; z\right] \longrightarrow \frac{(b-1)}{a-1} {}_rF_s\left[\begin{matrix} a-1, (A) \\ b-1, (B) \end{matrix}; z\right] + \frac{(a-b)}{a-1} {}_rF_s\left[\begin{matrix} a-1, (A) \\ b, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C35[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C36

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ b, (B) \end{matrix}; z\right] \longrightarrow \frac{(b-a)}{b} {}_rF_s\left[\begin{matrix} a, (A) \\ 1+b, (B) \end{matrix}; z\right] + \frac{a}{b} {}_rF_s\left[\begin{matrix} a+1, (A) \\ b+1, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C36[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C40

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ b, (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a-1, (A) \\ b-1, (B) \end{matrix}; z\right] + \frac{(b-a)\, z}{(b-1)\, b} \frac{\prod_{i=1}^{r-1} A_i}{\prod_{i=1}^{s-1} B_i} {}_rF_s\left[\begin{matrix} a, (A+1) \\ b+1, (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C40[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C41

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1,(A) \\ b+1,(B) \end{matrix};z\right] - \frac{(b-a)\,z}{b\,(1+b)}\frac{\prod_{i=1}^{r-1} A_i}{\prod_{i=1}^{s-1} B_i}\,{}_rF_s\left[\begin{matrix} a+1,(A+1) \\ b+2,(B+1) \end{matrix};z\right] $$

Usage: `Expr/.C41[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C42

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};z\right] \longrightarrow \frac{(b-2)\,(b-1)}{(b-a-1)\,z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}\,{}_rF_s\left[\begin{matrix} a,(A-1) \\ b-1,(B-1) \end{matrix};z\right] - \frac{(b-2)\,(b-1)}{(b-a-1)\,z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}\,{}_rF_s\left[\begin{matrix} a-1,(A-1) \\ b-2,(B-1) \end{matrix};z\right] $$

Usage: `Expr/.C42[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C43

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a-1,(A) \\ b+1,(B) \end{matrix};z\right] + \frac{(a+b)\,z}{b\,(1+b)}\frac{\prod_{i=1}^{r-1} A_i}{\prod_{i=1}^{s-1} B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix} a,1+a+b,(A+1) \\ b+2,a+b,(B+1) \end{matrix};z\right] $$

Usage: `Expr/.C43[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C44

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ b,(B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1,(A) \\ b-1,(B) \end{matrix};z\right] - \frac{(a+b)\,z}{(b-1)\,b}\frac{\prod_{i=1}^{r-1} A_i}{\prod_{i=1}^{s-1} B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+a,a+b+1,(A+1) \\ b+1,a+b,(B+1) \end{matrix};z\right] $$

Usage: `Expr/.C44[m1,n1]`.
`m1` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C45

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};z\right] \longrightarrow \frac{(a-1)}{a-b}\,{}_rF_s\left[\begin{matrix}(A)\\ -1+a,b,(B)\end{matrix};z\right] + \frac{(b-1)}{b-a}\,{}_rF_s\left[\begin{matrix}(A)\\ a,-1+b,(B)\end{matrix};z\right] $$

Usage: `Expr/.C45[n1,n2]`.
`n1`, `n2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C46

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};z\right] \longrightarrow \frac{(b-1)}{a}\,{}_rF_s\left[\begin{matrix}(A)\\ a+1,b-1,(B)\end{matrix};z\right] + \frac{(1+a-b)}{a}\,{}_rF_s\left[\begin{matrix}(A)\\ a+1,b,(B)\end{matrix};z\right] $$

Usage: `Expr/.C46[n1,n2]`.
`n1`, `n2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C49

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix}(A)\\ a+1,b-1,(B)\end{matrix};z\right] + \frac{(b-a-1)\,z}{a\,(1+a)\,(b-1)\,b}\,\frac{\prod_{i=1}^{r} A_i}{\prod_{i=1}^{s-2} B_i}\,{}_rF_s\left[\begin{matrix}(A+1)\\ a+2,b+1,(B+1)\end{matrix};z\right] $$

Usage: `Expr/.C49[n1,n2]`.
`n1`, `n2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C50

Description: Contiguous relation in form of a rule.

$$ \begin{aligned} {}_rF_s\left[\begin{matrix}(A)\\ a,b,(B)\end{matrix};z\right] \longrightarrow{}& \frac{(a-2)\,(a-1)\,(b-2)\,(b-1)}{(b-a)\,z}\,\frac{\prod_{i=1}^{s-2}(B_i-1)}{\prod_{i=1}^{r}(A_i-1)}\,{}_rF_s\left[\begin{matrix}(A-1)\\ -2+a,b-1,(B-1)\end{matrix};z\right] \\ & - \frac{(a-2)\,(a-1)\,(b-2)\,(b-1)}{(b-a)\,z}\,\frac{\prod_{i=1}^{s-2}(B_i-1)}{\prod_{i=1}^{r}(A_i-1)}\,{}_rF_s\left[\begin{matrix}(A-1)\\ a-1,b-2,(B-1)\end{matrix};z\right] \end{aligned} $$

Usage: `Expr/.C50[n1,n2]`.
`n1`, `n2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C51

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}(A)\\a,b,(B)\end{matrix};z\right]\longrightarrow {}_rF_s\left[\begin{matrix}(A)\\a+1,b+1,(B)\end{matrix};z\right]+\frac{(1+a+b)\,z}{a\,(1+a)\,b\,(1+b)}\frac{\prod\limits_{i=1}^{r}A_i}{\prod\limits_{i=1}^{s-2}B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix}a+b+2,(A+1)\\a+2,b+2,a+b+1,(B+1)\end{matrix};z\right]
$$

Usage: `Expr/.C51[n1,n2]`.
`n1`, `n2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C52

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}(A)\\a,b,(B)\end{matrix};z\right]\longrightarrow {}_rF_s\left[\begin{matrix}(A)\\a-1,b-1,(B)\end{matrix};z\right]
-\frac{(a+b-1)\,z}{(a-1)\,a\,(b-1)\,b}\frac{\prod\limits_{i=1}^{r}A_i}{\prod\limits_{i=1}^{s-2}B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix}a+b,(A+1)\\a+1,1+b,b-a+1,(B+1)\end{matrix};z\right]
$$

Usage: `Expr/.C52[n1,n2]`.
`n1`, `n2` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C53

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a,b,c,(A)\\(B)\end{matrix};z\right]\longrightarrow\frac{b\,(c-a-1)}{(b-a)\,(c-1)}\,{}_rF_s\left[\begin{matrix}a,b+1,c-1,(A)\\(B)\end{matrix};z\right]+\frac{a\,(c-b-1)}{(a-b)\,(c-1)}\,{}_rF_s\left[\begin{matrix}a+1,b,c-1,(A)\\(B)\end{matrix};z\right]
$$

Usage: `Expr/.C53[m1,m2,m3]`.
`m1`, `m2`, `m3` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C54

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a,b,(A)\\c,(B)\end{matrix};z\right]\longrightarrow\frac{b\,(c-a)}{(b-a)\,c}\,{}_rF_s\left[\begin{matrix}a,b+1,(A)\\c+1,(B)\end{matrix};z\right]+\frac{a\,(c-b)}{(a-b)\,c}\,{}_rF_s\left[\begin{matrix}a+1,b,(A)\\c+1,(B)\end{matrix};z\right]
$$

Usage: `Expr/.C54[m1,m2,n1]`.
`m1`, `m2` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C55

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,b,(A) \\ c,(B) \end{matrix};z\right] \longrightarrow \frac{(1-a+b)\,(c-1)}{(a-1)\,(1+b-c)}\,{}_rF_s\left[\begin{matrix} a-1,b,(A) \\ c-1,(B) \end{matrix};z\right] + \frac{b\,(c-a)}{(a-1)\,(c-b-1)}\,{}_rF_s\left[\begin{matrix} a-1,b+1,(A) \\ c,(B) \end{matrix};z\right] $$

Usage: `Expr/.C55[m1,m2,n1]`.
`m1`, `m2` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C56

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ b,c,(B) \end{matrix};z\right] \longrightarrow \frac{(b-1)\,(c-a)}{(b-a-1)\,c}\,{}_rF_s\left[\begin{matrix} a,(A) \\ b-1,c+1,(B) \end{matrix};z\right] + \frac{a\,(1-b+c)}{(1+a-b)\,c}\,{}_rF_s\left[\begin{matrix} a+1,(A) \\ b,c+1,(B) \end{matrix};z\right] $$

Usage: `Expr/.C56[m1,n1,n2]`.
`m1` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C57

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,(A) \\ b,c,(B) \end{matrix};z\right] \longrightarrow \frac{(b-1)\,(c-a)}{(a-1)\,(c-b)}\,{}_rF_s\left[\begin{matrix} a-1,(A) \\ b-1,c,(B) \end{matrix};z\right] + \frac{(b-a)\,(c-1)}{(a-1)\,(b-c)}\,{}_rF_s\left[\begin{matrix} a-1,(A) \\ b,c-1,(B) \end{matrix};z\right] $$

Usage: `Expr/.C57[m1,n1,n2]`.
`m1` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C58

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} (A) \\ a,b,c,(B) \end{matrix};z\right] \longrightarrow \frac{(a-1)\,(1-b+c)}{(a-b)\,c}\,{}_rF_s\left[\begin{matrix} (A) \\ a-1,b,c+1,(B) \end{matrix};z\right] + \frac{(b-1)\,(1-a+c)}{(b-a)\,c}\,{}_rF_s\left[\begin{matrix} (A) \\ a,b-1,c+1,(B) \end{matrix};z\right] $$

Usage: `Expr/.C58[n1,n2,n3]`.
`n1`, `n2`, `n3` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C59

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, 2-a+c, b, c-b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow \frac{(a-b-1)\,(1-a-b+c)}{(a-1)\,(1-a+c)}\,{}_rF_s\left[\begin{matrix} a-1, 1-a+c, b, c-b, (A) \\ (B) \end{matrix}; z\right] + \frac{b\,(c-b)}{(a-1)\,(1-a+c)}\,{}_rF_s\left[\begin{matrix} a-1, 1-a+c, b+1, 1-b+c, (A) \\ (B) \end{matrix}; z\right] $$

Usage: `Expr/.C59[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C60

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, c-a, b, c-b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)}{(a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix} a+1, 1-a+c, b, c-b, (A) \\ (B) \end{matrix}; z\right] - \frac{b\,(c-b)}{(a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix} a, c-a, b+1, 1-b+c, (A) \\ (B) \end{matrix}; z\right] $$

Usage: `Expr/.C60[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C61

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, c-a, b, c-b, (A) \\ (B) \end{matrix}; z\right] \longrightarrow \frac{ab}{(c-a-1)\,(c-b-1)}\,{}_rF_s\left[\begin{matrix} a+1, c-a-1, b+1, -1-b+c, (A) \\ (B) \end{matrix}; z\right] + \frac{(c-1)\,(-1-a-b+c)}{(c-a-1)\,(c-b-1)}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+\frac{c-1}{2}, a, -1-a+c, b, c-b-1, (A) \\ \frac{c-1}{2}, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C61[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C62

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, c-a, b, 2-b+c, (A) \\ (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1, 1-a+c, b-1, 1+c-b, (A) \\ (B) \end{matrix}; z\right] + (a-b+1)\,(1+c)\,(c-a-b+1)\,z\,\frac{\prod_{i=1}^{r-4} A_i}{\prod_{i=1}^{s} B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix} a+1, 1-a+c, b, 2-b+c, c+2, (A+1) \\ c+1, (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C62[m1,m2,m3,m4]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C63

Description: Contiguous relation in form of a rule.

$$
\begin{aligned}
&{}_rF_s\left[\begin{matrix} a, 2-a+c, b, c-b, (A) \\ (B) \end{matrix}; z\right] \\
&\qquad \longrightarrow \frac{b\,(c-b)\,(a-d-1)\,(1-a+c-d)}{(a-1)\,(1-a+c)\,(b-d)\,(c-b-d)}\,{}_rF_s\left[\begin{matrix} a-1, 1-a+c, b+1, 1-b+c, (A) \\ (B) \end{matrix}; z\right] \\
&\qquad + \frac{(1-a+b)\,(1-a-b+c)\,(c-d)\,d}{(a-1)\,(1-a+c)\,(b-d)\,(c-b-d)}\,{}_{r+2}F_{s+2}\left[\begin{matrix} a-1, 1-a+c, b, -b+c, d+1, 1+c-d, (A) \\ d, c-d, (B) \end{matrix}; z\right]
\end{aligned}
$$

Usage: `Expr/.C63[m1,m2,m3,m4,`d`]`.
`m1`, `m2`, `m3`, `m4` are the positions of the special upper parameters, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C64

Description: Contiguous relation in form of a rule.

$$
\begin{aligned}
{}_rF_s\left[\begin{matrix} a, 1-a+c, b, (A) \\ c-b, (B) \end{matrix}; z\right] \longrightarrow &\frac{(a-b-1)\,(c-a-b)}{(a-1)\,(c-a)}\,{}_rF_s\left[\begin{matrix} a-1, c-a, b, (A) \\ c-b, (B) \end{matrix}; z\right] \\
&+ \frac{b\,(c-b-1)}{(a-1)\,(c-a)}\,{}_rF_s\left[\begin{matrix} a-1, c-a, b+1, (A) \\ -1-b+c, (B) \end{matrix}; z\right]
\end{aligned}
$$

Usage: `Expr/.C64[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

Example(s):

```
In[1]:= F[{A,B,1+C,D},{E,A+C-D,F},z]
```

```
              A, B, 1 + C, D
Out[1]=  F                     ; z
        4 3   E, A + C - D, F
```

```
In[2]:= %/.C64[3,1,4,2]
```

```
             [ C, -1 + A, D, B          ]
            F[                          ] (-1 + A - D)  (C - D)
             [                    ; z   ]
          4 3[ A + C - D, E, F          ]             1        1
Out[2]= ---------------------------------------------- +
                        (-1 + A)  (C)
                                1    1

         [ C, -1 + A, 1 + D, B            ]
        F[                                ] (-1 + A + C - D)  (D)
         [                        ; z     ]
      4 3[ -1 + A + C - D, E, F           ]                 1    1
>       ------------------------------------------------------
                         (-1 + A)  (C)
                                 1    1

In[3]:= %/.paufl

                                    [ C, -1 + A, D, B       ]
         (-1 + A - D) (C - D)  F    [                 ; z   ]
                             4 3    [ A + C - D, E, F       ]
Out[3]= ---------------------------------------------- +
                          (-1 + A) C

                                 [ C, -1 + A, 1 + D, B      ]
       (-1 + A + C - D) D  F     [                    ; z   ]
                         4 3     [ -1 + A + C - D, E, F     ]
>       ------------------------------------------------
                          (-1 + A) C
```

The third, first, and fourth upper parameters in `Out[1]` are `1+C`, `A`, and `D`, respectively, the second lower parameter in `Out[1]` is `A+C-D`. Hence `C64` can be applied with the replacements $a \to$ `1+C`, $b \to$ `D` and $c \to$ `A+C`.

See also: `ContigListe, Ers, PosListe.`

C65

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, c-a, b, (A) \\ 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)}{(b-1)\,(1-b+c)}\,{}_rF_s\left[\begin{matrix} a+1, 1-a+c, b-1, (A) \\ 2-b+c, (B) \end{matrix}; z\right] - \frac{(1+a-b)\,(1-a-b+c)}{(b-1)\,(1-b+c)}\,{}_rF_s\left[\begin{matrix} a, c-a, b-1, (A) \\ 2-b+c, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C65[m1,m2,m3,n1].`

m1, m2, m3 and n1 are the positions of the special upper and lower parameters, respectively.

See also: C64, ContigListe, Ers, PosListe.

C66

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, c-a, b, (A) \\ 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)}{(a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix} a+1, 1-a+c, b, (A) \\ 1-b+c, (B) \end{matrix}; z\right] - \frac{b\,(c-b)}{(a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix} a, c-a, b+1, (A) \\ c-b, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C66[m1,m2,m3,n1]`.
m1, m2, m3 and n1 are the positions of the special upper and lower parameters, respectively.

See also: C64, ContigListe, Ers, PosListe.

C67

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, 1-a+c, b, (A) \\ c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{ab}{(c-a)\,(c-b)}\,{}_rF_s\left[\begin{matrix} a+1, c-a, b+1, (A) \\ 1-b+c, (B) \end{matrix}; z\right] + \frac{c\,(c-a-b)}{(c-a)\,(c-b)}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+\frac{c}{2}, a, c-a, b, (A) \\ \frac{c}{2}, 1-b+c, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C67[m1,m2,m3,n1]`.
m1, m2, m3 and n1 are the positions of the special upper and lower parameters, respectively.

See also: C64, ContigListe, Ers, PosListe.

C68

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, c-a, b, (A) \\ 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{(c-a)\,(c-b)}{(a-1)\,(b-1)}\,{}_rF_s\left[\begin{matrix} a-1, 1-a+c, b-1, (A) \\ c-b, (B) \end{matrix}; z\right] - \frac{(c-1)\,(1-a-b+c)}{(a-1)\,(b-1)}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+\frac{c-1}{2}, -1+a, c-a, b-1, (A) \\ \frac{c-1}{2}, 1-b+c, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C68[m1,m2,m3,n1]`.
m1, m2, m3 and n1 are the positions of the special upper and lower parameters, respectively.

See also: C64, ContigListe, Ers, PosListe.

C69

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, c-a, b, (A) \\ 1-b+c, (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1, 1-a+c, b-1, (A) \\ 2-b+c, (B) \end{matrix}; z\right] + \frac{(1+a-b)\,(1+c)\,(1-a-b+c)\,z}{(1-b+c)\,(2-b+c)} \frac{\prod_{i=1}^{r-3} A_i}{\prod_{i=1}^{s-1} B_i} {}_{r+1}F_{s+1}\left[\begin{matrix} a+1, 1-a+c, b, c+2, (A+1) \\ 3-b+c, c+1, (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C69[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C70

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, 1-a+c, b, (A) \\ c-b, (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a-1, c-a, b+1, (A) \\ -1-b+c, (B) \end{matrix}; z\right] - \frac{(a-b-1)\,c\,(c-a-b)\,z}{(c-b-1)\,(c-b)} \frac{\prod_{i=1}^{r-3} A_i}{\prod_{i=1}^{s-1} B_i} {}_{r+1}F_{s+1}\left[\begin{matrix} a, 1-a+c, b+1, c+1, (A+1) \\ 1-b+c, c, (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C70[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C71

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, 1-a+c, b, (A) \\ c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{b\,(c-b-1)\,(a-d-1)\,(c-a-d)}{(a-1)\,(c-a)\,(b-d)\,(c-b-1-d)} {}_rF_s\left[\begin{matrix} a-1, c-a, b+1, (A) \\ -1-b+c, (B) \end{matrix}; z\right] + \frac{(1-a+b)\,(c-a-b)\,(c-d-1)\,d}{(a-1)\,(c-a)\,(b-d)\,(c-b-1-d)} {}_{r+2}F_{s+2}\left[\begin{matrix} a-1, c-a, b, d+1, c-d, (A) \\ c-b, d, c-d-1, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C71[m1,m2,m3,n1,`d`]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C72

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, c-a, b, (A) \\ 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)\,(b-d-1)\,(1-b+c-d)}{(b-1)\,(1-b+c)\,(a-d)\,(c-a-d)}\,{}_rF_s\left[\begin{matrix} a+1, 1-a+c, b-1, (A) \\ 2-b+c, (B) \end{matrix}; z\right]
$$
$$
-\frac{(b-a-1)\,(1-a-b+c)\,(c-d)\,d}{(b-1)\,(1-b+c)\,(a-d)\,(c-a-d)}\,{}_{r+2}F_{s+2}\left[\begin{matrix} a, c-a, b-1, d+1, 1+c-d, (A) \\ 2-b+c, d, c-d, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C72[m1,m2,m3,n1,`d`]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C73

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, b, a+b, (A) \\ b-a+1, (B) \end{matrix}; z\right] \longrightarrow \frac{1}{(a+b-1)\,z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r-3}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, b, (A-1) \\ (B-1) \end{matrix}; z\right]
$$
$$
-\frac{1}{(a+b-1)\,z}\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r-3}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a-1, b-1, (A-1) \\ (B-1) \end{matrix}; z\right]
$$

Usage: `Expr/.C73[m1,m2,m3,n1]`.
`m1`, `m2`, `m3` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C74

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, b, (A) \\ c-a, c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{(a-b-1)\,(c-a-b)}{(a-1)\,(c-a)}\,{}_rF_s\left[\begin{matrix} a-1, b, (A) \\ 1-a+c, -b+c, (B) \end{matrix}; z\right]
$$
$$
+\frac{b\,(c-b-1)}{(a-1)\,(c-a)}\,{}_rF_s\left[\begin{matrix} a-1, b+1, (A) \\ 1-a+c, c-b-1, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C74[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C75

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ c-a, c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a-1)}{(a-b)\,(-1-a-b+c)}\,{}_rF_s\left[\begin{matrix} a+1, b, (A) \\ c-a-1, -b+c, (B) \end{matrix}; z\right] - \frac{b\,(c-b-1)}{(a-b)\,(-1-a-b+c)}\,{}_rF_s\left[\begin{matrix} a, b+1, (A) \\ c-a, -1-b+c, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C75[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C76

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ c-a, c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{ab}{(c-a)\,(c-b)}\,{}_rF_s\left[\begin{matrix} a+1, b+1, (A) \\ 1-a+c, 1-b+c, (B) \end{matrix}; z\right] + \frac{c\,(c-a-b)}{(c-a)\,(c-b)}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+\frac{c}{2}, a, b, (A) \\ \frac{c}{2}, 1-a+c, 1-b+c, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C76[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C77

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ c-a, c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{(c-a-1)\,(c-b-1)}{(a-1)\,(b-1)}\,{}_rF_s\left[\begin{matrix} a-1, b-1, (A) \\ c-a-1, c-b-1, (B) \end{matrix}; z\right] - \frac{(c-2)\,(c-a-b)}{(a-1)\,(b-1)}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+\frac{c-2}{2}, -1+a, b-1, (A) \\ \frac{c-2}{2}, c-a, c-b, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C77[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C78

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, b, (A) \\ c-a, c-b, (B) \end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1, b-1, (A) \\ c-a-1, 1-b+c, (B) \end{matrix}; z\right] + \frac{(1+a-b)\,c\,(c-a-b)\,z}{(c-a-1)\,(c-a)\,(c-b)\,(1-b+c)}\,\frac{\prod_{i=1}^{r-2} A_i}{\prod_{i=1}^{s-2} B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix} a+1, b, c+1, (A+1) \\ 1-a+c, 2-b+c, c, (B+1) \end{matrix}; z\right] $$

Usage: `Expr/.C78[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C79

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,b,(A) \\ c-a,c-b,(B) \end{matrix};z\right] \longrightarrow \frac{b\,(c-b-1)\,(a-d-1)\,(c-a-d)}{(a-1)\,(c-a)\,(b-d)\,(c-b-1-d)}\,{}_rF_s\left[\begin{matrix} a-1,b+1,(A) \\ 1-a+c,c-b-1,(B) \end{matrix};z\right] + \frac{(1-a+b)\,(c-a-b)\,(c-d-1)\,d}{(a-1)\,(c-a)\,(b-d)\,(c-b-1-d)}\,{}_{r+2}F_{s+2}\left[\begin{matrix} a-1,b,d+1,c-d,(A) \\ 1-a+c,c-b,d,c-d-1,(B) \end{matrix};z\right] $$

Usage: `Expr/.C79[m1,m2,n1,n2,`d`]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

Example(s):

```
In[1]:= F[{A,B,B+F-E,D},{E,F},z]

             A, B, B - E + F, D
Out[1]=  F                          ; z
        4 2         E, F

In[2]:= %/.C79[2,3,2,1,x]

Out[2]= ((-1 + E) (B - E + F) (-1 + B - x) (F - x)

              -1 + B, 1 + B - E + F, A, D
>         F                                  ; z ) /
         4 2         1 + F, -1 + E

>      ((-1 + B) F (-1 + E - x) (B - E + F - x)) +

>     ((-B + E) (1 - E + F) (-1 + B + F - x) x

              -1 + B, B - E + F, 1 + x, B + F - x, A, D
>         F                                              ; z ) /
         6 4         1 + F, E, x, -1 + B + F - x

>      ((-1 + B) F (-1 + E - x) (B - E + F - x))
```

The second and third upper parameters in `Out[1]` are `B`, and `B+F-E`, respectively, the second and first lower parameters in `Out[1]` are `F` and `E`, respectively. Hence `C79` can be applied with the replacements $a \to$ `B`, $b \to$ `B+F-E` and $c \to$ `B+F`. The `x` replaces d in the right-hand side expression.

See also: `C79, ContigListe, Ers, PosListe.`

C80

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b, c-b, (B)\end{matrix}; z\right] \longrightarrow \frac{(b-1)\,(c-b-1)}{(a-1)\,(c-a-1)}\,{}_rF_s\left[\begin{matrix}a-1, c-a-1, (A)\\ b-1, c-b-1, (B)\end{matrix}; z\right] + \frac{(a-b)\,(c-a-b)}{(a-1)\,(c-a-1)}\,{}_rF_s\left[\begin{matrix}a-1, c-a-1, (A)\\ b, -b+c, (B)\end{matrix}; z\right] $$

Usage: `Expr/.C80[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C81

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b, c-b, (B)\end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)}{b\,(c-b)}\,{}_rF_s\left[\begin{matrix}a+1, 1-a+c, (A)\\ b+1, 1-b+c, (B)\end{matrix}; z\right] - \frac{(a-b)\,(c-a-b)}{b\,(c-b)}\,{}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b+1, 1-b+c, (B)\end{matrix}; z\right] $$

Usage: `Expr/.C81[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C82

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b, 2-b+c, (B)\end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)}{(1+a-b)\,(1-a-b+c)}\,{}_rF_s\left[\begin{matrix}a+1, 1-a+c, (A)\\ b, 2-b+c, (B)\end{matrix}; z\right] - \frac{(b-1)\,(1-b+c)}{(1+a-b)\,(1-a-b+c)}\,{}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b-1, 1-b+c, (B)\end{matrix}; z\right] $$

Usage: `Expr/.C82[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C83

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b, c-b, (B)\end{matrix}; z\right] \longrightarrow \frac{a\,(b-1)}{(c-a-1)\,(c-b)}\,{}_rF_s\left[\begin{matrix}a+1, -1-a+c, (A)\\ b-1, 1-b+c, (B)\end{matrix}; z\right] + \frac{(c-1)\,(c-a-b)}{(c-a-1)\,(c-b)}\,{}_{r+1}F_{s+1}\left[\begin{matrix}1+\frac{c-1}{2}, a, -1-a+c, (A)\\ \frac{c-1}{2}, b, 1-b+c, (B)\end{matrix}; z\right] $$

Usage: `Expr/.C83[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C84

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b, c-b, (B)\end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix}a+1, 1-a+c, (A)\\ b+1, 1-b+c, (B)\end{matrix}; z\right]
$$
$$
+\frac{(a-b)\,(1+c)\,(c-a-b)\,z}{b\,(1+b)\,(c-b)\,(1-b+c)}\frac{\prod_{i=1}^{r-2} A_i}{\prod_{i=1}^{s-2} B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix}a+1, 1-a+c, c+2, (A+1)\\ b+2, 2-b+c, c+1, (B+1)\end{matrix}; z\right]
$$

Usage: `Expr/.C84[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C85

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b, c-b, (B)\end{matrix}; z\right] \longrightarrow {}_rF_s\left[\begin{matrix}a-1, c-a-1, (A)\\ b-1, -1-b+c, (B)\end{matrix}; z\right]
$$
$$
-\frac{(a-b)\,(c-1)\,(c-a-b)\,z}{(b-1)\,b\,(c-b-1)\,(c-b)}\frac{\prod_{i=1}^{r-2} A_i}{\prod_{i=1}^{s-2} B_i}\,{}_{r+1}F_{s+1}\left[\begin{matrix}a, c-a, c, (A+1)\\ 1+b, 1-b+c, c-1, (B+1)\end{matrix}; z\right]
$$

Usage: `Expr/.C85[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C86

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a, c-a, (A)\\ b, c-b, (B)\end{matrix}; z\right] \longrightarrow \frac{(b-1)\,(c-b-1)\,(a-d-1)\,(c-a-1-d)}{(a-1)\,(c-a-1)\,(b-d-1)\,(c-b-1-d)}\,{}_rF_s\left[\begin{matrix}a-1, c-a-1, (A)\\ b-1, c-b-1, (B)\end{matrix}; z\right]
$$
$$
+\frac{(b-a)\,(c-a-b)\,(c-d-2)\,d}{(a-1)\,(c-a-1)\,(b-d-1)\,(c-b-1-d)}\,{}_{r+2}F_{s+2}\left[\begin{matrix}a-1, c-a-1, d+1, c-d-1, (A)\\ b, c-b, d, c-d-2, (B)\end{matrix}; z\right]
$$

Usage: `Expr/.C86[m1,m2,n1,n2,`d`]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C87

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, c-a, (A) \\ b, c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)\,(b-d)\,(c-b-d)}{b\,(c-b)\,(a-d)\,(c-a-d)}\,{}_rF_s\left[\begin{matrix} a+1, 1-a+c, (A) \\ b+1, 1-b+c, (B) \end{matrix}; z\right]
$$
$$
- \frac{(b-a)\,(c-a-b)\,(c-d)\,d}{b\,(c-b)\,(a-d)\,(c-a-d)}\,{}_{r+2}F_{s+2}\left[\begin{matrix} a, c-a, d+1, 1+c-d, (A) \\ b+1, 1-b+c, d, c-d, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C87[m1,m2,n1,n2,`d`]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C88

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, a+b, (A) \\ b+1, a+b-1, (B) \end{matrix}; z\right] \longrightarrow \frac{(b-1)\,b}{(a+b-1)\,z}\frac{\prod_{i=1}^{s-2}(B_i-1)}{\prod_{i=1}^{r-2}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, (A-1) \\ b-1, (B-1) \end{matrix}; z\right]
$$
$$
- \frac{(b-1)\,b}{(a+b-1)\,z}\frac{\prod_{i=1}^{s-2}(B_i-1)}{\prod_{i=1}^{r-2}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a-1, (A-1) \\ b, (B-1) \end{matrix}; z\right]
$$

Usage: `Expr/.C88[m1,m2,n1,n2]`.
`m1`, `m2` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C89

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, (A) \\ c-a, b, 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{(b-1)\,(c-b)}{(a-1)\,(c-a)}\,{}_rF_s\left[\begin{matrix} a-1, (A) \\ 1-a+c, b-1, c-b, (B) \end{matrix}; z\right]
$$
$$
+ \frac{(a-b)\,(1-a-b+c)}{(a-1)\,(c-a)}\,{}_rF_s\left[\begin{matrix} a-1, (A) \\ 1-a+c, b, 1-b+c, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C89[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C90

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ 1-a+c, b, c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a)}{b\,(c-b)}\,{}_rF_s\left[\begin{matrix} a+1, (A) \\ c-a, b+1, 1-b+c, (B) \end{matrix}; z\right] - \frac{(a-b)\,(c-a-b)}{b\,(c-b)}\,{}_rF_s\left[\begin{matrix} a, (A) \\ 1-a+c, b+1, 1-b+c, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C90[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C91

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ c-a, b, 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(c-a-1)}{(1+a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix} a+1, (A) \\ c-a-1, b, 1-b+c, (B) \end{matrix}; z\right] - \frac{(b-1)\,(c-b)}{(1+a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix} a, (A) \\ c-a, b-1, c-b, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C91[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C92

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ c-a, b, 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{a\,(b-1)}{(c-a)\,(1-b+c)}\,{}_rF_s\left[\begin{matrix} a+1, (A) \\ 1-a+c, b-1, 2-b+c, (B) \end{matrix}; z\right] + \frac{c\,(1-a-b+c)}{(c-a)\,(1-b+c)}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+\frac{c}{2}, a, (A) \\ \frac{c}{2}, 1-a+c, b, 2-b+c, (B) \end{matrix}; z\right] $$

Usage: `Expr/.C92[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C93

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a, (A) \\ 1-a+c, b, c-b, (B) \end{matrix}; z\right] \longrightarrow \frac{(c-a)\,(c-b-1)}{(a-1)\,b}\,{}_rF_s\left[\begin{matrix} a-1, (A) \\ c-a, b+1, -1-b+c, (B) \end{matrix}; z\right] - \frac{(c-1)\,(c-a-b)}{(a-1)\,b}\,{}_{r+1}F_{s+1}\left[\begin{matrix} 1+\frac{c-1}{2}, -1+a, (A) \\ \frac{c-1}{2}, 1-a+c, b+1, c-b, (B) \end{matrix}; z\right] \longrightarrow $$

Usage: `Expr/.C93[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C94

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a,(A) \\ 1-a+c,b,c-b,(B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a+1,(A) \\ c-a,b+1,1-b+c,(B) \end{matrix};z\right]
$$
$$
+\frac{(a-b)\,(1+c)\,(c-a-b)\,z}{b\,(1+b)\,(c-a)\,(1-a+c)\,(c-b)\,(1-b+c)}\frac{\prod\limits_{i=1}^{r-1}A_i}{\prod\limits_{i=1}^{s-3}B_i}{}_{r+1}F_{s+1}\left[\begin{matrix} a+1,c+2,(A+1) \\ 2-a+c,b+2,2-b+c,c+1,(B+1) \end{matrix};z\right]
$$

Usage: `Expr/.C94[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C95

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a,(A) \\ c-a,b,1-b+c,(B) \end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix} a-1,(A) \\ 1-a+c,b-1,-b+c,(B) \end{matrix};z\right]
$$
$$
-\frac{(a-b)\,c\,(1-a-b+c)\,z}{(b-1)\,b\,(c-a)\,(1-a+c)\,(c-b)\,(1-b+c)}\frac{\prod\limits_{i=1}^{r-1}A_i}{\prod\limits_{i=1}^{s-3}B_i}{}_{r+1}F_{s+1}\left[\begin{matrix} a,c+1,(A+1) \\ 2-a+c,b+1,2-b+c,c,(B+1) \end{matrix};z\right]
$$

Usage: `Expr/.C95[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C96

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a,(A) \\ c-a,b,1-b+c,(B) \end{matrix};z\right] \longrightarrow \frac{(b-1)\,(c-b)\,(a-d-1)\,(c-a-d)}{(a-1)\,(c-a)\,(b-d-1)\,(c-b-d)}{}_rF_s\left[\begin{matrix} a-1,(A) \\ 1-a+c,b-1,c-b,(B) \end{matrix};z\right]
$$
$$
+\frac{(b-a)\,(1-a-b+c)\,(c-d-1)\,d}{(a-1)\,(c-a)\,(b-d-1)\,(c-b-d)}{}_{r+2}F_{s+2}\left[\begin{matrix} a-1,d+1,c-d,(A) \\ 1-a+c,b,1-b+c,d,c-d-1,(B) \end{matrix};z\right]
$$

Usage: `Expr/.C96[m1,n1,n2,n3,`d`]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C97

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a,(A) \\ 1-a+c,b,c-b,(B)\end{matrix};z\right] \longrightarrow \frac{a\,(c-a)\,(b-d)\,(c-b-d)}{b\,(c-b)\,(a-d)\,(c-a-d)}\,{}_rF_s\left[\begin{matrix} a+1,(A) \\ c-a,b+1,1-b+c,(B)\end{matrix};z\right]
$$
$$
-\frac{(b-a)\,(c-a-b)\,(c-d)\,d}{b\,(c-b)\,(a-d)\,(c-a-d)}\,{}_{r+2}F_{s+2}\left[\begin{matrix} a,d+1,1+c-d,(A) \\ 1-a+c,b+1,1-b+c,d,c-d,(B)\end{matrix};z\right]
$$

Usage: `Expr/.C97[m1,n1,n2,n3,`d`]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C98

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a+b,(A) \\ a+1,b+1,b-a+1,(B)\end{matrix};z\right] \longrightarrow \frac{(a-1)\,a\,(b-1)\,b}{(a+b-1)\,z}\,\frac{\prod_{i=1}^{s-3}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} (A-1) \\ a-1,b-1,(B-1)\end{matrix};z\right]
$$
$$
-\frac{(a-1)\,a\,(b-1)\,b}{(a+b-1)\,z}\,\frac{\prod_{i=1}^{s-3}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} (A-1) \\ a,b,(B-1)\end{matrix};z\right]
$$

Usage: `Expr/.C98[m1,n1,n2,n3]`.
`m1` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C99

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} (A) \\ a,c-a,b,2-b+c,(B)\end{matrix};z\right] \longrightarrow \frac{(b-1)\,(1-b+c)}{a\,(c-a)}\,{}_rF_s\left[\begin{matrix} (A) \\ a+1,1-a+c,b-1,1-b+c,(B)\end{matrix};z\right]
$$
$$
+\frac{(1+a-b)\,(1-a-b+c)}{a\,(c-a)}\,{}_rF_s\left[\begin{matrix} (A) \\ a+1,1-a+c,b,2-b+c,(B)\end{matrix};z\right]
$$

Usage: `Expr/.C99[n1,n2,n3,n4]`.
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C100

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}(A)\\ a, c-a, b, c-b, (B)\end{matrix};z\right] \longrightarrow \frac{(a-1)\,(c-a-1)}{(a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix}(A)\\ a-1, c-a-1, b, -b+c, (B)\end{matrix};z\right]
- \frac{(b-1)\,(c-b-1)}{(a-b)\,(c-a-b)}\,{}_rF_s\left[\begin{matrix}(A)\\ a, c-a, b-1, -1-b+c, (B)\end{matrix};z\right]
$$

Usage: `Expr/.C100[n1,n2,n3,n4]`.
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C101

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}(A)\\ a, c-a, b, c-b, (B)\end{matrix};z\right] \longrightarrow \frac{(a-1)\,(b-1)}{(c-a)\,(c-b)}\,{}_rF_s\left[\begin{matrix}(A)\\ a-1, 1-a+c, b-1, 1-b+c, (B)\end{matrix};z\right]
+ \frac{(c-1)\,(1-a-b+c)}{(c-a)\,(c-b)}\,{}_{r+1}F_{s+1}\left[\begin{matrix}1+\frac{c-1}{2}, (A)\\ \frac{c-1}{2}, a, 1-a+c, b, 1-b+c, (B)\end{matrix};z\right]
$$

Usage: `Expr/.C101[n1,n2,n3,n4]`.
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C102

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}(A)\\ a, 2-a+c, b, c-b, (B)\end{matrix};z\right] \longrightarrow {}_rF_s\left[\begin{matrix}(A)\\ a-1, 1-a+c, b+1, 1-b+c, (B)\end{matrix};z\right]
+ \frac{(a-b-1)\,(1+c)\,(1-a-b+c)\,z}{(a-1)\,ab\,(1+b)\,(1-a+c)\,(2-a+c)\,(c-b)\,(1-b+c)}\,\frac{\prod_{i=1}^{r} A_i}{\prod_{i=1}^{s-4} B_i}
\times {}_{r+1}F_{s+1}\left[\begin{matrix}c+2, (A+1)\\ a+1, 3-a+c, b+2, 2-b+c, c+1, (B+1)\end{matrix};z\right]
$$

Usage: `Expr/.C102[n1,n2,n3,n4]`.
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters.

See also: `C64, ContigListe, Ers, PosListe.`

C103

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}(A)\\ a, c-a, b, 2-b+c, (B)\end{matrix};z\right]
\longrightarrow \frac{(b-1)\,(1-b+c)\,(a-d)\,(c-a-d)}{a\,(c-a)\,(b-d-1)\,(1-b+c-d)}\,{}_rF_s\left[\begin{matrix}(A)\\ a+1, 1-a+c, b-1, 1-b+c, (B)\end{matrix};z\right]
+\frac{(b-a-1)\,(1-a-b+c)\,(c-d)\,d}{a\,(c-a)\,(b-d-1)\,(1-b+c-d)}\,{}_{r+2}F_{s+2}\left[\begin{matrix}d+1, 1+c-d, (A)\\ 1+a, 1-a+c, b, 2-b+c, d, c-d, (B)\end{matrix};z\right]
$$

Usage: `Expr/.C103[n1,n2,n3,n4,`d`].`
`n1`, `n2`, `n3`, `n4` are the positions of the special lower parameters, d is the additional parameter at the right hand side.

See also: `C79, ContigListe, Ers, PosListe.`

C104

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a, c-a, b, c-b, c, (A)\\ c-1, (B)\end{matrix};z\right]
\longrightarrow \frac{1}{(a-b)\,(c-1)\,(c-a-b)\,z}\,\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r-5}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix}a-1, c-a-1, b, -b+c, (A-1)\\ (B-1)\end{matrix};z\right]
-\frac{1}{(a-b)\,(c-1)\,(c-a-b)\,z}\,\frac{\prod_{i=1}^{s-1}(B_i-1)}{\prod_{i=1}^{r-5}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix}a, c-a, b-1, -1-b+c, (A-1)\\ (B-1)\end{matrix};z\right]
$$

Usage: `Expr/.C104[m1,m2,m3,m4,m5,n1].`
`m1`, `m2`, `m3`, `m4`, `m5` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C110

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}1+\frac{c}{2}, a, c-a, b, c-b, (A)\\ \frac{c}{2}, (B)\end{matrix};z\right]
\longrightarrow \frac{(c-a)\,(c-b)}{c\,(c-a-b)}\,{}_{r-1}F_{s-1}\left[\begin{matrix}a, 1-a+c, b, 1-b+c, (A)\\ (B)\end{matrix};z\right]
-\frac{ab}{c\,(c-a-b)}\,{}_{r-1}F_{s-1}\left[\begin{matrix}a+1, c-a, b+1, c-b, (A)\\ (B)\end{matrix};z\right]
$$

Usage: `Expr/.C110[m1,m2,m3,m4,m5,n1].`
`m1`, `m2`, `m3`, `m4`, `m5` and `n1` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C105

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, c-a, b, c, (A) \\ 1-b+c, -1+c, (B) \end{matrix}; z\right] \longrightarrow \frac{(c-b-1)\,(c-b)}{(a-b)\,(c-1)\,(c-a-b)\,z}\frac{\prod_{i=1}^{s-2}(B_i-1)}{\prod_{i=1}^{r-4}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a-1, c-a-1, b, (A-1) \\ c-b-1, (B-1) \end{matrix}; z\right]
$$
$$
- \frac{(c-b-1)\,(c-b)}{(a-b)\,(c-1)\,(c-a-b)\,z}\frac{\prod_{i=1}^{s-2}(B_i-1)}{\prod_{i=1}^{r-4}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, c-a, b-1, (A-1) \\ c-b, (B-1) \end{matrix}; z\right]
$$

Usage: `Expr/.C105[m1,m2,m3,m4,n1,n2]`.
`m1`, `m2`, `m3`, `m4` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C112

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} 1+\frac{c}{2}, a, c-a, b, (A) \\ \frac{c}{2}, 1-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{(c-a)\,(c-b)}{c\,(c-a-b)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, 1-a+c, b, (A) \\ c-b, (B) \end{matrix}; z\right]
$$
$$
- \frac{ab}{c\,(c-a-b)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a+1, c-a, b+1, (A) \\ 1-b+c, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C112[m1,m2,m3,m4,n1,n2]`.
`m1`, `m2`, `m3`, `m4` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C106

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, b, c, (A) \\ 1-a+c, 1-b+c, -1+c, (B) \end{matrix}; z\right]
$$
$$
\longrightarrow \frac{(c-a-1)\,(c-a)\,(c-b-1)\,(c-b)}{(a-b)\,(c-1)\,(c-a-b)\,z}\frac{\prod_{i=1}^{s-3}(B_i-1)}{\prod_{i=1}^{r-3}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a-1, b, (A-1) \\ -a+c, c-b-1, (B-1) \end{matrix}; z\right]
$$
$$
- \frac{(c-a-1)\,(c-a)\,(c-b-1)\,(c-b)}{(a-b)\,(c-1)\,(c-a-b)\,z}\frac{\prod_{i=1}^{s-3}(B_i-1)}{\prod_{i=1}^{r-3}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, b-1, (A-1) \\ -1-a+c, c-b, (B-1) \end{matrix}; z\right]
$$

Usage: `Expr/.C106[m1,m2,m3,n1,n2,n3]`.
`m1`, `m2`, `m3` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C107

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, c-a, c, (A) \\ b+1, 1-b+c, -1+c, (B) \end{matrix}; z\right]
\longrightarrow \frac{(b-1)\,b\,(c-b-1)\,(c-b)}{(a-b)\,(c-1)\,(c-a-b)\,z}\frac{\prod_{i=1}^{s-3}(B_i-1)}{\prod_{i=1}^{r-3}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} -1+a, c-a-1, (A-1) \\ b-1, c-b-1, (B-1) \end{matrix}; z\right]
- \frac{(b-1)\,b\,(c-b-1)\,(c-b)}{(a-b)\,(c-1)\,(c-a-b)\,z}\frac{\prod_{i=1}^{s-3}(B_i-1)}{\prod_{i=1}^{r-3}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, c-a, (A-1) \\ b, c-b, (B-1) \end{matrix}; z\right]
$$

Usage: `Expr/.C107[m1,m2,m3,n1,n2,n3]`.
`m1`, `m2`, `m3` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C114

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} 1+\frac{c}{2}, a, b, (A) \\ \frac{c}{2}, 1-a+c, 1-b+c, (B) \end{matrix}; z\right]
\longrightarrow \frac{(c-a)\,(c-b)}{c\,(c-a-b)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, b, (A) \\ c-a, -b+c, (B) \end{matrix}; z\right]
- \frac{ab}{c\,(c-a-b)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a+1, b+1, (A) \\ 1-a+c, 1-b+c, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C114[m1,m2,m3,n1,n2,n3]`.
`m1`, `m2`, `m3` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C115

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} 1+\frac{c}{2}, a, c-a, (A) \\ \frac{c}{2}, b, 2-b+c, (B) \end{matrix}; z\right]
\longrightarrow \frac{(c-a)\,(1-b+c)}{c\,(1-a-b+c)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, 1-a+c, (A) \\ b, 1-b+c, (B) \end{matrix}; z\right]
- \frac{a\,(b-1)}{c\,(1-a-b+c)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a+1, c-a, (A) \\ -1+b, 2-b+c, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C115[m1,m2,m3,n1,n2,n3]`.
`m1`, `m2`, `m3` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C108

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} a, c, (A) \\ 1-a+c, b, 2-b+c, -1+c, (B) \end{matrix}; z\right]
$$

$$
\longrightarrow \frac{(b-2)\,(b-1)\,(c-a-1)\,(c-a)\,(c-b)\,(1-b+c)}{(1+a-b)\,(c-1)\,(1-a-b+c)\,z}\,\frac{\prod_{i=1}^{s-4}(B_i-1)}{\prod_{i=1}^{r-2}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a-1, (A-1) \\ c-a, b-2, c-b, (B-1) \end{matrix}; z\right]
$$

$$
-\frac{(b-2)\,(b-1)\,(c-a-1)\,(c-a)\,(c-b)\,(1-b+c)}{(1+a-b)\,(c-1)\,(1-a-b+c)\,z}\,\frac{\prod_{i=1}^{s-4}(B_i-1)}{\prod_{i=1}^{r-2}(A_i-1)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, (A-1) \\ c-a-1, b-1, 1-b+c, (B-1) \end{matrix}; z\right]
$$

Usage: `Expr/.C108[m1,m2,n1,n2,n3,n4].`
`m1`, `m2` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C118

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} 1+\frac{c}{2}, a, (A) \\ \frac{c}{2}, 1-a+c, b, 2-b+c, (B) \end{matrix}; z\right] \longrightarrow \frac{(c-a)\,(1-b+c)}{c\,(1-a-b+c)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a, (A) \\ c-a, b, 1-b+c, (B) \end{matrix}; z\right]
$$

$$
-\frac{a\,(b-1)}{c\,(1-a-b+c)}\,{}_{r-1}F_{s-1}\left[\begin{matrix} a+1, (A) \\ 1-a+c, b-1, 2-b+c, (B) \end{matrix}; z\right]
$$

Usage: `Expr/.C118[m1,m2,n1,n2,n3,n4].`
`m1`, `m2` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C109

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix} c, (A) \\ a, 2-a+c, b, 2-b+c, -1+c, (B) \end{matrix}; z\right]
$$

$$
\longrightarrow \frac{(a-2)\,(a-1)\,(b-2)\,(b-1)\,(c-a)\,(1-a+c)\,(c-b)\,(1-b+c)}{(a-b)\,(c-1)\,(2-a-b+c)\,z}\,\frac{\prod_{i=1}^{s-5}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}
$$

$$
\times\,{}_{r-1}F_{s-1}\left[\begin{matrix} (A-1) \\ a-1, 1-a+c, b-2, c-b, (B-1) \end{matrix}; z\right]
$$

$$
-\frac{(a-2)\,(a-1)\,(b-2)\,(b-1)\,(c-a)\,(1-a+c)\,(c-b)\,(1-b+c)}{(a-b)\,(c-1)\,(2-a-b+c)\,z}\,\frac{\prod_{i=1}^{s-5}(B_i-1)}{\prod_{i=1}^{r-1}(A_i-1)}
$$

$$
\times\,{}_{r-1}F_{s-1}\left[\begin{matrix} (A-1) \\ a-2, c-a, b-1, 1-b+c, (B-1) \end{matrix}; z\right]
$$

Usage: `Expr/.C109[m1,n1,n2,n3,n4,n5]`.
`m1` and `n1`, `n2`, `n3`, `n4`, `n5` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C120

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}1+\frac{c}{2},(A)\\ \frac{c}{2},a,2-a+c,b,2-b+c,(B)\end{matrix};z\right]\longrightarrow\frac{(1-a+c)\,(1-b+c)}{c\,(2-a-b+c)}{}_{r-1}F_{s-1}\left[\begin{matrix}(A)\\ a,1-a+c,b,1-b+c,(B)\end{matrix};z\right]
$$
$$
-\frac{(a-1)\,(b-1)}{c\,(2-a-b+c)}{}_{r-1}F_{s-1}\left[\begin{matrix}(A)\\ a-1,2-a+c,b-1,2-b+c,(B)\end{matrix};z\right]
$$

Usage: `Expr/.C120[m1,n1,n2,n3,n4,n5]`.
`m1` and `n1`, `n2`, `n3`, `n4`, `n5` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C111

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a,c-a,b,c-b,d+1,1+c-d,(A)\\ d,c-d,(B)\end{matrix};z\right]
$$
$$
\longrightarrow\frac{a\,(c-a)\,(b-d)\,(c-b-d)}{(b-a)\,(c-a-b)\,(c-d)\,d}{}_{r-2}F_{s-2}\left[\begin{matrix}a+1,1-a+c,b,-b+c,(A)\\ (B)\end{matrix};z\right]
$$
$$
-\frac{b\,(c-b)\,(a-d)\,(c-a-d)}{(b-a)\,(c-a-b)\,(c-d)\,d}{}_{r-2}F_{s-2}\left[\begin{matrix}a,c-a,b+1,1-b+c,(A)\\ (B)\end{matrix};z\right]
$$

Usage: `Expr/.C111[m1,m2,m3,m4,m5,m6,n1,n2]`.
`m1`, `m2`, `m3`, `m4`, `m5`, `m6` and `n1`, `n2` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C113

Description: Contiguous relation in form of a rule.

$$
{}_rF_s\left[\begin{matrix}a,c-a,b,d+1,1+c-d,(A)\\ 1-b+c,d,c-d,(B)\end{matrix};z\right]\longrightarrow\frac{a\,(c-a)\,(b-d)\,(c-b-d)}{(b-a)\,(c-a-b)\,(c-d)\,d}{}_{r-2}F_{s-2}\left[\begin{matrix}a+1,1-a+c,b,(A)\\ 1-b+c,(B)\end{matrix};z\right]
$$
$$
-\frac{b\,(c-b)\,(a-d)\,(c-a-d)}{(b-a)\,(c-a-b)\,(c-d)\,d}{}_{r-2}F_{s-2}\left[\begin{matrix}a,c-a,b+1,(A)\\ c-b,(B)\end{matrix};z\right]
$$

Usage: `Expr/.C113[m1,m2,m3,m4,m5,n1,n2,n3]`.
`m1`, `m2`, `m3`, `m4`, `m5` and `n1`, `n2`, `n3` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C116

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,b,d+1,1+c-d,(A) \\ 1-a+c,1-b+c,d,c-d,(B) \end{matrix};z\right] \longrightarrow \frac{a\,(c-a)\,(b-d)\,(c-b-d)}{(b-a)\,(c-a-b)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} a+1,b,(A) \\ c-a,1-b+c,(B) \end{matrix};z\right] - \frac{b\,(c-b)\,(a-d)\,(c-a-d)}{(b-a)\,(c-a-b)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} a,b+1,(A) \\ 1-a+c,c-b,(B) \end{matrix};z\right] $$

Usage: `Expr/.C116[m1,m2,m3,m4,n1,n2,n3,n4].`
`m1`, `m2`, `m3`, `m4` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C117

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,c-a,d+1,1+c-d,(A) \\ b,2-b+c,d,c-d,(B) \end{matrix};z\right] \longrightarrow \frac{a\,(c-a)\,(b-d-1)\,(1-b+c-d)}{(b-a-1)\,(1-a-b+c)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} a+1,1-a+c,(A) \\ b,2-b+c,(B) \end{matrix};z\right] - \frac{(b-1)\,(1-b+c)\,(a-d)\,(c-a-d)}{(b-a-1)\,(1-a-b+c)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} a,c-a,(A) \\ -1+b,1-b+c,(B) \end{matrix};z\right] $$

Usage: `Expr/.C117[m1,m2,m3,m4,n1,n2,n3,n4].`
`m1`, `m2`, `m3`, `m4` and `n1`, `n2`, `n3`, `n4` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C119

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} a,d+1,1+c-d,(A) \\ 1-a+c,b,2-b+c,d,c-d,(B) \end{matrix};z\right] \longrightarrow \frac{a\,(c-a)\,(b-d-1)\,(1-b+c-d)}{(b-a-1)\,(1-a-b+c)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} a+1,(A) \\ c-a,b,2-b+c,(B) \end{matrix};z\right] - \frac{(b-1)\,(1-b+c)\,(a-d)\,(c-a-d)}{(b-a-1)\,(1-a-b+c)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} a,(A) \\ 1-a+c,-1+b,1-b+c,(B) \end{matrix};z\right] $$

Usage: `Expr/.C119[m1,m2,m3,n1,n2,n3,n4,n5].`
`m1`, `m2`, `m3` and `n1`, `n2`, `n3`, `n4`, `n5` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

C121

Description: Contiguous relation in form of a rule.

$$ {}_rF_s\left[\begin{matrix} d+1,1+c-d,(A) \\ a,2-a+c,b,2-b+c,d,c-d,(B) \end{matrix};z\right] \longrightarrow \frac{(a-1)\,(1-a+c)\,(b-d-1)\,(1-b+c-d)}{(b-a)\,(2-a-b+c)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} (A) \\ a-1,1-a+c,b,2-b+c,(B) \end{matrix};z\right] - \frac{(b-1)\,(1-b+c)\,(a-d-1)\,(1-a+c-d)}{(b-a)\,(2-a-b+c)\,(c-d)\,d}\,{}_{r-2}F_{s-2}\left[\begin{matrix} (A) \\ a,2-a+c,-1+b,1-b+c,(B) \end{matrix};z\right] $$

Usage: `Expr/.C121[m1,m2,n1,n2,n3,n4,n5,n6].`
`m1`, `m2` and `n1`, `n2`, `n3`, `n4`, `n5`, `n6` are the positions of the special upper and lower parameters, respectively.

See also: `C64, ContigListe, Ers, PosListe.`

ContigListe

Description: List of all contiguous relations.

Usage: `ContigListe`.

Div

Description: Function that divides `Gleichung` by `Expr`.

Usage: `Div[Expr]`.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n

             [                 ]    (-a + c)
             [ a, -n           ]            n
Out[1]=  F   [         ;  1    ] == ---------
        2 1  [   c             ]      (c)
                                         n

In[2]:= Div[a^n]

             [                 ]
             [ a, -n           ]
         F   [         ;  1    ]
        2 1  [   c             ]    (-a + c)
                                            n
Out[2]= --------------- == ---------
              n              n
             a              a   (c)
                                   n

In[3]:= Gleichung

             [                 ]
             [ a, -n           ]
         F   [         ;  1    ]
        2 1  [   c             ]    (-a + c)
                                            n
Out[3]= --------------- == ---------
              n              n
             a              a   (c)
                                   n
```

See also: `Gleichung`, `SumListe$gl`, `TransListe$gl`, `LS`, `RS`, `Mal`, `Add`, `Sub`, `Hoch`, `GlTausche`, `Ers`.

Drucke

Description: Function that directly sends an expression `Expr` in the Form `PrintedForm` to the printer. This function only works for DOS-machines with a printer directly connected. `PrintedForm` is an optional parameter which can be any of the format types (`InputForm`, `OutputForm`, `TeXForm`, ...). The default is `OutputForm`.

Usage: `Drucke[Expr,PrintedForm].`

See also: `TeXMat, AmSTeX, AmSLaTeX, LaTeX, TeX, TeXFV.`

Ers

Description: Function for controlled application of rules and functions.

Usage: `Ers[Expr,Rules,PosList].`

`Rules` can be a rule, a list of rules, or a function. `PosList` must be a list of positions in `Expr` to which `Rules` should be applied. For instance, if `PosList=`$\{\{1,2\},\{4\}\}$, then `Rules` is applied to `Expr[[1,2]]` and `Expr[[4]]` in `Expr`. If `PosList=`$\{2,3\}$, then `Rules` is applied to `Expr[[2]]` and `Expr[[3]]` in `Expr`. The positions of subexpressions can be determined by the function `PosListe`. If `Ers` is applied to an equation then the new left-hand and right-hand sides are automatically stored in the variables `LS` and `RS`.

There is an exceptional usage of `Ers`, namely

`Ers[Rules].`

In this case the `Rules` are applied to both sides of the equation that is currently stored in `Gleichung`. Again, the new left-hand and right-hand sides are automatically stored in the variables `LS` and `RS`.

Example(s):

```
In[1]:= (-1)^n*p[a+1,n]*p[c,k]/p[-b,m]/p[1-d,l]

            n
        (-1)   (1 + a)   (c)
                      n    k
Out[1]= -------------------
          (-b)   (1 - d)
              m         l

In[2]:= PosListe[%]

             n                                 1
Out[2]= {{(-1)  , {{1}}}, {(1 + a)  , {{2}}}, {-----, {{3}}}, {(c)  , {{4}}},
                                  n           (-b)                k
                                                  m

           1
>     {--------, {{5}}}}
       (1 - d)
             l

In[3]:= Ers[%%,neg1,{5}]
```

```
                 n
        (-1)  (1 + a)  (c)  (1 - d + l)
                     n    k            -l
Out[3]= --------------------------------
                      (-b)
                          m

In[4]:= Ers[%%%,neg1,{2,4}]


                               n
                          (-1)
Out[4]= --------------------------------------
        (-b)  (1 - d)  (c + k)   (1 + a + n)
            m        l        -k            -n

In[5]:= SUM[%,{k,0,Infinity}]


         ∞
        ┌──┐                        n
         \                      (-1)
Out[5]=   〉  --------------------------------------
         /   (-b)  (1 - d)  (c + k)   (1 + a + n)
        └──┘     m        l        -k            -n
         k=0

In[6]:= PosListe[%,2]


                                   n
Out[6]= {{0, {{2, 2}}}, {(-1) , {{1, 1}}}, {k, {{2, 1}}}, {∞, {{2, 3}}},

          1                   1                     1
〉    {-----, {{1, 2}}}, {--------, {{1, 3}}}, {---------, {{1, 4}}},
       (-b)                 (1 - d)              (c + k)
           m                       l                    -k

              1
〉    {-------------, {{1, 5}}}}
       (1 + a + n)
                  -n

In[7]:= Ers[%%,neg1,{{1,3}}]
```

```
          ∞             n
         ____
                   (-1)   (1 - d + l)
          \                                -l
Out[7]=    >   -----------------------------
          /    (-b)   (c + k)    (1 + a + n)
         ____
                   m         -k              -n
          k=0
```

```
In[8]:= Ers[%%%,neg1,{{1,4},{1,5}}]
```

```
          ∞       n
         ____
              (-1)   (1 + a)   (c)
          \                 n     k
Out[8]=    >   -------------------
          /      (-b)   (1 - d)
         ____
                     m         l
          k=0
```

```
In[9]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
```

```
                                (-a + c)
              [ a, -n        ]          n
Out[9]=  F    [         ; 1  ] == ---------
          2 1 [   c          ]       (c)
                                        n
```

```
In[10]:= Ers[a->1-a]
```

```
                                       (-1 + a + c)
               [ 1 - a, -n        ]                n
Out[10]=  F    [             ; 1  ] == -------------
           2 1 [     c            ]        (c)
                                              n
```

```
In[11]:= Gleichung
```

```
                                       (-1 + a + c)
               [ 1 - a, -n        ]                n
Out[11]=  F    [             ; 1  ] == -------------
           2 1 [     c            ]        (c)
                                              n
```

```
In[12]:= P

In[13]:= p[a,4]+1/p[b,3]
```

```
                                                     1
Out[13]= a (1 + a) (2 + a) (3 + a) + -----------------
                                     b (1 + b) (2 + b)

In[14]:= Ers[%,Expand,{1}]

                   2      3    4            1
Out[14]= 6 a + 11 a  + 6 a  + a  + -----------------
                                   b (1 + b) (2 + b)
```

See also: PosListe, ManipulationsListe, Subst.

---

erw1

Description: $(a)_n \to (a)_{m+n}/(a+n)_m$,
$(a)_n \to \Gamma(a+n)/\Gamma(a)$.

The parameter m has to be entered on request. To apply the second rule, m has to be Infinity.

Usage: Expr/.erw1.

Example(s):

```
In[1]:= p[a,n+1]

Out[1]= (a)
           1 + n

In[2]:= %/.erw1
top-extend by:  m-1

            (a)
               m + n
Out[2]= -----------------
        (1 + a + n)
                   -1 + m

In[3]:= p[-a+1,m]

Out[3]= (1 - a)
               m

In[4]:= %/.erw1
top-extend by:  Infinity

        Γ(1 - a + m)
Out[4]= ------------
          Γ(1 - a)
```

See also: erw2, zus1, zus2, zus3, Ers, PosListe, ManipulationsListe.

---

erw2

Description: $(a)_n \to (a-m)_{m+n}/(a-m)_m$,
$\Gamma(a) \to \Gamma(a-m)\cdot(a-m)_m$.

The parameter m has to be entered on request.

Usage: `Expr/.erw2.`

Example(s):

```
In[1]:= p[a-1,n+1]

Out[1]= (-1 + a)
                1 + n

In[2]:= %/.erw2
bottom-extend by:  m-1

        (a - m)
               m + n
Out[2]= -------------
        (a - m)
               -1 + m

In[3]:= GAMMA[-b]

Out[3]= Γ(-b)

In[4]:= %/.erw2
bottom-extend by:  n

Out[4]= Γ(-b - n) (-b - n)
                          n
```

See also: `erw1, zus1, zus2, zus3, Ers, PosListe, ManipulationsListe.`

Expandq

Description: Rule that expands all the exponents in powers.

Usage: `Expr/.Expandq.`

Example(s):

```
In[1]:= (-1)^m*p[a,m+1]

            m
Out[1]= (-1)  (a)
                 1 + m

In[2]:= %/.baszerl
split into ?  terms:  2
```

```
             m   1 + m   a              1 + a
Out[2]= (-1)   2       (-)            (-----)
                        2 (1 + m)/2    2   (1 + m)/2
```

```
In[3]:= Ers[%,trans,{3}]
```

```
             m + (1 + m)/2  1 + m  1 + a                  a   1 + m
Out[3]= (-1)                2      (-----)            (1 - - - -----)
                                    2   (1 + m)/2       2     2   (1 + m)/2
```

```
In[4]:= %/.Expandq
```

```
             1/2 + (3 m)/2  1 + m  1 + a                  a   1 + m
Out[4]= (-1)                2      (-----)            (1 - - - -----)
                                    2   (1 + m)/2       2     2   (1 + m)/2
```

See also: SimplifyP, MinusOne, SUMExpand, Ers, PosListe.

F

Description: F[List1,List2,z] is the hypergeometric series with upper parameters List1, lower parameters List2, and argument z.

Usage: F[List1,List2,z].

Example(s):

```
In[1]:= F[{a,b},{c},z]
```

```
               [ a, b        ]
Out[1]=  F     [        ; z  ]
        2 1    [  c          ]
```

```
In[2]:= F[{a,b,c},{d,e,f},z]
```

```
               [ a, b, c        ]
Out[2]=  F     [           ; z  ]
        3 3    [ d, e, f        ]
```

See also: SListe, TListe, SUMRegeln, SUMF, FSUM, H, p, GAMMA, FCancel, FOrdne, FPerm, FTausche, P, FFormat.

Factorialp

Description: `Factorialp[n,k]` is the usual factorial, written in terms of factorial symbols (Pochhammer symbols) `p`.

Usage: `Factorialp[n]`.

Example(s):

```
In[1]:= Factorialp[n]

Out[1]= (1)
           n

In[2]:= Factorialp[5]

Out[2]= (1)
           5
```

See also: `Binomialp, Multinomialp`.

FCancel

Description: Switch that activates automatic cancelling of the upper and lower parameters in `F[]` and `H[]`, or makes it inactive, respectively. By default automatic cancelling is active.

Usage: `FCancel`.

Example(s):

```
In[1]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[2]:= F[{a,b},{a,c},z]

            [ b      ]
Out[2]=  F  [   ; z  ]
        1 1 [ c      ]

In[3]:= p[{a,b},{a,c},n]

        (b)
           n
Out[3]= ----
        (c)
           n

In[4]:= GAMMA[{a,b},{a,c}]
```

```
           [ b ]
Out[4]= Γ  [   ]
           [ c ]

In[5]:= FCancel

In[6]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is inactive.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[7]:= F[{a,b},{a,c},z]

             [ a, b       ]
Out[7]=  F   [       ; z  ]
        2 2  [ a, c       ]

In[8]:= p[{a,b},{a,c},n]

        (b)
           n
Out[8]= ----
        (c)
           n

In[9]:= GAMMA[{a,b},{a,c}]

           [ b ]
Out[9]= Γ  [   ]
           [ c ]

In[10]:= FCancel
```

```
In[11]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.
```

See also: F, H, V, hypAttributes.

FEinf

Description: Rule that inactivates automatic cancelling in F[] and then adds a parameter which has to be entered on request to the upper and lower parameters of F[].

Usage: Expr/.FEinf.

Example(s):

```
In[1]:= F[{b,c,1},{1+a-c,1+a-b},z]
```

$$\text{Out[1]=}\ {}_{3}F_{2}\left[\begin{matrix} b,\ c,\ 1 \\ 1+a-c,\ 1+a-b \end{matrix};\ z\right]$$

```
In[2]:= %/.FEinf
Add the parameter:  a
```

$$\text{Out[2]=}\ {}_{4}F_{3}\left[\begin{matrix} a,\ b,\ c,\ 1 \\ a,\ 1+a-c,\ 1+a-b \end{matrix};\ z\right]$$

See also: FCancel, FOrdne, FPerm, FTausche, PSort, SUMRegeln, SUMUmkehr, Ers, PosListe.

FFormat

Description: Switch that activates hypergeometric output, or makes it inactive, respectively. By default hypergeometric output is active.

Usage: FFormat.

Example(s):

```
In[1]:= p[a,n]/GAMMA[b/2]*F[{c,d},{c*d},z]
```

```
            [ c, d         ]
           F[              ] (a)
            [      ; z     ]
         2 1[ c d          ]    n
Out[1]= -------------------
                  b
                Γ(-)
                  2

In[2]:= Tgl5402
Do you want to set values for the equation?  [y|n]:  n


Format::toobig:  Expression too big for output.
                 Enter "FFormat" and retry.


In[3]:= FFormat

In[4]:= %%

Out[4]= F[{-n, b, c, d, e}, {1 - b - n, 1 - c - n, 1 - d - n,

>       -2 + 2 b + 2 c + 2 d + e + 2 n}, 1] ==

                               3   b   c   d
>     F[{1 - b - c - d - 2 n, - - - - - - - - n, 1 - c - d - n, 1 - b - d - n,
                               2   2   2   2

                         -n  1   n
>       1 - b - c - n, --, - - -, e, 3 - 2 b - 2 c - 2 d - e - 3 n},
                         2   2   2

        1   b   c   d
>      {- - - - - - - - n, 1 - b - n, 1 - c - n, 1 - d - n,
        2   2   2   2

                          3 n  3               3 n
>       2 - b - c - d - ---, - - b - c - d - ---, 2 - b - c - d - e - 2 n,
                           2   2                2

>       -1 + b + c + d + e + n}, 1]

>      p[{2 - b - c - d - e - 2 n, 3 - 2 b - 2 c - 2 d - 3 n},

>      {2 - b - c - d - 2 n, 3 - 2 b - 2 c - 2 d - e - 3 n}, n]
```

```
In[5]:= %1

        F[{c, d}, {c d}, z] p[a, n]
Out[5]= ---------------------------
                      b
                 GAMMA[-]
                      2

In[6]:= FFormat

In[7]:= %1

          [ c, d       ]
         F[            ]
          [      ; z   ] (a)
        2 1[ c d       ]    n

Out[7]= -------------------
                  b
                Γ(-)
                  2
```

FH

Description: Rule that transforms a `F[]` into a difference of a `H[]` and a `F[]`.

Usage: `Expr/.FH.`

Example(s):

```
In[1]:= F[{a,b},{c},z]

           [ a, b     ]
Out[1]=  F [     ; z  ]
         2 1[  c      ]

In[2]:= %/.FH

                              [ 2 - c, 1, 1   1 ]  (-1 + c, 0)
                             F[             ; - ]             1
                              [                 ]  -----------------
                            3 2[ 2 - a, 2 - b  z ] (-1 + a, -1 + b)
           [ a, b     ]                                           1
Out[2]=  H [     ; z  ] - -----------------------------------------
         2 2[ c, 1    ]                       z

In[3]:= %/.paufl
```

```
                [ a, b         ]
Out[3]=  H      [       ; z    ]
         2 2    [ c, 1         ]
```

```
In[4]:= F[{a,b,1},{c,d},1]
```

```
                [ a, b, 1          ]
Out[4]=  F      [          ; 1     ]
         3 2    [  c, d            ]
```

```
In[5]:= %/.FH
```

```
                                                          (-1 + c, -1 + d)
                [ a, b      ]         [ 2 - c, 2 - d, 1       ]          1
Out[5]=  H      [     ; 1   ]  -  F   [                  ; 1  ] -----------------
         2 2    [ c, d      ]    3 2  [  2 - a, 2 - b         ] (-1 + a, -1 + b)
                                                                         1
```

See also: `F, H, HF, Ers, PosListe.`

FOrdne

Description: Rule that tries to order the parameters of a hypergeometric series in “well-poised” order. If the parameters could be paired such that the sum of each pair equals `A`$+1$, however `A` is missing in the upper parameters, then you have to add `A` to the upper and lower parameters by `FEinf` before applying `FOrdne`.

Usage: `Expr/.FOrdne.`

Example(s):

```
In[1]:= F[{-n,b,1+a/2,a},{a+1-b,a/2,a+1+n},z]
```

```
                            a
                [    -n, b, 1 + -, a         ]
                                2
Out[1]=  F      [                       ; z  ]
         4 3    [               a            ]
                [ 1 + a - b, -, 1 + a + n    ]
                                2
```

```
In[2]:= %/.FOrdne
```

```
                     a
             a, 1 + -, -n, b
                     2
Out[2]=  F                               ; z
       4 3  a
            -, 1 + a + n, 1 + a - b
            2

In[3]:= F[{b,c,1},{a+1-c,a+1-b},z]


               b, c, 1
Out[3]=  F                        ; z
       3 2  1 + a - c, 1 + a - b


In[4]:= %/.FOrdne


               b, c, 1
Out[4]=  F                        ; z
       3 2  1 + a - c, 1 + a - b


In[5]:= %/.FEinf
Add the parameter:  a


               a, b, c, 1
Out[5]=  F                           ; z
       4 3  a, 1 + a - c, 1 + a - b


In[6]:= %/.FOrdne


               a, b, c, 1
Out[6]=  F                           ; z
       4 3  1 + a - b, 1 + a - c, a


In[7]:= FCancel

In[8]:= %%/.FOrdne
```

```
                    b, c, 1
Out[8]=  F                                ; z
        3 2   1 + a - b, 1 + a - c
```

See also: FEinf, FPerm, FTausche, F, V, PSort, Ers, PosListe.

FPerm

Description: Rule for permuting parameters in basic hypergeometric series.

Usage: `Expr/.FPerm[⟨Permutation⟩,x]`.
`x` can be `u`, `l`, `b`. `u` causes a permutation of upper parameters, `l` causes a permutation of lower parameters, `b` causes a simultaneous permutation of respective upper and lower parameters. `Permutation` must be a sequence of positive numbers forming a permutation. Under the options `u` and `l` the effect is that the new parameter at position `i` is the old parameter from position `Permutation[i]`. However, the behaviour of `FPerm` under the option `b` is special. The option `b` is especially designed for the permutation of parameters of *well-poised* series. Hence, the first upper parameter is not moved, whereas the new *upper* parameter at position `i+1` is the old upper parameter from position `Permutation[i]+1`, and the new *lower* parameter at position `i` is the old lower parameter from position `Permutation[i]`.

Example(s):

```
In[1]:= F[{a,b,c,d},{e,f,g},z]

              a, b, c, d
Out[1]=  F                ; z
        4 3    e, f, g

In[2]:= %1/.FPerm[3,2,1,u]

              c, b, a, d
Out[2]=  F                ; z
        4 3    e, f, g

In[3]:= %1/.FPerm[3,2,1,l]

              a, b, c, d
Out[3]=  F                ; z
        4 3    g, f, e

In[4]:= %1/.FPerm[2,1,b]
```

```
              ┌                ┐
              │ a, c, b, d     │
Out[4]=   F   │           ; z  │
         4 3  │ f, e, g        │
              └                ┘
```

See also: `FTausche`, `FOrdne`, `F`, `V`, `PSort`, `Ers`, `PosListe`.

FSUM

Description: Rule that transforms a `F[]` into a `SUM[]`.

Usage: `Expr/.FSUM`.

Example(s):

```
In[1]:= F[{a,b},{c},z]

              ┌          ┐
              │ a, b     │
Out[1]=   F   │     ; z  │
         2 1  │ c        │
              └          ┘

In[2]:= %/.FSUM
Is -a a nonnegative integer?
[y|n]:    n
Is -b a nonnegative integer?
[y|n]:    n
A hypergeometric series is converted into a sum.
Enter a variable for the summation index:  k

          ∞   k
        ┌───┐z  (a)   (b)
         \          k     k
Out[2]=   〉  ------------
         /     (1)   (c)
        └───┘       k     k
         k=0

In[3]:= %1/.FSUM
Is -a a nonnegative integer?
[y|n]:    n
Is -b a nonnegative integer?
[y|n]:    y
A hypergeometric series is converted into a sum.
Enter a variable for the summation index:  j
```

```
         -b   j
        ┌───┐z  (a)  (b)
         \         j    j
Out[3]=   >   ------------
         /    (1)  (c)
        └───┘      j    j
         j=0
```

See also: `F, SUM, SUMF, Ers, PosListe.`

FTausche

Description: Rule for reordering parameters in hypergeometric series.

Usage: `Expr/.FTausche[n1,n2,x].`
`x` can be `u`, `l`, `b`. `u` causes a reordering of upper parameters, `l` causes a reordering of lower parameters, `b` causes a simultaneous reordering of respective upper and lower parameters. `n1` is the position of the parameter to be reordered, `n2` is the new position.

Example(s):

```
In[1]:= F[{-n,b,1+a/2,a},{a+1-b,a/2,a+1+n},z]

                         a
              -n, b, 1 + -, a
                         2
Out[1]=  F                              ; z
        4 3               a
          1 + a - b, -, 1 + a + n
                     2

In[2]:= %/.FTausche[1,4,u]

                     a
              b, 1 + -, a, -n
                     2
Out[2]=  F                              ; z
        4 3               a
          1 + a - b, -, 1 + a + n
                     2

In[3]:= %/.FTausche[3,2,l]

                     a
              b, 1 + -, a, -n
                     2
Out[3]=  F                              ; z
        4 3                      a
          1 + a - b, 1 + a + n, -
                                 2
```

```
In[4]:= %/.FTausche[1,3,b]

                          a
            b, a, -n, 1 + -
                          2
Out[4]=  F                        ; z
        4 3            a
        1 + a + n, -, 1 + a - b
                       2
```

See also: `FPerm, FOrdne, F, V, PSort, Ers, PosListe.`

GAMMA

Description: `GAMMA[x]` is the Gamma function $\Gamma(x)$. `GAMMA[List1,List2]` is also provided as the usual abbreviation for the quotient of Gamma functions (cf. the example for `Gzerl`).

Usage: `GAMMA[x]`
or: `GAMMA[List1,List2].`

Example(s):

```
In[1]:= GAMMA[2*a+1]

Out[1]= Γ(1 + 2 a)

In[2]:= GAMMA[{a,b},{c,d}]

            a, b
Out[2]= Γ[      ]
            c, d
```

See also: `p, P, Gzerl, Gzus, FFormat.`

Gleichung

Description: Is a variable which stores equations. The equation `Gleichung` can be manipulated using the functions `Add`, `Sub`, `Mal`, `Div`, `Hoch`, `GlTausche`, `Ers`, and `SUM[k,m,n]`, where `m` and `n` are integers or variables. The last command causes the equation to be summed over `k` from `m` to `n`. The parameter `k` is optional. It will be set `kk`, `ii`, `jj`, `ll`, `mm`, or `nn`, if it is omitted.

Usage: `Gleichung.`

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
```

Out[1]= $${}_2F_1\left[\begin{matrix}a,\ -n\\ c\end{matrix};\ 1\right] == \frac{(-a+c)_n}{(c)_n}$$

In[2]:= Gleichung

Out[2]= $${}_2F_1\left[\begin{matrix}a,\ -n\\ c\end{matrix};\ 1\right] == \frac{(-a+c)_n}{(c)_n}$$

In[3]:= Add[1]

Out[3]= $$1 + {}_2F_1\left[\begin{matrix}a,\ -n\\ c\end{matrix};\ 1\right] == 1 + \frac{(-a+c)_n}{(c)_n}$$

In[4]:= Gleichung

Out[4]= $$1 + {}_2F_1\left[\begin{matrix}a,\ -n\\ c\end{matrix};\ 1\right] == 1 + \frac{(-a+c)_n}{(c)_n}$$

In[5]:= LS=5

Out[5]= 5

In[6]:= Gleichung

Out[6]= $$5 == 1 + \frac{(-a+c)_n}{(c)_n}$$

In[7]:= SUM[n,0,m]

Out[7]= $$\sum_{n=0}^{m} 5 == \sum_{n=0}^{m} 1 + \sum_{n=0}^{m} \frac{(-a + c)_n}{(c)_n}$$

In[8]:= SUM[0,M]

Out[8]= $$\sum_{kk=0}^{M} \sum_{n=0}^{m} 5 == \sum_{kk=0}^{M} \sum_{n=0}^{m} 1 + \sum_{kk=0}^{M} \sum_{n=0}^{m} \frac{(-a + c)_n}{(c)_n}$$

In[9]:= Gleichung

Out[9]= $$\sum_{kk=0}^{M} \sum_{n=0}^{m} 5 == \sum_{kk=0}^{M} \sum_{n=0}^{m} 1 + \frac{(-a + c)_n}{(c)_n}$$

<u>See also</u>: `SumListe$gl`, `TransListe$gl`, `LS`, `RS`, `Mal`, `Add`, `Div`, `Sub`, `Hoch`, `GlTausche`, `Ers`, `Subst`, `PSort`.

GlTausche

<u>Description</u>: `GlTausche` interchanges right-hand and left-hand sides in `Gleichung`.

<u>Usage</u>: `GlTausche.`

<u>Example(s)</u>:

```
In[1]:= Sgl3201
Do you want to set values for the equation?  [y|n]:  n
```

Out[1]= $${}_3F_2\left[\begin{matrix} a,\ b,\ -n \\ c,\ 1 + a + b - c - n \end{matrix} ;\ 1\right] == \frac{(-a + c)_n \ (-b + c)_n}{(c)_n \ (-a - b + c)_n}$$

In[2]:= Gleichung

$$\text{Out[2]=}\ {}_3F_2\left[\begin{matrix} a,\ b,\ -n \\ c,\ 1 + a + b - c - n \end{matrix};\ 1\right] == \frac{(-a + c)_n\ (-b + c)_n}{(c)_n\ (-a - b + c)_n}$$

In[3]:= GlTausche

$$\text{Out[3]=}\ \frac{(-a + c)_n\ (-b + c)_n}{(c)_n\ (-a - b + c)_n} == {}_3F_2\left[\begin{matrix} a,\ b,\ -n \\ c,\ 1 + a + b - c - n \end{matrix};\ 1\right]$$

In[4]:= Gleichung

$$\text{Out[4]=}\ \frac{(-a + c)_n\ (-b + c)_n}{(c)_n\ (-a - b + c)_n} == {}_3F_2\left[\begin{matrix} a,\ b,\ -n \\ c,\ 1 + a + b - c - n \end{matrix};\ 1\right]$$

See also: Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Add, Div, Sub, Hoch, Ers, Subst.

GOSPER

Description: Rule that does symbolic summation using Gosper's algorithm [3].

Here a call is made to the function `Gosper` of the *Mathematica* implementation of Gosper's and Zeilberger's algorithms written by Peter Paule and Markus Schorn. The current version 1.1 or updates can be received via e-mail request to `peter.paule@risc.uni-linz.ac.at`. This implemenation provides the user with the objects

`Zb`, `Gosper`, `RunMode`, `FileName`, `SolAmount`, `Fnk`, `GoRat`, `GoSol`, `Cert`, `DegBound`, `System`, `SystemDimension`.

Also within the package HYP, all these objects work as described in the documentation of this implementation. Therefore the user is referred to this documentation and the description [5] in order to learn about the various features of these objects.

The package HYP provides two additional objects, `ZB` and `GOSPER`. The rule `GOSPER` allows to apply Gosper's algorithm directly to an expression containing a `SUM` or a hypergeometric series.

Usage: `Expr/.GOSPER[]`

or: `Expr/.GOSPER[order]`.

(Runs Gosper's algorithm trying an additional polynomial Ansatz of degree 'order'.)

Example(s):

In[1]:= SUM[1/(k*(k+1)),{k,1,n}]

```
         n
        ____
        \       1
Out[1]=  )   ---------
        /    k (1 + k)
        ----
         k=1

In[2]:= %/.GOSPER[]

    Peter Paule and Markus Schorn's implementation of the
                Gosper algorithm.  (Version 1.1)

                  1                           1
Out[2]= {SUM[---------, {k, 1, n}] == 1 - -----}
              k (1 + k)                     1 + n
In[3]:= F[{1,1},{3},1]/2

              1, 1
         F  [       ; 1 ]
        2 1    3
Out[3]= --------------
              2

In[4]:= %/.GOSPER[]

    Peter Paule and Markus Schorn's implementation of the
                Gosper algorithm.  (Version 1.1)

                kk!
Out[4]= {SUM[---------, {kk, 0, ∞}] == 1}
             (2 + kk)!
```

See also: ZB.

Gzerl

Description: Rule that splits `GAMMA[List1,List2]` into a quotient of products of Gamma functions.

Usage: `Expr/.Gzerl`.

Example(s):

```
In[1]:= GAMMA[{a,b},{c,d}]
```

```
                [ a, b ]
Out[1]= Γ       [      ]
                [ c, d ]
```

```
In[2]:= %/.Gzerl
```

```
        Γ(a) Γ(b)
Out[2]= ---------
        Γ(c) Γ(d)
```

See also: paufl, pzerl, pzus, GAMMA, Gzus, Ers, PosListe.

Gzus

Description: Rule that collects several Gammma functions GAMMA[x_i] to an expression GAMMA[List1,List2].

Usage: Expr/.Gzus.

Example(s):

```
In[1]:= GAMMA[a]*GAMMA[b]/GAMMA[c]/GAMMA[d]
```

```
        Γ(a) Γ(b)
Out[1]= ---------
        Γ(c) Γ(d)
```

```
In[2]:= %/.Gzus
```

```
                [ a, b ]
Out[2]= Γ       [      ]
                [ c, d ]
```

See also: paufl, pzerl, pzus, GAMMA, Gzerl, Ers, PosListe.

H

Description: H[List1,List2,z] is the bilateral hypergeometric series with upper parameters List1, lower parameters List2, and argument z.

Usage: H[List1,List2,z].

Example(s):

```
In[1]:= H[{a,b,c},{d,e,f},1]
```

```
            a, b, c
Out[1]=  H           ; 1
        3 3 d, e, f
```

See also: SListe, TListe, SUMRegeln, SUMH, HSUM, F, p, GAMMA, FCancel, FOrdne, FPerm, FTausche, P, FFormat.

HEinf

Description: Rule that inactivates automatic cancelling in `H[]` and then adds a parameter which has to be entered on request to the upper and lower parameters of `H[]`.

Usage: `Expr/.HEinf`.

Example(s):

```
In[1]:= H[{b,c},{1+a-c,1+a-b},1]

                 b, c
Out[1]=  H                       ; 1
        2 2 1 + a - c, 1 + a - b

In[2]:= %/.HEinf
Add the parameter:  a

                 a, b, c
Out[2]=  H                          ; 1
        3 3 a, 1 + a - c, 1 + a - b
```

See also: FCancel, HOrdne, HPerm, PSort, SUMRegeln, SUMUmkehr, Ers, PosListe.

HF

Description: Rule that transforms a `H[]` into a sum of two `F[]`'s.

$$ {}_rH_s\left[\begin{matrix} a_1,\ldots,a_r \\ b_1,\ldots,b_s \end{matrix};z\right] \rightarrow \sum_{n=-\infty}^{m-1} \frac{(a_1)_n\cdots(a_r)_n}{(b_1)_n\cdots(b_s)_n} z^n + \sum_{n=m}^{\infty} \frac{(a_1)_n\cdots(a_r)_n}{(b_1)_n\cdots(b_s)_n} z^n . $$

The parameter `m` has to be entered on request.

Usage: `Expr/.HF`.

Example(s):

```
In[1]:= H[{a,b},{c,d},1]
```

```
Out[1]=  H  [ a, b      ]
        2 2 [     ; 1   ]
            [ c, d      ]

In[2]:= %/.HF
Split at:  0

Out[2]=  F  [ a, b, 1      ]     F  [ 2 - c, 2 - d, 1      ] (a, b)
        3 2 [        ; 1   ] +  3 2 [                 ; 1  ]       -1
            [  c, d        ]        [  2 - a, 2 - b        ] --------
                                                               (c, d)
                                                                     -1

In[3]:= H[{a,b},{c,d},1]

Out[3]=  H  [ a, b      ]
        2 2 [     ; 1   ]
            [ c, d      ]

In[4]:= %/.HF
Split at:  5

Out[4]=  F  [ -3 - c, -3 - d, 1      ] (a, b)
        3 2 [                   ; 1  ]       4
            [  -3 - a, -3 - b        ] ------- +
                                       (c, d)
                                             4

>       F  [ 5 + a, 5 + b, 1      ] (a, b)
       3 2 [               ; 1    ]       5
           [  5 + c, 5 + d        ] -------
                                    (c, d)
                                          5
```

See also: `H, F, FH, Ers, PosListe.`

Hoch

Description: Function that takes `Gleichung` to the `Expr`-th power.

Usage: `Hoch[Expr]`.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
```

$$\text{Out[1]= } {}_2F_1\left[\begin{matrix}a,\ -n\\ c\end{matrix};\ 1\right] == \frac{(-a+c)_n}{(c)_n}$$

```
In[2]:= Hoch[3]
```

$$\text{Out[2]= } {}_2F_1\left[\begin{matrix}a,\ -n\\ c\end{matrix};\ 1\right]^3 == \frac{(-a+c)_n^3}{(c)_n^3}$$

```
In[3]:= Gleichung
```

$$\text{Out[3]= } {}_2F_1\left[\begin{matrix}a,\ -n\\ c\end{matrix};\ 1\right]^3 == \frac{(-a+c)_n^3}{(c)_n^3}$$

<u>See also</u>: `Gleichung`, `SumListe$gl`, `TransListe$gl`, `LS`, `RS`, `Mal`, `Add`, `Div`, `Sub`, `GlTausche`, `Ers`.

HOrdne

<u>Description</u>: Rule that tries to order the parameters of a bilateral hypergeometric series in “well-poised” order. If there is an upper parameter of the form $-n$, where n is a nonnegative integer, then it is put at the very last place in the upper list.

<u>Usage</u>: `Expr/.HOrdne`.

<u>Example(s)</u>:

```
In[1]:= H[{-n,b,1+a/2},{a+1-b,a/2,a+1+n},z]
```

$$\text{Out[1]= } {}_3H_3\left[\begin{matrix}-n,\ b,\ 1+\frac{a}{2}\\ 1+a-b,\ \frac{a}{2},\ 1+a+n\end{matrix};\ z\right]$$

```
In[2]:= %/.HOrdne
```

```
                  a
              1 + -, -n, b
                  2
Out[2]=  H                          ; z
        3 3  a
             -, 1 + a + n, 1 + a - b
             2
```

See also: `HEinf, HPerm, H, PSort, Ers, PosListe.`

HPerm

Description: Rule for permuting parameters in bilateral hypergeometric series.

Usage: : `Expr/.HPerm[⟨Permutation⟩,x].`
`x` can be `u`, `l`, `b`. `u` causes a permutation of upper parameters, `l` causes a permutation of lower parameters, `b` causes a simultaneous permutation of respective upper and lower parameters. `Permutation` must be a sequence of positive numbers forming a permutation. The effect is that the new parameter at position `i` is the old parameter from position `Permutation[i]`.

Example(s):

```
In[1]:= H[{a,b,c},{d,e,f},1]
```

```
             a, b, c
Out[1]=  H           ; 1
        3 3  d, e, f
```

```
In[2]:= %1/.HPerm[3,2,1,u]
```

```
             c, b, a
Out[2]=  H           ; 1
        3 3  d, e, f
```

```
In[3]:= %1/.HPerm[3,2,1,l]
```

```
             a, b, c
Out[3]=  H           ; 1
        3 3  f, e, d
```

```
In[4]:= %1/.HPerm[3,2,1,b]
```

```
                 [ c, b, a          ]
Out[4]=  H       [             ; 1  ]
        3 3      [ f, e, d          ]
```

See also: HOrdne, H, PSort, Ers, PosListe, FPerm.

HShift

Description: Rule that shifts the summation index in a bilateral hypergeometric series.

$$ {}_rH_s\left[\begin{matrix} a_1,\ldots,a_r \\ b_1,\ldots,b_s \end{matrix};z\right] \to z^m \frac{\prod_{i=1}^r (a_i)_m}{\prod_{i=1}^s (b_i)_m} {}_rH_s\left[\begin{matrix} a_1+m,\ldots,a_r+m \\ b_1+m,\ldots,b_s+m \end{matrix};z\right]. $$

The parameter m has to be entered on request.

Usage: Expr/.HShift.

Example(s):

```
In[1]:= H[{a,b,c},{d,e,f},1]

                 [ a, b, c          ]
Out[1]=  H       [             ; 1  ]
        3 3      [ d, e, f          ]

In[2]:= %/.HShift
shift by:  4

                 [                             ]  (a, b, c)
                 [ 4 + a, 4 + b, 4 + c         ]          4
Out[2]=  H       [                       ; 1   ]  ----------
        3 3      [ 4 + d, 4 + e, 4 + f         ]  (d, e, f)
                 [                             ]          4
```

See also: HEinf, HPerm, H, PSort, Ers, PosListe.

HSUM

Description: Rule that transforms a H[] into a SUM[].

Usage: Expr/.HSUM.

Example(s):

```
In[1]:= H[{a,b},{c,d},1]
```

```
Out[1]=  H [ a, b
             ; 1 ]
        2 2  c, d

In[2]:= %/.HSUM
Is -a a nonnegative integer?
[y|n]:    n
Is -b a nonnegative integer?
[y|n]:    n
Is c a nonnegative integer?
[y|n]:    n
Is d a nonnegative integer?
[y|n]:    n
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  k

          ∞
         ___ (a)  (b)
         \      k    k
Out[2]=   )  ---------
         /   (c)  (d)
         ---    k    k
        k=-∞

In[3]:= %1/.HSUM
Is -a a nonnegative integer?
[y|n]:    y
Is -b a nonnegative integer?
[y|n]:    n
Is c a nonnegative integer?
[y|n]:    y
Is d a nonnegative integer?
[y|n]:    n
A basic hypergeometric series is converted into a sum.
Enter a variable for the summation index:  k

           -a
          ___  (a)  (b)
          \       k    k
Out[3]=    )   ---------
          /    (c)  (d)
          ---     k    k
        k=1 - c
```

<u>See also</u>: H, SUM, SUMH, Ers, PosListe.

hypAttributes

Description: Shows the current setup of the session. The setup can be changed by the switches `P`, `FCancel`, `AmSTeX`, `AmSLaTeX`, `LaTeX`, `TeX`, and `TeXFV`. The default-setup is shown in the following Example.

Usage: `hypAttributes.`

Example(s):

```
In[1]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.
```

See also: `P, FCancel, AmSTeX, AmSLaTeX, LaTeX, TeX, TeXFV.`

---

inv

Description: $\Gamma(a) \to \dfrac{\pi}{\sin(\pi a)} \dfrac{1}{\Gamma(1-a)}$.

Usage: `Expr/.inv.`

Example(s):

```
In[1]:= GAMMA[2*a+1]

Out[1]= Γ(1 + 2 a)

In[2]:= %/.inv

        π Csc[(1 + 2 a) π]
Out[2]= ------------------
             Γ(-2 a)
```

See also: `inv, Ers, PosListe, ManipulationsListe.`

LaTeX

Description: Switch that changes the output of TeXForm to be usable with Plain-TEX and LATEX. By default the output of TeXForm is usable with AMS-TEX.

Usage: `LaTeX.`

Example(s):

```
In[1]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised basic hypergeometric series.
```

```
In[2]:= TeXForm[F[{a,b},{c},z]]

Out[2]//TeXForm=
  {} _{2} F _{1} \!\left [ \matrix { a, b}\\ { c}\endmatrix ; {\displaystyle
   z}\right ]

In[3]:= LaTeX

In[4]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with Plain-TeX and LaTeX.
TeXForm uses V[] for very well-poised basic hypergeometric series.

In[5]:= TeXForm[F[{a,b},{c},z]]

Out[5]//TeXForm=
  {} _{2} F _{1} \!\left [ \matrix { a, b\cr c} ; {\displaystyle z} \right ]
```

See also: `AmSTeX`, `AmSLaTeX`, `TeX`, `TeXMat`, `TeXFV`.

Limes

Description: Function for doing formal limits of hypergeometric expressions. If required for taking the limit, you will be asked whether or not the absolute value of some variable or expression is smaller than 1. You will be offered three options, `[y|n|u]`. If the absolute value of the variable is smaller than 1 then enter `y`, if it is greater than 1 then enter `n`, if you do not want to make an explicit declaration then enter `u` (for "undetermined"). Your decision, if explicit, is stored for the rest of your MATHEMATICA session. If you want to change your decision later, use `AbsGreater`, `AbsSmaller`, or `AbsUndetermined`, respectively.

Warning: This function uses primitive algebraic techniques to do the limit. There is no check if taking the limit is actually allowed. So it is left to you to check the validity of a result of `Limes`.

Usage: `Limes[Expr, x->x0]`.

Example(s):

The derivation of the Vandermonde summation (see `S2101`) from the Pfaff–Saalschütz summation.

```
In[1]:= Sgl3201
Do you want to set values for the equation?  [y|n]:  n
```

$$
{}_3F_2\left[\begin{matrix} a,\ b,\ -n \\ c,\ 1+a+b-c-n \end{matrix};1\right] == \frac{(-a+c)_n\,(-b+c)_n}{(c)_n\,(-a-b+c)_n}
$$

```
In[2]:= Limes[%,b->Infinity]
```

```
                [              ]   (-a + c)
                [ a, -n        ]           n
Out[2]=  F      [        ; 1   ] == ---------
        2 1     [  c           ]     (c)
                [              ]        n
```

The derivation of the Gauss summation (see S2103) from the Pfaff–Saalschütz summation.

```
In[3]:= Limes[%%,n->Infinity]
```

```
                [              ]
                [ a, b         ]     Γ(c) Γ(-a - b + c)
Out[3]=  F      [       ; 1    ] == -------------------
        2 1     [  c           ]    Γ(-a + c) Γ(-b + c)
                [              ]
```

See also: AbsGreater, AbsSmaller, AbsUndetermined, MinusOne.

---

lina1

Description: $(a)_n \to a\,(a+1)_{n-1}$,
$\Gamma(a) \to (a-1)\Gamma(a-1)$.

Usage: Expr/.lina1.

Example(s):

```
In[1]:= p[a-1,m]

Out[1]= (-1 + a)
                m

In[2]:= %/.lina1

Out[2]= (-1 + a) (a)
                    -1 + m

In[3]:= 1/GAMMA[2*a]

           1
Out[3]= ------
        Γ(2 a)

In[4]:= %/.lina1
```

```
                  1
Out[4]= ----------------------
        (-1 + 2 a) Γ(-1 + 2 a)
```

See also: lina2, linz, Ers, PosListe, ManipulationsListe.

---

lina2

Description: $(a)_n \to (a+n-1)(a)_{n-1}$.

Usage: Expr/.lina2.

Example(s):

```
In[1]:= p[a,m]

Out[1]= (a)
           m

In[2]:= %/.lina2

Out[2]= (-1 + a + m) (a)
                        -1 + m
```

See also: lina1, linz, Ers, PosListe, ManipulationsListe.

---

linz

Description: Rule that absorbs linear terms.

Usage: Expr/.linz.

Example(s):

```
In[1]:= a*p[a+1,m]/(a/2+m-1)/p[a/2,m-1]

              a (1 + a)
                       m
Out[1]= ----------------------
              a       a
        (-1 + - + m) (-)
              2       2 -1 + m

In[2]:= %/.linz

               (a)
                  1 + m
Out[2]= ----------------------
              a       a
        (-1 + - + m) (-)
              2       2 -1 + m

In[3]:= %/.linz
```

```
        (a)
           1 + m
Out[3]= --------
           a
          (-)
           2 m

In[4]:= 1/(1-b)/GAMMA[b-1]

                1
Out[4]= -----------------
        (1 - b) Γ(-1 + b)

In[5]:= %/.linz

           1
Out[5]= -(----)
          Γ(b)
```

See also: `lina1, lina2, Ers, PosListe, ManipulationsListe.`

LS

Description: `LS` is the left-hand side in `Gleichung`.

Usage: `LS.`

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n

                                (-a + c)
               a, -n                    n
Out[1]=  F   [       ; 1 ]  == ---------
        2 1    c                  (c)
                                     n

In[2]:= LS

               a, -n
Out[2]=  F   [       ; 1 ]
        2 1    c

In[3]:= Add[1]
```

```
                                         (-a + c)
                 [ a, -n      ]                  n
Out[3]= 1 +  F   [       ; 1  ] == 1 + ---------
            2 1  [   c        ]            (c)
                                              n
```

```
In[4]:= LS
```

```
                 [ a, -n      ]
Out[4]= 1 +  F   [       ; 1  ]
            2 1  [   c        ]
```

```
In[5]:= LS=p[a,m]
```

```
Out[5]= (a)
           m
```

```
In[6]:= Gleichung
```

```
                         (-a + c)
                                 n
Out[6]= (a)  == 1 + ---------
           m           (c)
                          n
```

See also: `Gleichung`, `SumListe$gl`, `TransListe$gl`, `RS`, `Mal`, `Add`, `Div`, `Sub`, `Hoch`, `GlTausche`, `Ers`, `Subst`.

Mal

Description: Function that multiplies `Gleichung` by `Expr`.

Usage: `Mal[Expr]`.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n
```

```
                                (-a + c)
             [ a, -n      ]             n
Out[1]=  F   [       ; 1  ] == ---------
        2 1  [   c        ]       (c)
                                     n
```

```
In[2]:= Mal[p[c,n]]
```

```
                  a, -n
Out[2]=  F              ; 1   (c)   == (-a + c)
        2 1       c              n              n
```

```
In[3]:= Gleichung
```

```
                  a, -n
Out[3]=  F              ; 1   (c)   == (-a + c)
        2 1       c              n              n
```

See also: `Gleichung, SumListe$gl, TransListe$gl, LS, RS, Add, Div, Sub, Hoch, GlTausche, Ers.`

ManipulationsListe

Description: Gives a list of all available rules for manipulating factorial symbols and Gamma functions.

Usage: `ManipulationsListe.`

MinusOne

Description: : Rule for getting rid of expressions of the form $(-1)^N$ where N is an even or odd integer.

Usage: `Expr/.MinusOne.`

```
In[1]:= p[a,2*n]

Out[1]= (a)
           2 n

In[2]:= %/.trans

            2 n
Out[2]= (-1)    (1 - a - 2 n)
                             2 n

In[3]:= %/.MinusOne
Is 2 n even, odd, or neither of both?
[e|o|n]:  e

Out[3]= (1 - a - 2 n)
                     2 n
```

See also: `SimplifyP, Expandq, SUMExpand.`

Multinomialp

Description: : `Multinomialp[n1,n2, ... ]` is the multinomial coefficient $\binom{\sum_i n_i}{n_1,n_2,\ldots}$, written in terms of factorial symbols (Pochhammer symbols) `p`.

Usage: `Multinomial[n1,n2, ... ]`.

```
In[1]:= Multinomialp[a,b,c]

          (1)
             a + b + c
Out[1]= --------------
         (1)   (1)   (1)
            a     b     c

In[2]:= Multinomialp[3,2,6]

             (1)
                11
Out[2]= --------------
         (1)   (1)   (1)
            2     3     6
```

See also: `Binomialp, Factorialp.`

---

neg1

Description: $(a)_n \to 1/(a+n)_{-n}$.

Usage: `Expr/.neg1.`

Example(s):

```
In[1]:= p[a,-n]

Out[1]= (a)
           -n

In[2]:= %/.neg1

           1
Out[2]= --------
        (a - n)
               n
```

See also: `neg2, Ers, PosListe, ManipulationsListe.`

---

neg2

Description: $(a)_n \to (-1)^n/(1-a)_{-n}$.

Usage: `Expr/.neg2`.

Example(s):

```
In[1]:= p[a,-n]

Out[1]= (a)
           -n

In[2]:= %/.neg2

              n
         (-1)
Out[2]= --------
        (1 - a)
               n
```

See also: `neg1, Ers, PosListe, ManipulationsListe.`

---

p

Description: `p[x,n]` is the factorial symbol (Pochhammer symbol) $(x)_n$. `p[List1,List2,n]` is also provided as the usual abbreviation for the quotient of factorial symbols (see [2]).

Usage: `p[x,n]`
or: `p[List1,List2,n]`.

Example(s):

```
In[1]:= p[a,n]

Out[1]= (a)
           n

In[2]:= p[{a,b},{c,d},2*m]

        (a, b)
             2 m
Out[2]= ---------
        (c, d)
             2 m
```

See also: `GAMMA, Binomialp, Multinomialp, Factorialp, P, paufl, pzerl, pzus, FFormat.`

P

Description: Is a switch that activates automatic evaluating of factorial symbols (Pochhammer symbols) `p` and hypergeometric series `F`, `H`, or makes it inactive, respectively. By default automatic evaluating is inactive.

Usage: `P`.

Example(s):

```
In[1]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[2]:= p[a,5]

Out[2]= (a)
           5

In[3]:= F[{a,b},{c},z]
```

$$\text{Out[3]=}\ {}_2F_1\left[\begin{matrix} a,\ b \\ c \end{matrix}\ ;\ z\right]$$

```
In[4]:= F[{-n,b},{c},z]
```

$$\text{Out[4]=}\ {}_2F_1\left[\begin{matrix} -n,\ b \\ c \end{matrix}\ ;\ z\right]$$

```
In[5]:= F[{-3,b},{c},z]
```

$$\text{Out[5]=}\ {}_2F_1\left[\begin{matrix} -3,\ b \\ c \end{matrix}\ ;\ z\right]$$

```
In[6]:= P

In[7]:= hypAttributes
```

```
Automatic evaluation of p and F is active.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.

In[8]:= p[a,5]

Out[8]= a (1 + a) (2 + a) (3 + a) (4 + a)

In[9]:= F[{a,b},{c},z]
Is -a a nonnegative integer?
[y|n]:    n
Is -b a nonnegative integer?
[y|n]:    n
A hypergeometric series is converted into a sum.
Enter a variable for the summation index:  k

          ∞   k
        ----- z  (a)  (b)
         \          k    k
Out[9]=   〉  ------------
         /     (1)  (c)
        -----     k    k
         k=0

In[10]:= F[{-n,b},{c},z]
Is n a nonnegative integer?
[y|n]:    y
Is -b a nonnegative integer?
[y|n]:    n
A hypergeometric series is converted into a sum.
Enter a variable for the summation index:  j

           n   j
         ----- z  (b)  (-n)
          \          j     j
Out[10]=   〉  -------------
          /     (1)  (c)
         -----     j    j
          j=0

In[11]:= F[{-3,b},{c},z]
Is -b a nonnegative integer?
[y|n]:    n
A hypergeometric series is converted into a sum.
Enter a variable for the summation index:  s
```

```
                                 2                      3
          3 b z   3 b (1 + b) z    b (1 + b) (2 + b) z
Out[11]= 1 - ----- + -------------- - --------------------
            c       c (1 + c)       c (1 + c) (2 + c)

In[12]:= P

In[13]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised hypergeometric series.
```

See also: `F, H, p, hypAttributes.`

---

paufl

Description: Rule that writes $(x)_n$ as the defining product $\prod_{i=0}^{n-1}(x+i)$, if n is an integer.

Usage: `Expr/.paufl.`

Example(s):

```
In[1]:= p[a,-2]/p[b,3]*p[c,1]

        (a)   (c)
           -2    1
Out[1]= ----------
           (b)
              3

In[2]:= %/.paufl

                          c
Out[2]= ----------------------------------
        (-2 + a) (-1 + a) b (1 + b) (2 + b)

In[3]:= F[{a,b,c},{d,e,f},z]

            [ a, b, c        ]
Out[3]=  F  [           ; z  ]
        3 3 [ d, e, f        ]

In[4]:= %/.C01
```

```
                  [ 1, 1 + a, 1 + b, 1 + c          ]
           z   F  [                              ; z ] (a)  (b)  (c)
              4 4 [ 2, 1 + d, 1 + e, 1 + f          ]    1    1    1

Out[4]= 1 + -------------------------------------------------
                                (d)  (e)  (f)
                                   1    1    1
```

```
In[5]:= %/.paufl
```

```
                       [ 1, 1 + a, 1 + b, 1 + c        ]
           a b c z   F [                            ; z ]
                    4 4[ 2, 1 + d, 1 + e, 1 + f        ]

Out[5]= 1 + ----------------------------------------
                                d e f
```

See also: pzerl, pzus, p, Gzerl, Gzus, Ers, PosListe.

PosListe

Description: Function that provides a list of subexpressions of Expr together with the respective positions in Expr. This helps to use controlled application of rules or functions by means of Ers.

Usage: PosListe[Expr].

Example(s):

```
In[1]:= p[a,n]/p[1,n]*SUM[p[b,k]/p[c,k+1]*a^k,{k,0,Infinity}]
```

```
                 ∞    k
                ┌───┐a   (b)
                 \          k
        (a)   (  〉  --------)
           n    /    (c)
                └───┘     1 + k
                k=0
Out[1]= --------------------
                (1)
                   n
```

```
In[2]:= PosListe[%]
```

```
                                             ∞   k
                                            ┏━━┓a  (b)
          1                                 \         k
Out[2]= {{----, {{1}}}, {(a)  , {{2}}}, {   〉  --------, {{3}}}}
         (1)                n               /   (c)
            n                               ┗━━┛   1 + k
                                            k=0

In[3]:= PosListe[%%,2]

Out[3]= {{-1, {{1, 2}}}, {a, {{2, 1}}}, {n, {{2, 2}}}, {{k, 0, ∞}, {{3, 2}}},

                           k
                          a  (b)
                               k
〉   {(1)  , {{1, 1}}}, {--------, {{3, 1}}}}
        n               (c)
                           1 + k

In[4]:= PosListe[%%%,3]

                                              k
Out[4]= {{0, {{3, 2, 2}}}, {1, {{1, 1, 1}}}, {a , {{3, 1, 1}}},

〉   {k, {{3, 2, 1}}}, {n, {{1, 1, 2}}}, {∞, {{3, 2, 3}}},

                              1
〉   {(b)  , {{3, 1, 2}}}, {--------, {{3, 1, 3}}}}
        k                 (c)
                             1 + k
```

See also: `Ers`, `Subst`.

PSort

Description: Rule that orders the parameters of hypergeometric series `F[List1,List2,z]`, `H[ ... ]`, `V[ ... ]`, of "multiple" Pochhammer symbols `p[List1,List2,n]`, and of "multiple" Gamma functions `GAMMA[List1,List2]` in a standard order. For instance, this function can be used for a quick test if two expressions agree. It is recommended to apply `Gzus` and `pzus[n]` first.

Usage: `Expr/.PSort`.

Example(s):

```
In[1]:= p[{c,c-a,b},{a-b-c,b+1},n]*GAMMA[{b-a,c},{a+b+c,-b}]*
        F[{b-c,b+a,a},{b,a},z]
```

```
                                            (c, -a + c, b)
             b - c, a + b        -a + b, c                 n
Out[1]=  F                ; z  Γ             --------------------
        2 1      b               a + b + c, -b   (a - b - c, 1 + b)
                                                                 n
```

```
In[2]:= %/.PSort
```

```
                                            (b, c, -a + c)
             a + b, b - c        -a + b, c                 n
Out[2]=  F                ; z  Γ             --------------------
        2 1      b               -b, a + b + c   (1 + b, a - b - c)
                                                                 n
```

See also: `SimplifyP`, `SUMExpand`, `FEinf`, `FOrdne`, `FPerm`, `FTausche`, `F`, `H`, `V`, `p`, `GAMMA`.

---

pzerl

Description: Rule that splits `p[List1,List2,n]` into a quotient of products of factorial symbols (Pochhammer symbols).

Usage: `Expr/.pzerl`.

Example(s):

```
In[1]:= p[{a,b},{c,d},n]

        (a, b)
              n
Out[1]= -------
        (c, d)
              n

In[2]:= %/.pzerl

        (a)  (b)
           n    n
Out[2]= ---------
        (c)  (d)
           n    n
```

See also: `paufl`, `pzus`, `p`, `Gzerl`, `Gzus`, `Ers`, `PosListe`.

---

pzus

Description: Rule that collects several factorial symbols (Pochhammer symbols) `p[`x_i`,n]` to an expression `p[List1,List2,n]`.

Usage: `Expr/.pzus[n]`.

Example(s):

```
In[1]:= p[a,n]*p[b,m]/p[c,n]/p[d,m]
```

```
        (a)  (b)
           n    m
Out[1]= ---------
        (c)  (d)
           n    m

In[2]:= %/.pzus[n]

             (a)
                n
        (b)  ----
           m (c)
                n
Out[2]= ---------
          (d)
             m

In[3]:= %/.pzus[m]

        (a)  (b)
           n    m
Out[3]= ---- ----
        (c)  (d)
           n    m
```

See also: paufl, pzerl, p, Gzus, Gzerl, Ers, PosListe.

RS

Description: RS is the right-hand side in Gleichung.

Usage: RS.

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n

                                 (-a + c)
           [ a, -n          ]            n
Out[1]=  F [          ; 1   ] == ---------
        2 1[   c            ]      (c)
                                      n

In[2]:= RS

        (-a + c)
                n
Out[2]= ---------
          (c)
             n
```

```
In[3]:= Add[1]

                                          (-a + c)
                 [ a, -n       ]                  n
Out[3]= 1 +  F   [        ; 1  ] == 1 + ---------
            2 1  [   c         ]          (c)
                                             n

In[4]:= RS

             (-a + c)
                     n
Out[4]= 1 + ---------
              (c)
                 n

In[5]:= RS=1/p[1,m]

          1
Out[5]= ----
        (1)
           m

In[6]:= Gleichung

                 [ a, -n       ]       1
Out[6]= 1 +  F   [        ; 1  ] ==  ----
            2 1  [   c         ]      (1)
                                         m
```

See also: `Gleichung`, `SumListe$gl`, `TransListe$gl`, `LS`, `Mal`, `Add`, `Div`, `Sub`, `Hoch`, `GlTausche`, `Ers`, `Subst`.

S1001

Description: Summation formula ([7], Appendix (III.1)) in form of a rule. A q-analogue is HYPQ's `S1001`.

$$ {}_1F_0\left[\begin{matrix} a \\ - \end{matrix}; z\right] \longrightarrow (1-z)^{-a} $$

See also: `S2103`, `S3201`, `SListe`, `SumListe`, `Ers`, `PosListe`.

S2101

Description: Summation formula ([7], (1.7.7); Appendix (III.4)) in form of a rule. A q-analogue is HYPQ's `S2101`.

$$ {}_2F_1\left[\begin{matrix} a, -n \\ c \end{matrix}; 1\right] \longrightarrow \frac{(c-a)_n}{(c)_n} $$

where n is a nonnegative integer.

See also: `S2103`, `S3201`, `SListe`, `SumListe`, `Ers`, `PosListe`.

S2103

Description: Summation formula ([7], (1.7.6); Appendix (III.3)) in form of a rule. A q-analogue is HYPQ's `S2103`.

$${}_2F_1\left[\begin{matrix} a, b \\ c \end{matrix}; 1\right] \longrightarrow \Gamma\left[\begin{matrix} c, c-a-b \\ c-a, -b+c \end{matrix}\right]$$

Example(s):

```
In[1]:= F[{a,b},{2+a+b},1]
```

$$\text{Out[1]= } {}_2F_1\left[\begin{matrix} a,\ b \\ 2 + a + b \end{matrix}; 1\right]$$

```
In[2]:= %/.S2103
```

$$\text{Out[2]= } \Gamma\left[\begin{matrix} 2 + a + b,\ 2 \\ 2 + b,\ 2 + a \end{matrix}\right]$$

See also: `S3201, SListe, SumListe, Ers, PosListe.`

S2104

Description: Summation formula ([7], (1.7.1.6), corrected, (2.3.2.9); Appendix (III.5)) in form of a rule. A q-analogue is HYPQ's `S2104`.

$${}_2F_1\left[\begin{matrix} a, b \\ 1+a-b \end{matrix}; -1\right] \longrightarrow \Gamma\left[\begin{matrix} 1+\frac{a}{2}, 1+a-b \\ 1+a, 1+\frac{a}{2}-b \end{matrix}\right]$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2105

Description: Summation formula ([2], Ex. 1.6(i), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `S2105`.

$${}_2F_1\left[\begin{matrix} -\frac{n}{2}, \frac{1}{2}-\frac{n}{2} \\ \frac{1}{2}+b \end{matrix}; 1\right] \longrightarrow \frac{2^n (b)_n}{(2b)_n}$$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2106

Description: Summation formula ([7], (1.5.21)) in form of a rule. q-Analogues are HYPQ's `S2106` and `S3203`.

$${}_2V_1(a; -; z) \longrightarrow (1-z)^{-1-a}(1+z)$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2131

Description: Summation formula ([7], (1.7.1.8); Appendix (III.7)) in form of a rule. A q-analogue is HYPQ's `S2201`.

$$ {}_2F_1\left[\begin{matrix} a, 1-a \\ b \end{matrix}; \frac{1}{2}\right] \longrightarrow \Gamma\left[\begin{matrix} \frac{b}{2}, \frac{1}{2}+\frac{b}{2} \\ \frac{a}{2}+\frac{b}{2}, \frac{1}{2}-\frac{a}{2}+\frac{b}{2} \end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2132

Description: Summation formula ([7], (1.7.1.9); Appendix (III.6)) in form of a rule. A q-analogue is HYPQ's `S2202`.

$$ {}_2F_1\left[\begin{matrix} 2a, 2b \\ \frac{1}{2}+a+b \end{matrix}; \frac{1}{2}\right] \longrightarrow \Gamma\left[\begin{matrix} \frac{1}{2}, \frac{1}{2}+a+b \\ \frac{1}{2}+a, \frac{1}{2}+b \end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2210

Description: Summation formula ([7], (6.1.2.6), $d \to -\infty$) in form of a rule. A q-analogue is HYPQ's `S2210`.

$$ {}_2H_2\left[\begin{matrix} b, c \\ 1+a-b, 1+a-c \end{matrix}; -1\right] \longrightarrow \Gamma\left[\begin{matrix} 1-\frac{a}{2}, 1+\frac{a}{2}, 1-b, 1+a-b, 1-c, 1+a-c \\ 1-a, 1+a, 1+\frac{a}{2}-b, 1+\frac{a}{2}-c, 1+a-b-c \end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S2240

Description: Summation formula ([7], (6.1.2.1), Appendix (III.28)) in form of a rule. A q-analogue is HYPQ's `S3310`.

$$ {}_2H_2\left[\begin{matrix} a, b \\ c, d \end{matrix}; 1\right] \longrightarrow \Gamma\left[\begin{matrix} c, d, 1-a, 1-b, -1-a-b+c+d \\ -a+c, -a+d, -b+c, -b+d \end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3201

Description: Summation formula ([7], (2.3.1.3); Appendix (III.2)) in form of a rule. A q-analogue is HYPQ's `S3201`.

$$ {}_3F_2\left[\begin{matrix} a, b, -n \\ c, 1+a+b-c-n \end{matrix}; 1\right] \longrightarrow \frac{(c-a)_n(c-b)_n}{(c)_n(c-a-b)_n} $$

where n is a nonnegative integer.

Example(s):

```
In[1]:= F[{a,b,-n},{a+b-n,1},1]
```

```
                 a, b, -n
Out[1]=  F                  ; 1
        3 2  a + b - n, 1
```

```
In[2]:= %/.S3201
Is n a nonnegative integer?
[y|n]:    y

        (a - n)  (b - n)
               n        n
Out[2]= ------------------
        (a + b - n)  (-n)
                   n     n

In[3]:= %%/.S3201
Is n a nonnegative integer?
[y|n]:    n
Is -b a nonnegative integer?
[y|n]:    n
Is -a a nonnegative integer?
[y|n]:    n
```

$$\text{Out[3]= } {}_3F_2\left[\begin{matrix} a,\ b,\ -n \\ a+b-n,\ 1 \end{matrix};\ 1\right]$$

```
In[4]:= F[{a,-m,-n},{a-m-n,1},1]
```

$$\text{Out[4]= } {}_3F_2\left[\begin{matrix} a,\ -m,\ -n \\ a-m-n,\ 1 \end{matrix};\ 1\right]$$

```
In[5]:= %/.S3201
Is n a nonnegative integer?
[y|n]:    n
Is m a nonnegative integer?
[y|n]:    y

        (a - m)  (-m - n)
               m         m
Out[5]= ------------------
        (-m)  (a - m - n)
            m            m
```

See also: `S2103, SListe, SumListe, Ers, PosListe.`

S3202

Description: Summation formula ([7], (2.3.3.5); Appendix (III.8); terminated in the first variable) in form of a rule. A q-analogue is HYPQ's `S3202`.

$$ {}_3F_2\left[\begin{matrix}-2n, b, c\\ 1-b-2n, 1-c-2n\end{matrix};1\right] \longrightarrow \frac{(1)_{2n}(b)_n(c)_n(b+c)_{2n}}{(1)_n(b)_{2n}(c)_{2n}(b+c)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3204

Description: Summation formula ([2], Ex. 3.9, $q \to 1$) in form of a rule. q-Analogues are HYPQ's `S3204` and `S8704`.

$$ {}_3F_2\left[\begin{matrix}a, 1+\frac{\lambda}{2}, b\\ \frac{\lambda}{2}, 1-b+\lambda\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}\lambda, 1-a+\lambda, -a-2b+\lambda, 1-b+\lambda\\ 1+\lambda, -a+\lambda, -2b+\lambda, 1-a-b+\lambda\end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3231

Description: Summation formula ([7], (2.3.3.5); Appendix (III.8)) in form of a rule. A q-analogue is HYPQ's `S4301`.

$$ {}_3F_2\left[\begin{matrix}a, b, c\\ 1+a-b, 1+a-c\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}1+a-b, 1+a-c, 1+\frac{a}{2}, 1+\frac{a}{2}-b-c\\ 1+a, 1+\frac{a}{2}-b, 1+\frac{a}{2}-c, 1+a-b-c\end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3232

Description: Summation formula ([7], (2.3.3.6); Appendix (III.9)) in form of a rule. A q-analogue is HYPQ's `S4302`.

$$ {}_3F_2\left[\begin{matrix}a, b, -n\\ 1+a-b, 1+a+n\end{matrix};1\right] \longrightarrow \frac{(1+a)_n(1+\frac{a}{2}-b)_n}{(1+\frac{a}{2})_n(1+a-b)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3233

Description: Summation formula ([7], (2.3.3.13); Appendix (III.23)) in form of a rule. A q-analogue is HYPQ's `S8702`, a terminating q-analogue is HYPQ's `S4303`.

$$ {}_3F_2\left[\begin{matrix}a, b, c\\ \frac{1+a+b}{2}, 2c\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}\frac{1}{2}, \frac{1}{2}+c, \frac{1}{2}+\frac{a}{2}+\frac{b}{2}, \frac{1}{2}-\frac{a}{2}-\frac{b}{2}+c\\ \frac{1}{2}+\frac{a}{2}, \frac{1}{2}+\frac{b}{2}, \frac{1}{2}-\frac{a}{2}+c, \frac{1}{2}-\frac{b}{2}+c\end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3234

Description: Summation formula ([7], (2.3.3.14); Appendix (III.24)) in form of a rule. A q-analogue is HYPQ's `S8703`, a terminating q-analogue is HYPQ's `S4304`.

$$ {}_3F_2\left[\begin{matrix} a, 1-a, c \\ d, 1+2c-d \end{matrix}; 1\right] \longrightarrow 2^{1-2c}\pi\Gamma\left[\begin{matrix} d, 1+2c-d \\ \frac{1}{2}+\frac{a}{2}+c-\frac{d}{2}, \frac{a}{2}+\frac{d}{2}, 1-\frac{a}{2}+c-\frac{d}{2}, \frac{1}{2}-\frac{a}{2}+\frac{d}{2} \end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3235

Description: Summation formula ([7], (2.4.2.5); Appendix (III.16)) in form of a rule. A q-analogue is HYPQ's `S4308`.

$$ {}_3F_2\left[\begin{matrix} a, b, -n \\ 1+a-b, 1+2b-n \end{matrix}; 1\right] \longrightarrow \frac{(1+\frac{a}{2}-b)_n(a-2b)_n(-b)_n}{(\frac{a}{2}-b)_n(1+a-b)_n(-2b)_n} $$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3261

Description: Summation formula ([7], (2.4.4.4); Appendix (III.31)) in form of a rule. A q-analogue is HYPQ's `S3261`.

$$ {}_3F_2\left[\begin{matrix} a, b, c \\ d, 1+a+b+c-d \end{matrix}; 1\right] \longrightarrow \Gamma\left[\begin{matrix} 1+a-d, 1+b-d, 1+c-d, 1+a+b+c-d \\ 1-d, 1+b+c-d, 1+a+c-d, 1+a+b-d \end{matrix}\right] $$
$$ -\Gamma\left[\begin{matrix} d-1, 1+a-d, 1+b-d, 1+c-d, 1+a+b+c-d \\ 1-d, a, b, c, 2+a+b+c-2d \end{matrix}\right] {}_3F_2\left[\begin{matrix} 1+a-d, 1+b-d, 1+c-d \\ 2-d, 2+a+b+c-2d \end{matrix}; 1\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3291

Description: Summation formula ([2], Ex. 2.9, $q \to 1$) in form of a rule. A q-analogue is HYPQ's `S4361`.

$$ {}_3F_2\left[\begin{matrix} a, b, 1+a-2c \\ 1+a-c, 2b \end{matrix}; 1\right] \longrightarrow \Gamma\left[\begin{matrix} \frac{1}{2}, \frac{1}{2}+\frac{a}{2}-b, 1+a-c, 1+\frac{a}{2}-b-c \\ \frac{1}{2}+\frac{a}{2}, \frac{1}{2}-b, 1+\frac{a}{2}-c, 1+a-b-c \end{matrix}\right] $$
$$ -{}_3F_2\left[\begin{matrix} 1-b, 1+a-2b, 2+a-2b-2c \\ 2+a-2b-c, 2-2b \end{matrix}; 1\right] \Gamma\left[\begin{matrix} \frac{1}{2}+\frac{a}{2}-b, 1+\frac{a}{2}-b, -\frac{1}{2}+b, 1+a-c, 1+\frac{a}{2}-b-c, \frac{3}{2}+\frac{a}{2}-b-c \\ \frac{1}{2}+\frac{a}{2}, \frac{a}{2}, \frac{1}{2}-b, \frac{1}{2}+\frac{a}{2}-c, 1+\frac{a}{2}-c, 2+a-2b-c \end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S3340

Description: Summation formula ([7], (6.1.2.6), Appendix (III.30)) in form of a rule. A q-analogue is HYPQ's `S4410`.

$$ {}_3H_3\left[\begin{matrix} b, c, d \\ 1+a-b, 1+a-c, 1+a-d \end{matrix}; 1\right] $$
$$ \longrightarrow \Gamma\left[\begin{matrix} 1-\frac{a}{2}, 1+\frac{a}{2}, 1-b, 1+a-b, 1-c, 1+a-c, 1-d, 1+a-d, 1+\frac{3a}{2}-b-c-d \\ 1-a, 1+a, 1+\frac{a}{2}-b, 1+\frac{a}{2}-c, 1+a-b-c, 1+\frac{a}{2}-d, 1+a-b-d, 1+a-c-d \end{matrix}\right] $$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4306

Description: Summation formula ([7], (2.4.2.6); Appendix (III.17)) in form of a rule. A q-analogue is HYPQ's `S4306`.

$${}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, -n \\ \frac{a}{2}, 1+a-b, 1+2b-n \end{matrix}; 1\right] \longrightarrow \frac{(a-2b, -b)_n}{(1+a-b, -2b)_n}$$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4307

Description: Summation formula ([7], (2.3.4.6); Appendix (III.10)) in form of a rule. A q-analogue is HYPQ's `S4307`.

$${}_4V_3(a; b, c; -1) \longrightarrow \Gamma\left[\begin{matrix} 1+a-b, 1+a-c \\ 1+a, 1+a-b-c \end{matrix}\right]$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4331

Description: Summation formula ([7], Appendix (III.22)) in form of a rule. q-Analogues are HYPQ's `S5401` and `S5501`.

$${}_4V_3(a; b, c; 1) \longrightarrow \Gamma\left[\begin{matrix} 1+a-b, 1+a-c, \frac{1}{2}+\frac{a}{2}, \frac{1}{2}+\frac{a}{2}-b-c \\ 1+a, 1+a-b-c, \frac{1}{2}+\frac{a}{2}-b, \frac{1}{2}+\frac{a}{2}-c \end{matrix}\right]$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S4332

Description: Summation formula ([7], (2.4.2.7); Appendix (III.18)) in form of a rule. A q-analogue is HYPQ's `S5402`.

$${}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, -n \\ \frac{a}{2}, 1+a-b, 2+2b-n \end{matrix}; 1\right] \longrightarrow \frac{(\frac{1}{2}+\frac{a}{2}-b)_n(-1+a-2b)_n(-1-b)_n}{(-\frac{1}{2}+\frac{a}{2}-b)_n(1+a-b)_n(-1-2b)_n}$$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S5431

Description: Summation formula ([7], (2.3.4.5); Appendix (III.12)) in form of a rule. A q-analogue is HYPQ's `S6501`.

$${}_5V_4(a; b, c, d; 1) \longrightarrow \Gamma\left[\begin{matrix} 1+a-b, 1+a-c, 1+a-d, 1+a-b-c-d \\ 1+a, 1+a-b-c, 1+a-b-d, 1+a-c-d \end{matrix}\right]$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S5432

Description: Summation formula ([7], (2.3.4.6); Appendix (III.13)) in form of a rule. A q-analogue is HYPQ's `S6502`.

$${}_5V_4(a; b, c, -n; 1) \longrightarrow \frac{(1+a)_n(1+a-b-c)_n}{(1+a-b)_n(1+a-c)_n}$$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S5540

Description: Summation formula ([7], (6.1.2.5), Appendix (III.29)) in form of a rule. A q-analogue is HYPQ's `S6610`.

$$
{}_5H_5\left[\begin{matrix} 1+\frac{a}{2}, b, c, d, e \\ \frac{a}{2}, 1+a-b, 1+a-c, 1+a-d, 1+a-e \end{matrix}; 1\right]
\longrightarrow \Gamma\left[\begin{matrix} 1-b, 1+a-b, 1-c, 1+a-c, 1-d, 1+a-d, 1-e, 1+a-e, 1+2a-b-c-d-e \\ 1-a, 1+a, 1+a-b-c, 1+a-b-d, 1+a-c-d, 1+a-b-e, 1+a-c-e, 1+a-d-e \end{matrix}\right]
$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S6531

Description: Summation formula ([8], (1.8)) in form of a rule. A q-analogue is HYPQ's `S8702`.

$$
{}_6V_5(-\frac{1}{2}+\frac{a}{2}+\frac{b}{2}+c; a, b, c, \frac{1}{2}+\frac{a}{2}+\frac{b}{2}-c; -1) \longrightarrow \Gamma\left[\begin{matrix} \frac{1}{2}, \frac{1}{2}+\frac{a}{2}+\frac{b}{2}, \frac{1}{2}+c, \frac{1}{2}+\frac{a}{2}-\frac{b}{2}+c, \frac{1}{2}-\frac{a}{2}+\frac{b}{2}+c \\ \frac{1}{2}+\frac{a}{2}, \frac{1}{2}+\frac{b}{2}, \frac{1}{2}-\frac{a}{2}+c, \frac{1}{2}-\frac{b}{2}+c, \frac{1}{2}+\frac{a}{2}+\frac{b}{2}+c \end{matrix}\right]
$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S6532

Description: Summation formula ([7], Appendix (III.27), corrected; [8], (1.7)) in form of a rule. A q-analogue is HYPQ's `S8703`.

$$
{}_6V_5(a; b, 1-b, d, 1-d; -1) \longrightarrow 2^{2b}\Gamma\left[\begin{matrix} 1+a-b, a+b, 1+a-d, 1+\frac{a}{2}+\frac{b}{2}-\frac{d}{2}, \frac{1}{2}+\frac{a}{2}+\frac{b}{2}+\frac{d}{2}, a+d \\ a, 1+a, 1+a+b-d, 1+\frac{a}{2}-\frac{b}{2}-\frac{d}{2}, \frac{1}{2}+\frac{a}{2}-\frac{b}{2}+\frac{d}{2}, a+b+d \end{matrix}\right]
$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S7631

Description: Summation formula ([7], (2.3.4.4); Appendix (III.14)) in form of a rule. A q-analogue is HYPQ's `S8701`.

$$
{}_7V_6(a; b, c, d, 1+2a-b-c-d+n, -n; 1) \longrightarrow \frac{(1+a)_n(1+a-b-c)_n(1+a-b-d)_n(1+a-c-d)_n}{(1+a-b)_n(1+a-c)_n(1+a-d)_n(1+a-b-c-d)_n}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S7632

Description: Summation formula ([7], (2.4.1.5); Appendix (III.19)) in form of a rule. A q-analogue is HYPQ's `S10901`.

$$
{}_7V_6(a; b, \frac{1}{2}+b, a-2b, 1+2a-2b+n, -n; 1) \longrightarrow \frac{(1+a, 1+2a-4b)_n}{(1+a-2b, 1+2a-2b)_n}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

S7691

Description: Summation formula ([7], (4.2.3.8); Appendix (III.32)) in form of a rule. A q-analogue is HYPQ's `S8761`.

$$
\begin{aligned}
&{}_7V_6(a; b, c, d, e, 1+2a-b-c-d-e; 1)\\
&\quad\longrightarrow \Gamma\left[\begin{matrix}1+a-c, 1+a-d, 1+a-e, -a+b+c+d+e, -a+b+c, -a+b+d, -a+b+e, 1+a-c-d-e\\ 1+a, -a+b, 1+a-c-d, 1+a-c-e, -a+b+d+e, 1+a-d-e, -a+b+c+e, -a+b+c+d\end{matrix}\right]\\
&\qquad - \Gamma\left[\begin{matrix}a-b, 1+a-c, 1+a-d, 1+a-e, -a+b+c+d+e, -a+b+c, -a+b+d, -a+b+e,\\ 1+a, c, d, e, 1+2a-b-c-d-e, -a+b, 1+b-c, 1+b-d,\end{matrix}\right.\\
&\qquad\qquad \left.\begin{matrix}1+a-c-d-e, 1-a+2b\\ 1+b-e, -2a+2b+c+d+e\end{matrix}\right] {}_7V_6(-a+2b; b, -a+b+c, -a+b+d, -a+b+e, 1+a-c-d-e; 1)
\end{aligned}
$$

See also: `S2103, S3201, SListe, SumListe, Ers, PosListe.`

SchreibeZahl

Description: Variable that counts the number of expressions already written by using `TeXMat`. Can be reset by defining a new value.

Usage: `SchreibeZahl=n_Integer.`

Example(s):

```
In[1]:= SchreibeZahl

Out[1]= 0

In[2]:= TeXMat[p[a,n],filename]

In[3]:= !type filename.m
A[1]:=
p[a, n]

In[3]:= !type filename.tex
A[1]:=
({ \textstyle a}) _{n}

In[3]:= SchreibeZahl=4

Out[3]= 4

In[4]:= TeXMat[p[a,n],filename]

In[5]:= !type filename.m
A[1]:=
p[a, n]
A[5]:=
p[a, n]
```

```
In[5]:= !type filename.tex
A[1]:=
({ \textstyle a}) _{n}
A[5]:=
({ \textstyle a}) _{n}
```

See also: `TeXMat`.

Sgl1001

Description: Summation formula ([7], Appendix (III.1)) in form of an equation. It is the same summation as that in `S1001`.

See also: `Sgl2101, SumListe$gl, Gleichung`.

Sgl2101

Description: Summation formula ([7], Appendix (III.4)) in form of an equation. It is the same summation as that in `S2101`.

Example(s):

```
In[1]:= {a,c}

Out[1]= {a, c}

In[2]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n

                           (-a + c)
             [ a, -n         ]        n
Out[2]=  F   [        ;  1   ] == ---------
        2 1  [   c           ]      (c)
                                       n

In[3]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  y
a=2*a
c=-b
n=n

                               (-2 a - b)
             [ 2 a, -n         ]          n
Out[3]=  F   [          ;  1   ] == -----------
        2 1  [   -b            ]       (-b)
                                          n

In[4]:= {a,c}

Out[4]= {a, c}
```

```
In[5]:= a=2*b

Out[5]= 2 b

In[6]:= Sgl2101
Some variables have a value.  Should the variables
{a, c, n} be cleared?  Do you want to set
values for the equation (v)?  [y|n|yv|nv]:  nv
a=a
c=c
n=n
```

Out[6]= $${}_2F_1\left[\begin{matrix}2\ b,\ -n \\ c\end{matrix}\ ;\ 1\right] == \frac{(-2\ b + c)_n}{(c)_n}$$

```
In[7]:= {a,c}

Out[7]= {2 b, c}

In[8]:= Sgl2101
Some variables have a value.  Should the variables
{a, c, n} be cleared?  Do you want to set
values for the equation (v)?  [y|n|yv|nv]:  y
```

Out[8]= $${}_2F_1\left[\begin{matrix}a,\ -n \\ c\end{matrix}\ ;\ 1\right] == \frac{(-a + c)_n}{(c)_n}$$

```
In[9]:= {a,c}

Out[9]= {a, c}

In[10]:= c=3*e

Out[10]= 3 e

In[11]:= Sgl2101
Some variables have a value.  Should the variables
{a, c, n} be cleared?  Do you want to set
values for the equation (v)?  [y|n|yv|nv]:  yv
a=w+2
c=d
n=n
```

```
                                      (-2 + d - w)
          [ 2 + w, -n        ]                    n
Out[11]=  F [           ; 1  ] == -------------
        2 1 [     d          ]          (d)
                                           n
```

```
In[12]:= {a,c}

Out[12]= {a, c}
```

See also: `SumListe$gl`, `Gleichung`.

Sgl2103

Description: Summation formula ([7], Appendix (III.3)) in form of an equation. It is the same summation as that in `S2103`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl2104

Description: Summation formula ([7], Appendix (III.5)) in form of an equation. It is the same summation as that in `S2104`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl2105

Description: Summation formula ([2], Ex. 1.6(i), $q \to 1$) in form of an equation. It is the same summation as that in `S2105`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl2106

Description: Summation formula ([7], (1.5.21)) in form of an equation. It is the same summation as that in `S2106`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl2131

Description: Summation formula ([7], Appendix (III.7)) in form of an equation. It is the same summation as that in `S2131`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl2132

Description: Summation formula ([7], Appendix (III.6)) in form of an equation. It is the same summation as that in `S2132`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl2210

Description: Summation formula ([7], (6.1.2.6), $d \to -\infty$) in form of an equation. It is the same summation as that in S2210.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl2240

Description: Summation formula ([7], (6.1.2.1), Appendix (III.28)) in form of an equation. It is the same summation as that in S3310.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl3201

Description: Summation formula ([7], Appendix (III.2)) in form of an equation. It is the same summation as that in S3201.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl3202

Description: Summation formula ([7], Appendix (III.8), terminated in the first variable) in form of an equation. It is the same summation as that in S3202.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl3204

Description: Summation formula ([2], Ex. 3.9, $q \to 1$) in form of an equation. It is the same summation as that in S3204.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl3231

Description: Summation formula ([7], Appendix (III.8)) in form of an equation. It is the same summation as that in S3231.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl3232

Description: Summation formula ([7], Appendix (III.9)) in form of an equation. It is the same summation as that in S3232.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl3233

Description: Summation formula ([7], Appendix (III.23)) in form of an equation. It is the same summation as that in S3233.

See also: Sgl2101, SumListe$gl, Gleichung.

Sgl3234

Description: Summation formula ([7], Appendix (III.24)) in form of an equation. It is the same summation as that in `S3234`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl3235

Description: Summation formula ([7], (2.4.2.5); Appendix (III.16)) in form of an equation. It is the same summation as that in `S3235`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl3261

Description: Summation formula ([7], Appendix (III.31)) in form of an equation. It is the same summation as that in `S3261`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl3291

Description: Summation formula ([2], Ex. 2.9, $q \to 1$) in form of an equation. It is the same summation as that in `S3291`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl3340

Description: Summation formula ([7], (6.1.2.6), Appendix (III.30)) in form of an equation. It is the same summation as that in `S4410`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl4306

Description: Summation formula ([7], (2.4.2.6); Appendix (III.17)) in form of an equation. It is the same summation as that in `S4306`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl4307

Description: Summation formula ([7], (2.3.4.6); Appendix (III.10)) in form of an equation. It is the same summation as that in `S4307`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl4331

Description: Summation formula ([7], Appendix (III.22)) in form of an equation. It is the same summation as that in `S4331`.

See also: `Sgl2101`, `SumListe$gl`, `Gleichung`.

Sgl4332

Description: Summation formula ([7], (2.4.2.7); Appendix (III.18)) in form of an equation. It is the same summation as that in `S4332`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl5431

Description: Summation formula ([7], Appendix (III.12)) in form of an equation. It is the same summation as that in `S5431`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl5432

Description: Summation formula ([7], Appendix (III.13)) in form of an equation. It is the same summation as that in `S5432`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl5540

Description: Summation formula ([7], (6.1.2.5), Appendix (III.29)) in form of an equation. It is the same summation as that in `S6610`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl6531

Description: Summation formula ([8], (1.8)) in form of an equation. It is the same summation as that in `S6531`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl6532

Description: Summation formula ([7], Appendix (III.27), corrected; [8], (1.7)) in form of an equation. It is the same summation as that in `S6532`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl7631

Description: Summation formula ([7], Appendix (III.14)) in form of an equation. It is the same summation as that in `S7631`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl7632

Description: Summation formula ([7], (2.4.1.5); Appendix (III.19)) in form of an equation. It is the same summation as that in `S7632`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

Sgl7691

Description: Summation formula ([7], Appendix (III.32)) in form of an equation. It is the same summation as that in `S7691`.

See also: `Sgl2101, SumListe$gl, Gleichung.`

SimplifyP

Description: Rule that simplifies arguments in `p`, `GAMMA`, `F`, `H`, `V`, `SUM`, and expands exponents in powers.

Usage: `Expr/.SimplifyP.`

Example(s):

```
In[1]:= p[a-(d-c)/2,(k-1)*2]/GAMMA[(f-g)^2]*F[{-(d+c)/2,-n},{-d/2-c/2},
        (t-1)/(1-t)]


             -c - d
            [------, -n         ]
            [  2         -1 + t ]      c - d
         F  [          ; ------ ] (a + -----)
        2 1 [  -c   d    1 - t  ]       2   2 (k - 1)
            [  -- - -           ]
               2    2
Out[1]= ----------------------------------------------
                                  2
                         Γ((f - g) )
```

```
In[2]:= %/.SimplifyP
Is c/2 + d/2 a nonnegative integer?
[y|n]:    n


            [-n     ]      c   d
         F  [  ; -1 ] (a + - - -)
        1 0 [-      ]      2   2 2 k - 2 1
Out[2]= ----------------------------------
                   2            2
                Γ(f   - 2 f g + g )
```

```
In[3]:= Simplify[%1]

             -c - d
            [------, -n    ]
            [  2           ]      c   d
         F  [          ; -1]  (a + - - -)
        2 1 [  -c   d      ]      2   2 2 (k - 1)
            [  -- - -      ]
               2    2
Out[3]= ------------------------------------------
                                2
                       Γ((f - g) )
```

See also: `Expandq`, `MinusOne`, `SUMExpand`, `PSort`.

SListe

Description: Rule that gives for a hypergeometric series a list of applicable summation formulas. Each entry of this list has the format {`S⟨number⟩`}, where `S⟨number⟩` is the name of the summation in form of a rule which can be applied subsequently. You should be aware that `SListe` automatically applies `FOrdne` before checking which summation could be applied.

Important Note: If the value returned by `SListe` is the empty set this does *not* mean that no summation can be applied. You always must remember that the list of summations included in this package is a list of *basic* summations. There are numerous special cases of these summations which are not contained in this list as a separate summation. The examples below should illustrate these remarks.

Usage: `Expr/.SListe.`

Example(s):

```
In[1]:= F[{a,b},{c},1]

                a, b
Out[1]=   F  [        ; 1 ]
         2 1    c


In[2]:= %/.SListe
Is -b a nonnegative integer?
[y|n]:    n
Is -a a nonnegative integer?
[y|n]:    n

Be sure to apply "FOrdne" before using the following information!

Out[2]= {{S2103}}

In[3]:= F[{a,-n},{c},1]

                a, -n
Out[3]=   F  [         ; 1 ]
         2 1     c


In[4]:= %/.SListe
Is n a nonnegative integer?
[y|n]:    y

Be sure to apply "FOrdne" before using the following information!

Out[4]= {{S2101}, {S2103}}
```

Now we consider two examples illustrating the note above. Though none of the implemented summations can be applied, both series can be summed, the first by a special case of Dougall's sum, the second by a special case of the very well-poised ${}_5F_4$ sum. These facts are also observed by using this package.

```
In[5]:= F[{-n,b,c,1-b-c-n/2,-n},{1-b-n,1-c-n,b+c-n/2,1},1]

                                     n
            -n, b, c, 1 - b - c - -, -n
                                     2
Out[5]=  F                                       ; 1
        5 4                              n
          1 - b - n, 1 - c - n, b + c - -, 1
                                         2

In[6]:= %/.SListe

Out[6]= {}

In[7]:= Sgl7631
Do you want to set values for the equation?  [y|n]:  y
a=-n
b=b
c=c
d=-n/2
n=n

                                     n
            -n, b, c, 1 - b - c - -, -n
                                     2
Out[7]=  F                                       ; 1   ==
        5 4                              n
          1 - b - n, 1 - c - n, b + c - -, 1
                                         2

                                           n            n
     (1 - n)  (1 - b - c - n)  (1 - b - -)  (1 - c - -)
          n                 n          2 n          2 n
⟩    ---------------------------------------------------
                                       n                n
     (1 - b - n)  (1 - c - n)  (1 - -)  (1 - b - c - -)
                n             n      2 n              2 n

In[8]:= F[{1+a/2,1,b,c},{a/2,1+a-b,1+a-c},1]

                   a
              1 + -, 1, b, c
                   2
Out[8]=  F                             ; 1
        4 3  a
             -, 1 + a - b, 1 + a - c
             2
```

```
In[9]:= %/.SListe

Out[9]= {}

In[10]:= %%/.FEinf
Add the parameter:  a

                        a
                 a, 1 + -, 1, b, c
                        2
Out[10]=  F                                ; 1
         5 4      a
             a, -, 1 + a - b, 1 + a - c
                  2

In[11]:= %/.SListe
Is -c a nonnegative integer?
[y|n]:    n

Be sure to apply "FOrdne" before using the following information!

Out[11]= {{S5431}}
```

See also: `TListe, FPerm, FTausche, SumListe.`

Sub

Description: Function that subtracts `Expr` from `Gleichung`.

Usage: `Sub[Expr].`

Example(s):

```
In[1]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  n

                         (-a + c)
          a, -n                  n
Out[1]=  F      ; 1  ==  ---------
        2 1   c             (c)
                               n

In[2]:= Sub[p[a,n]/p[c-a,n]]

                           (a)              (a)         (-a + c)
          a, -n                n                n                n
Out[2]=  F      ; 1  - --------- == -(---------) + ---------
        2 1   c          (-a + c)       (-a + c)        (c)
                                 n              n          n

In[3]:= Gleichung
```

```
                                 (a)            (a)          (-a + c)
                                    n              n                 n
          ┌  a, -n        ┐
Out[3]=  F          ; 1   │ - --------- == -(---------) + ---------
        2 1   c           │   (-a + c)       (-a + c)      (c)
          └               ┘           n               n        n
```

See also: Gleichung, SumListe$gl, TransListe$gl, LS, RS, Mal, Add, Div, Hoch, GlTausche, Ers.

Subst

Description: Function that substitutes RS instead of LS at position Position in Expr. The parameters LS and RS are optional. If they are omitted, the right-hand side “RS” of Gleichung is substituted instead of the left-hand side “LS” of Gleichung.

Usage: Subst[Expr,Position,LS,RS].

Example(s):

```
In[1]:= SUM[p[a,k]/p[b,k]/k!,{k,0,Infinity}]

          ∞
         ┌──┐ (a)
          \       k
Out[1]=    〉  -------
          /   k!  (b)
         └──┘        k
          k=0

In[2]:= Subst[%,{1},p[a,k]/p[b,k],F[{b-a,-k},{b},1]]

                ┌                ┐
                │  -a + b, -k    │
          ∞   F │                │
                │            ; 1 │
         ┌──┐2 1│      b         │
          \     └                ┘
Out[2]=    〉  --------------------
          /             k!
         └──┘
          k=0

In[3]:= Sgl2101
Do you want to set values for the equation?  [y|n]:  y
a=b-a
c=b
n=k

                                 (a)
          ┌              ┐          k
          │ -a + b, -k   │
Out[3]=  F│          ; 1 │ == ----
        2 1     b        │    (b)
          └              ┘       k
```

```
In[4]:= GlTausche

         (a)
            k            -a + b, -k
Out[4]= ---- ==  F                  ; 1
         (b)      2 1        b
            k

In[5]:= Gleichung

         (a)
            k            -a + b, -k
Out[5]= ---- ==  F                  ; 1
         (b)      2 1        b
            k

In[6]:= Subst[%1,{1}]

                     -a + b, -k
           ∞    F                ; 1
          ____ 2 1       b
          \
Out[6]=    >    -------------------
          /             k!
          ----
          k=0
```

<u>See also</u>: `Gleichung`, `SumListe$gl`, `TransListe$gl`, `LS`, `RS`, `GlTausche`, `Ers`, `PosListe`.

SUM

<u>Description</u>: This is HYP's internal object for entering sums. It should be used instead of Mathematica's `Sum`.

<u>Usage</u>: `SUM[Summand,summation-index,lower-bound,upper-bound]`.

<u>Example(s)</u>:

See the examples for `SUMF` and `SUMInfinity`.

<u>See also</u>: `SUMRegeln`, `SUMErw1`, `SUMErw2`, `SUMZerl`, `SUMShift`, `SUMSammle`, `SUMTausche`, `SUMF`.

SUMErw1

<u>Description</u>: Rule that extends a `SUM[]` at the top.

$$\sum_{k=l}^{n} \text{Expr} \rightarrow \sum_{k=l}^{n+m} \text{Expr} - \sum_{k=n+1}^{n+m} \text{Expr}.$$

The parameter `m` has to be entered on request.

<u>Usage</u>: `Expr/.SUMErw1`.

Example(s):

```
In[1]:= SUM[a[k],{k,0,N}]

          N
        ┌───┐
         \
Out[1]=   〉  a[k]
         /
        └───┘
         k=0

In[2]:= %/.SUMErw1
top-extend by:  3

                                               3 + N
                                              ┌───┐
                                               \
Out[2]= -a[1 + N] - a[2 + N] - a[3 + N] +       〉  a[k]
                                               /
                                              └───┘
                                               k=0

In[3]:= %%/.SUMErw1
top-extend by:  M

        M + N       M + N
        ┌───┐       ┌───┐
         \           \
Out[3]=   〉  a[k] -   〉   a[k]
         /           /
        └───┘       └───┘
         k=0       k=1 + N
```

See also: `SUM`, `SUMErw2`, `SUMZerl`, `SUMShift`, `SUMTausche`, `SUMRegeln`, `SUMUmkehr`, `Ers`, `PosListe`.

SUMErw2

Description: Rule that extends a `SUM[]` at the bottom.

$$\sum_{k=l}^{n} \text{Expr} \to \sum_{k=l-m}^{n} \text{Expr} - \sum_{k=l-m}^{l-1} \text{Expr}.$$

The parameter `m` has to be entered on request.

Usage: `Expr/.SUMErw2`.

Example(s):

```
In[1]:= SUM[a[k],{k,0,N}]
```

```
          N
         ____
         \
Out[1]=   >   a[k]
         /
         ----
         k=0

In[2]:= %/.SUMErw2
bottom-extend by:  3

                                           N
                                          ____
                                          \
Out[2]= -a[-3] - a[-2] - a[-1] +           >   a[k]
                                          /
                                          ----
                                          k=-3

In[3]:= %%/.SUMErw2
bottom-extend by:  M

           -1            N
          ____          ____
          \             \
Out[3]= -( >   a[k]) +   >   a[k]
          /             /
          ----          ----
          k=-M          k=-M
```

See also: `SUM`, `SUMErw1`, `SUMZerl`, `SUMShift`, `SUMTausche`, `SUMRegeln`, `SUMUmkehr`, `Ers`, `PosListe`.

SUMExpand

Description: Rule that expands `SUM`s.

Usage: `Expr/.SUMExpand`.

Example(s):

```
In[1]:= SUM[(x[k]-y[k])^2,{k,0,(m+n)/2}]
```

```
         m + n
         -----
           2
        ____             2
Out[1]=  \    (x[k] - y[k])
          >
         /
        ____
         k=0
```

In[2]:= %/.SUMExpand

```
         m   n         m   n                m   n
         - + -         - + -                - + -
         2   2         2   2                2   2
        ____     2    ____                 ____     2
Out[2]=  \    x[k]  +  \    -2 x[k] y[k] +  \    y[k]
          >             >                    >
         /             /                    /
        ____          ____                 ____
         k=0           k=0                  k=0
```

See also: SUM, SimplifyP, Expandq, MinusOne, PSort.

SUMF

Description: Rule that transforms a SUM[] into hypergeometric notation, if possible. If the upper bound is not Infinity you have to apply SUMInfinity first (if allowed).

Usage: Expr/.SUMF.

Example(s):

In[1]:= SUM[p[-n,k]/p[1,k]*a^k,{k,0,Infinity}]

```
          ∞   k
        ____a  (-n)
         \          k
Out[1]=   >  --------
         /      (1)
        ____       k
         k=0
```

In[2]:= %/.SUMF

```
            [ -n      ]
Out[2]=  F  [    ; a  ]
        1 0 [ -       ]
```

```
In[3]:= SUM[(k+2)*p[{-n,a},{b,c,1},k+1]*z^k,{k,0,Infinity}]

          ∞
         ____                (-n, a)
         \              k          1 + k
Out[3]=   >   (2 + k) z  --------------
         /                  (b, c, 1)
         ‾‾‾‾                       1 + k
         k=0

In[4]:= %/.SUMF

             [ 3, 1 + a, 1 - n, 1     ]
         2  F[                     ; z ] (a)   (-n)
           4 4[ 2, 2, 1 + b, 1 + c     ]    1      1
Out[4]= -----------------------------------------
                         (1)   (b)   (c)
                            1     1     1
```

<u>See also</u>: `SUM, F, SUMRegeln, SUMH, SUMInfinity, FSUM, Ers, PosListe.`

SUMH

<u>Description</u>: Rule that transforms a bilateral `SUM[]` into hypergeometric notation, if possible. If the upper bound is not `Infinity` you have to apply `SUMInfinity` first (if allowed). If the lower bound is not `Infinity` then `SUMF` is applied.

<u>Usage</u>: `Expr/.SUMH.`

<u>Example(s)</u>:

```
In[1]:= SUM[p[a,k]/p[b,k],{k,-Infinity,Infinity}]

          ∞
         ____(a)
         \      k
Out[1]=   >   ----
         /    (b)
         ‾‾‾‾    k
        k=-∞

In[2]:= %/.SUMH
```

```
            ┌      ┐
            │ a    │
Out[2]=  H  │  ; 1 │
        1 1 │ b    │
            └      ┘
```

See also: `SUM, H, SUMRegeln, SUMF, SUMInfinity, HSUM, Ers, PosListe.`

SUMInfinity

Description: Rule that changes the upper bound of a `SUM[]` to `Infinity`.

Usage: `Expr/.SUMInfinity.`

Example(s):

```
In[1]:= SUM[p[-n,k]/p[1,k]*a^k,{k,0,n}]

           n   k
         ┌──┐a  (-n)
          \          k
Out[1]=    >  --------
          /     (1)
         └──┘      k
          k=0

In[2]:= %/.SUMInfinity

           ∞   k
         ┌──┐a  (-n)
          \          k
Out[2]=    >  --------
          /     (1)
         └──┘      k
          k=0
```

See also: `SUM, SUMF, SUMH, Ers, PosListe.`

SumListe

Description: List of all summation formulas.

Usage: `SumListe.`

See also: `SumListe$gl, SListe.`

SumListe$gl

Description: List of all summation formulas.

Usage: `SumListe$gl.`

See also: `SumListe.`

SUMRegeln

Description: Rule that transforms the expressions in a `SUM[]` into a form that could also be expressed in hypergeometric notation. This is useful, if you want to convert a `SUM[]` into hypergeometric notation but without using the `F[]`-notation. In particular, expressions of the form $(-1)^{dk}$, where d is an integer and k is the summation index, will simplify.

Usage: `Expr/.SUMRegeln.`

Example(s):

```
In[1]:= SUM[Binomial[n,i]*Binomial[m,k-i],{i,0,Infinity}]

           ∞
          ┌──┐(          ) (   )
          \   (    m     ) ( n )
Out[1]=    〉  (          ) (   )
          /   ( -i + k ) ( i )
          └──┘(          ) (   )
          i=0

In[2]:= %/.SUMRegeln

                          ∞
                        ┌──┐    (-k)   (-n)
                         \           i      i
         (1 - k + m)  (   〉   -----------------)
                    k    /    (1)   (1 - k + m)
                        └──┘     i              i
                         i=0
Out[2]= -------------------------------------
                            (1)
                               k
```

See also: `SUM, F, H, SUMF, SUMH, FSUM, HSUM, MinusOne, Ers, PosListe.`

SUMSammle

Description: Rule that causes all terms of an expression `Expr`, which involves a `SUM[]` to be put into the `SUM[]`.

Usage: `Expr/.SUMSammle.`

Example(s):

```
In[1]:= p[a,n]/p[b,m]*(-1)^n*SUM[1/p[1,k],{k,0,Infinity}]
```

```
                  ∞
                ┌───┐
     n           \     1
(-1)   (a)   (   〉  ----)
         n       /    (1)
                └───┘     k
                 k=0
Out[1]= ----------------------
              (b)
                 m
```

```
In[2]:= %/.SUMSammle
```

```
         ∞       n
        ┌───┐(-1)   (a)
         \             n
Out[2]=   〉  ----------
         /    (1)   (b)
        └───┘     k     m
         k=0
```

See also: `SUM`, `SUMRegeln`, `SUMErw1`, `SUMErw2`, `SUMZerl`, `SUMShift`, `SUMTausche`, `pzus`, `Gzus`, `Ers`, `PosListe`.

SUMShift

Description: Rule that shifts the index in a `SUM[]`.

$$\sum_{k=l}^{n} \mathrm{Expr}(k) \to \sum_{k=l-m}^{n-m} \mathrm{Expr}(k+m).$$

The parameter `m` has to be entered on request.

Usage: `Expr/.SUMShift`.

Example(s):

```
In[1]:= SUM[a[k],{k,3,N}]
```

```
          N
        ┌───┐
         \
Out[1]=   〉  a[k]
         /
        └───┘
         k=3
```

```
In[2]:= %/.SUMShift
shift summation index by:  3
```

```
          -3 + N
          ┌──┐
           \
Out[2]=     〉   a[3 + k]
           /
          └──┘
           k=0
```

See also: `SUM, SUMErw1, SUMErw2, SUMShift, SUMZerl, SUMRegeln, SUMUmkehr, Ers, PosListe.`

SUMTausche

Description: Rule that exchanges summations. You should apply `SUMSammle` before applying `SUMTausche`.

$$\sum_{k_1=l_1}^{n_1}\sum_{k_2=l_2}^{n_2} \text{Expr} \to \sum_{k_2=l_2}^{n_2}\sum_{k_1=l_1}^{n_1} \text{Expr}.$$

Usage: `Expr/.SUMTausche.`

Example(s):

```
In[1]:= SUM[SUM[Binomial[n,k+l],{k,0,n1}],{l,0,n2}]

          n2    n1
          ┌──┐ ┌──┐(         )
           \     \  (    n    )
Out[1]=     〉    〉 (         )
           /     /  ( k + l   )
          └──┘ └──┘(         )
           l=0   k=0

In[2]:= %/.SUMTausche

          n1    n2
          ┌──┐ ┌──┐(         )
           \     \  (    n    )
Out[2]=     〉    〉 (         )
           /     /  ( k + l   )
          └──┘ └──┘(         )
           k=0   l=0
```

See also: `SUM, SUMErw1, SUMErw2, SUMSammle, SUMShift, SUMZerl, SUMRegeln, SUMUmkehr, Ers, PosListe.`

SUMUmkehr

Description: Rule that reverses the order of summation. `SUMUmkehr` applies to `SUM[]` as well as `F[]`.

Usage: `Expr/.SUMUmkehr.`

Example(s):

```
In[1]:= F[{a,-n},{b},z]
```

```
               a, -n
Out[1]=  F           ; z
        2 1    b

In[2]:= %/.SUMUmkehr
Is -a a nonnegative integer?
[y|n]:    n
Is n a nonnegative integer?
[y|n]:    y

                                              (a)
            n  n      -n, 1 - b - n  1          n
Out[2]= (-1)  z   F                  ; -    ----
                 2 1    1 - a - n     z     (b)
                                              n

In[3]:= SUM[p[-n,k]/p[1,k],{k,0,Infinity}]

         ∞
        ____(-n)
        \       k
Out[3]=  >   -----
        /    (1)
        ‾‾‾‾    k
         k=0

In[4]:= %/.SUMUmkehr
Is n a nonnegative integer?
[y|n]:    y

           n
          ____(-n)
          \       k
        (  >   -----) (-n)
          /    (1)        n
          ‾‾‾‾    k
           k=0
Out[4]= ------------------
               (1)
                  n
```

See also: `SUM`, `SUMErw1`, `SUMErw2`, `SUMZerl`, `SUMShift`, `SUMTausche`, `SUMRegeln`, `Ers`, `PosListe`.

SUMZerl

Description: Rule that splits a `SUM[]` into two parts.

$$\sum_{k=l}^{n} \mathrm{Expr} \to \sum_{k=l}^{l+m-1} \mathrm{Expr} + \sum_{k=l+m}^{n} \mathrm{Expr}.$$

The parameter `m` has to be entered on request.

Usage: `Expr/.SUMZerl.`

Example(s):

```
In[1]:= SUM[a[k],{k,0,N}]

           N
         ┌───┐
          \
Out[1]=    〉  a[k]
          /
         └───┘
          k=0

In[2]:= %/.SUMZerl
bottom-split by:  3

                                        N
                                      ┌───┐
                                       \
Out[2]= a[0] + a[1] + a[2] +            〉  a[k]
                                       /
                                      └───┘
                                       k=3

In[3]:= %%/.SUMZerl
bottom-split by:  M

         -1 + M       N
         ┌───┐      ┌───┐
          \          \
Out[3]=    〉  a[k] +  〉  a[k]
          /          /
         └───┘      └───┘
          k=0        k=M
```

See also: `SUM`, `SUMErw1`, `SUMErw2`, `SUMShift`, `SUMTausche`, `SUMRegeln`, `SUMUmkehr`, `Ers`, `PosListe`.

T2103

Description: Transformation formula ([7], (1.3.15)) in form of a rule. A q-analogue is HYPQ's `T2103`.

$${}_2F_1\left[\begin{matrix} a, b \\ c \end{matrix}; z\right] \longrightarrow (1-z)^{c-a-b}\,{}_2F_1\left[\begin{matrix} c-a, c-b \\ c \end{matrix}; z\right]$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2104

Description: Transformation formula ([7], (1.7.1.3)) in form of a rule. A q-analogue is HYPQ's `T2104`.

$$ {}_2F_1\left[\begin{matrix}a, b\\ c\end{matrix}; z\right] \longrightarrow (1-z)^{-a}{}_2F_1\left[\begin{matrix}a, c-b\\ c\end{matrix}; -\frac{z}{1-z}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2106

Description: Transformation formula ([7], (1.7.1.3), sum reversed at the right-hand side) in form of a rule. A q-analogue is HYPQ's `T2106`.

$$ {}_2F_1\left[\begin{matrix}a, -n\\ c\end{matrix}; z\right] \longrightarrow z^n\frac{(c-a)_n}{(c)_n}{}_2F_1\left[\begin{matrix}-n, 1-c-n\\ 1+a-c-n\end{matrix}; -\frac{1-z}{z}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2107

Description: Transformation formula ([7], (1.8.10), terminating form) in form of a rule. A q-analogue is HYPQ's `T2107`.

$$ {}_2F_1\left[\begin{matrix}a, -n\\ c\end{matrix}; z\right] \longrightarrow \frac{(c-a)_n}{(c)_n}{}_2F_1\left[\begin{matrix}-n, a\\ 1+a-c-n\end{matrix}; 1-z\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2110

Description: Transformation formula ([6], (3.2)) in form of a rule. A q-analogue is HYPQ's `T2110`.

$$ {}_2F_1\left[\begin{matrix}a, b\\ 1+a-b\end{matrix}; z\right] \longrightarrow (1+z)^{-a}{}_2F_1\left[\begin{matrix}\frac{a}{2}, \frac{1}{2}+\frac{a}{2}\\ 1+a-b\end{matrix}; \frac{4z}{(1+z)^2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2112

Description: Transformation formula ([6], (5.10)) in form of a rule. A q-analogue is HYPQ's `T2112`.

$$ {}_2F_1\left[\begin{matrix}a, \frac{1}{2}+a\\ \frac{1}{2}+b\end{matrix}; z^2\right] \longrightarrow (1-z)^{-2a}{}_2F_1\left[\begin{matrix}2a, b\\ 2b\end{matrix}; \frac{2z}{-1+z}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2131

Description: Transformation formula ([7], (1.8.10), terminating form, sum reversed at the right-hand side) in form of a rule. A q-analogue is HYPQ's `T3202`.

$$ {}_2F_1\left[\begin{matrix} a,-n \\ c \end{matrix};z\right] \longrightarrow (1-z)^n \frac{(a)_n}{(c)_n} {}_2F_1\left[\begin{matrix} -n, c-a \\ 1-a-n \end{matrix};\frac{1}{1-z}\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2132

Description: Transformation formula ([7], (2.5.7)) in form of a rule. A q-analogue is HYPQ's `T4307`.

$$ {}_2F_1\left[\begin{matrix} a,b \\ \frac{1}{2}+a+b \end{matrix};z\right]^2 \longrightarrow {}_3F_2\left[\begin{matrix} 2a,2b,a+b \\ 2a+2b,\frac{1}{2}+a+b \end{matrix};z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2133

Description: Transformation formula ([6], (5.12)) in form of a rule. A q-analogue is HYPQ's `T2202`.

$$ {}_2F_1\left[\begin{matrix} a,b \\ \frac{1}{2}+a+b \end{matrix};\frac{z^2}{-1+z^2}\right] \longrightarrow \frac{(1-z)^a}{(1+z)^a} {}_2F_1\left[\begin{matrix} 2a,a+b \\ 2a+2b \end{matrix};\frac{2z}{1+z}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2134

Description: Transformation formula ([1], Ex. 4.(iii), p. 97, reversed) in form of a rule. A q-analogue is HYPQ's `T3211`.

$$ {}_2F_1\left[\begin{matrix} a,b \\ \frac{1}{2}+\frac{a}{2}+\frac{b}{2} \end{matrix};z\right] \longrightarrow {}_2F_1\left[\begin{matrix} \frac{a}{2},\frac{b}{2} \\ \frac{1}{2}+\frac{a}{2}+\frac{b}{2} \end{matrix};4z\,(1-z)\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2135

Description: Transformation formula ([1], Ex. 4.(iii), p. 97) in form of a rule. A q-analogue is HYPQ's `T3212`.

$$ {}_2F_1\left[\begin{matrix} a,b \\ \frac{1}{2}+a+b \end{matrix};z\right] \longrightarrow {}_2F_1\left[\begin{matrix} 2a,2b \\ \frac{1}{2}+a+b \end{matrix};\frac{1-\sqrt{1-z}}{2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2136

Description: Transformation formula ([6], (5.10), reversed) in form of a rule. A q-analogue is HYPQ's `T3213`.

$$ {}_2F_1\left[\begin{matrix} a,b \\ 2b \end{matrix};z\right] \longrightarrow \left(\frac{2}{2-z}\right)^a {}_2F_1\left[\begin{matrix} \frac{a}{2},\frac{1}{2}+\frac{a}{2} \\ \frac{1}{2}+b \end{matrix};\frac{z^2}{(z-2)^2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2137

Description: Transformation formula ([6], (5.12), reversed) in form of a rule. A q-analogue is HYPQ's `T3214`.

$$ {}_2F_1\left[\begin{matrix} a,b \\ 2b \end{matrix};z\right] \longrightarrow (1-z)^{-\frac{a}{2}}\,{}_2F_1\left[\begin{matrix} \frac{a}{2},-\frac{a}{2}+b \\ \frac{1}{2}+b \end{matrix};\frac{z^2}{4\,(z-1)}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2138

Description: Transformation formula ([6], (3.31), reversed) in form of a rule. A q-analogue is HYPQ's `T3215`.

$$ {}_2F_1\left[\begin{matrix} a,1-a \\ c \end{matrix};z\right] \longrightarrow (1-z)^{c-1}\,{}_2F_1\left[\begin{matrix} \frac{c}{2}-\frac{a}{2},\frac{a}{2}+\frac{c}{2}-\frac{1}{2} \\ c \end{matrix};4z\,(1-z)\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2139

Description: Transformation formula ([6], (3.31)) in form of a rule. A q-analogue is HYPQ's `T3216`.

$$ {}_2F_1\left[\begin{matrix} c,-\frac{1}{2}+a-c \\ a \end{matrix};z\right] \longrightarrow \left(\frac{1+\sqrt{1-z}}{2}\right)^{1-a}\,{}_2F_1\left[\begin{matrix} a-2c,1-a+2c \\ a \end{matrix};\frac{1-\sqrt{1-z}}{2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2140

Description: Transformation formula ([6], (3.2), reversed) in form of a rule. A q-analogue is HYPQ's `T8707`.

$$ {}_2F_1\left[\begin{matrix} a,\frac{1}{2}+a \\ b \end{matrix};\frac{4z}{(1+z)^2}\right] \longrightarrow (1+z)^{2a}\,{}_2F_1\left[\begin{matrix} 2a,1+2a-b \\ b \end{matrix};z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2141

Description: Transformation formula ([2], (3.4.8), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T8708`.

$$ {}_2F_1\left[\begin{matrix} a,\frac{1}{2}+a \\ b \end{matrix};\frac{4z}{(1+z)^2}\right] \longrightarrow \frac{(1+z)^{2a}}{1-z}\,{}_3V_2(-1+2a;2a-b;z) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2163

Description: Transformation formula ([2], Ex 3.8, $q \to 1$; [7], pp. 36/37) in form of a rule. A q-analogue is HYPQ's `T2163`.

$$ {}_2F_1\left[\begin{matrix} a,b \\ c \end{matrix};z\right] \longrightarrow (1-z)^{-b}\Gamma\left[\begin{matrix} a-b,c \\ a,-b+c \end{matrix}\right]{}_2F_1\left[\begin{matrix} b,-a+c \\ 1-a+b \end{matrix};\frac{1}{1-z}\right] + (1-z)^{-a}\Gamma\left[\begin{matrix} -a+b,c \\ b,-a+c \end{matrix}\right]{}_2F_1\left[\begin{matrix} a,-b+c \\ 1+a-b \end{matrix};\frac{1}{1-z}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2191

Description: Transformation formula ([7], (1.8.10)) in form of a rule.

$$ {}_2F_1\left[\begin{matrix}a,b\\c\end{matrix};z\right] \longrightarrow (1-z)^{c-a-b}\Gamma\left[\begin{matrix}c,a+b-c\\a,b\end{matrix}\right]{}_2F_1\left[\begin{matrix}c-b,c-a\\1-a-b+c\end{matrix};1-z\right] + \Gamma\left[\begin{matrix}c,c-a-b\\c-b,c-a\end{matrix}\right]{}_2F_1\left[\begin{matrix}a,b\\1+a+b-c\end{matrix};1-z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T2192

Description: Transformation formula ([2], Ex 3.8, $q \to 1$, reversed; [7], pp. 36/37) in form of a rule. A q-analogue is HYPQ's `T3269`.

$$ {}_2F_1\left[\begin{matrix}a,b\\c\end{matrix};z\right] \longrightarrow z^{-a}\Gamma\left[\begin{matrix}1+a-c,1+b-c\\1-c,1+a+b-c\end{matrix}\right]{}_2F_1\left[\begin{matrix}a,1+a-c\\1+a+b-c\end{matrix};\frac{z-1}{z}\right] - z^{1-c}\Gamma\left[\begin{matrix}1+a-c,1+b-c,-1+c\\a,b,1-c\end{matrix}\right]{}_2F_1\left[\begin{matrix}1+a-c,1+b-c\\2-c\end{matrix};z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3204

Description: Transformation formula ([1], Ex. 7, p. 98) in form of a rule. A q-analogue is HYPQ's `T3204`.

$$ {}_3F_2\left[\begin{matrix}a,b,c\\d,e\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}e,-a-b-c+d+e\\-a+e,-b-c+d+e\end{matrix}\right]{}_3F_2\left[\begin{matrix}a,-b+d,-c+d\\d,-b-c+d+e\end{matrix};1\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3205

Description: Transformation formula ([7], (2.3.3.7)) in form of a rule. A q-analogue is HYPQ's `T3205`.

$$ {}_3F_2\left[\begin{matrix}a,b,c\\d,e\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}d,e,-a-b-c+d+e\\b,-a-b+d+e,-b-c+d+e\end{matrix}\right]{}_3F_2\left[\begin{matrix}-b+d,-b+e,-a-b-c+d+e\\-a-b+d+e,-b-c+d+e\end{matrix};1\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3206

Description: Transformation formula ([1], Ex. 7, p. 98, terminating form) in form of a rule. A q-analogue is HYPQ's `T3206`.

$$ {}_3F_2\left[\begin{matrix}a,b,-n\\d,e\end{matrix};1\right] \longrightarrow \frac{(-a-b+d+e)_n}{(e)_n}{}_3F_2\left[\begin{matrix}-n,-a+d,-b+d\\d,-a-b+d+e\end{matrix};1\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3207

Description: Transformation formula (Thomae [1879] in [2] (3.1.1)) in form of a rule. A q-analogue is HYPQ's `T3207`.

$$ {}_3F_2\left[\begin{matrix} a, b, -n \\ d, e \end{matrix}; 1\right] \longrightarrow \frac{(-b+e)_n}{(e)_n} {}_3F_2\left[\begin{matrix} -n, b, -a+d \\ d, 1+b-e-n \end{matrix}; 1\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3217

Description: Transformation formula ([1] 4.4(2), reversed) in form of a rule. A q-analogue is HYPQ's `T3217`.

$$ {}_3F_2\left[\begin{matrix} a, b, c \\ d, e \end{matrix}; 1\right] \longrightarrow \Gamma\left[\begin{matrix} -a+d+e, -a-b-c+d+e \\ -a-b+d+e, -a-c+d+e \end{matrix}\right] {}_6V_5(-1-a+d+e; -a+e, -a+d, b, c; -1) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3231

Description: Transformation formula ([2], Appendix (III.21), $q \uparrow 1$) in form of a rule. A q-analogue is HYPQ's `T4305`.

$$ {}_3F_2\left[\begin{matrix} 2a, 2b, c \\ \frac{1}{2}+a+b, d \end{matrix}; 1\right] \longrightarrow {}_4F_3\left[\begin{matrix} a, b, c, -c+d \\ \frac{1}{2}+a+b, \frac{d}{2}, \frac{1+d}{2} \end{matrix}; 1\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3232

Description: Transformation formula ([1], Ex. 4.(iv), p.97, reversed, first form) in form of a rule. A q-analogue is HYPQ's `T5469`. A terminating q-analogue is HYPQ's `T5404`.

$$ {}_3F_2\left[\begin{matrix} \frac{a}{2}, \frac{1+a}{2}, 1+a-b-c \\ 1+a-b, 1+a-c \end{matrix}; \frac{-4\,z}{(1-z)^2}\right] \longrightarrow (1-z)^a \, {}_3F_2\left[\begin{matrix} a, b, c \\ 1+a-b, 1+a-c \end{matrix}; z\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3233

Description: Transformation formula ([1], Ex. 4.(iv), p.97, reversed, second form) in form of a rule. A q-analogue is HYPQ's `T5469`. A terminating q-analogue is HYPQ's `T5404`.

$$ {}_3F_2\left[\begin{matrix} \frac{a}{2}, \frac{1+a}{2}, 1+a-b-c \\ 1+a-b, 1+a-c \end{matrix}; \frac{-4\,z}{(1-z)^2}\right] \longrightarrow \left(1-\frac{1}{z}\right)^a {}_3F_2\left[\begin{matrix} a, b, c \\ 1+a-b, 1+a-c \end{matrix}; \frac{1}{z}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3234

Description: Transformation formula ([7], (2.5.7), reversed) in form of a rule. A q-analogue is HYPQ's `T5405`.

$$ {}_3F_2\left[\begin{matrix} 2\,a, 2\,b, a+b \\ 2\,a+2\,b, \frac{1}{2}+a+b \end{matrix}; z\right] \longrightarrow {}_2F_1\left[\begin{matrix} a, b \\ \frac{1}{2}+a+b \end{matrix}; z\right]^2 $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3235

Description: Transformation formula ([2], Ex 3.4, $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T4201`.

$$ {}_3F_2\left[\begin{matrix} a, b, -n \\ d, 2b \end{matrix}; 2\right] \longrightarrow \frac{(-a+d)_n}{(d)_n} {}_4F_3\left[\begin{matrix} \frac{a}{2}, \frac{1}{2}+\frac{a}{2}, \frac{1}{2}-\frac{n}{2}, \frac{-n}{2} \\ \frac{1}{2}+\frac{a}{2}-\frac{d}{2}-\frac{n}{2}, 1+\frac{a}{2}-\frac{d}{2}-\frac{n}{2}, \frac{1}{2}+b \end{matrix}; 1\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3236

Description: Transformation formula ([2], (3.4.8), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T4311`.

$$ {}_3V_2(a; b; z) \longrightarrow \frac{(1-z)}{(1+z)^{1+a}} {}_2F_1\left[\begin{matrix} \frac{1}{2}+\frac{a}{2}, 1+\frac{a}{2} \\ 1+a-b \end{matrix}; \frac{4z}{(1+z)^2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3237

Description: Transformation formula ([2], (3.10.4), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T5403`.

$$ {}_3F_2\left[\begin{matrix} x, y, -n \\ b, a \end{matrix}; 1\right] \longrightarrow \frac{(a-x, a-y)_n}{(a, a-x-y)_n} {}_6V_5(-a-n+x+y; 1-a-b-n+x+y, x, y, -n; -1) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3238

Description: Transformation formula ([1], Ex. 6, p. 97) in form of a rule. A q-analogue is HYPQ's `T5462`.

$$ {}_3F_2\left[\begin{matrix} \frac{1}{2}+\frac{a}{2}, 1+\frac{a}{2}, -1-a+b+c \\ b, c \end{matrix}; -\frac{4z}{(1-z)^2}\right] \longrightarrow \frac{(1-z)^{1+a}}{(1+z)} {}_4V_3(a; 1+a-b, 1+a-c; z) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3239

Description: Transformation formula ([1], Ex. 4.(iv), p. 97) in form of a rule. A q-analogue is HYPQ's T3265. A terminating q-analogue is HYPQ's T3209.

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ 1+a-b,1+a-c \end{matrix};z\right] \longrightarrow (1-z)^{-a}{}_3F_2\left[\begin{matrix} \frac{a}{2},\frac{1+a}{2},1+a-b-c \\ 1+a-b,1+a-c \end{matrix};\frac{-4z}{(1-z)^2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3240

Description: Transformation formula ([2], (3.5.10), $q\to 1$, reversed) in form of a rule. A q-analogue is HYPQ's T8711.

$$ {}_3F_2\left[\begin{matrix} c,b,d \\ a,a-b+d \end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix} 2a,2a-2b-c,a-b+d,a-c+d \\ 2a-2b,2a-c,a+d,a-b-c+d \end{matrix}\right] {}_7V_6(-\frac{1}{2}+a;b,\frac{c}{2},\frac{1}{2}+\frac{c}{2},\frac{a}{2}-\frac{d}{2},\frac{1}{2}+\frac{a}{2}-\frac{d}{2};1) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3261

Description: Transformation formula ([2], (III.33), $q\uparrow 1$) in form of a rule. A q-analogue is HYPQ's T3261.

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ d,e \end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix} c,1+c-d,1-a,-b-c+e \\ -b+e,-c+e,1-a+c,1-d \end{matrix}\right] {}_3F_2\left[\begin{matrix} c,-a+d,1+c-e \\ 1-a+c,1+b+c-e \end{matrix};1\right] $$
$$ -\Gamma\left[\begin{matrix} -1+d,e,1+b-d,1+c-d,1-a,-b-c+e,1+b+c-e \\ 1-d,1-d+e,b,c,-a+d,-1-b-c+d+e,2+b+c-d-e \end{matrix}\right] {}_3F_2\left[\begin{matrix} 1+a-d,1+b-d,1+c-d \\ 2-d,1-d+e \end{matrix};1\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3262

Description: Transformation formula ([7], (4.3.4.2)) in form of a rule. A q-analogue is HYPQ's T3262.

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ d,e \end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix} e,-b-c+e \\ -b+e,-c+e \end{matrix}\right] {}_3F_2\left[\begin{matrix} -a+d,b,c \\ d,1+b+c-e \end{matrix};1\right] $$
$$ +\Gamma\left[\begin{matrix} d,e,b+c-e,-a-b-c+d+e \\ -a+d,b,c,-b-c+d+e \end{matrix}\right] {}_3F_2\left[\begin{matrix} -b+e,-c+e,-a-b-c+d+e \\ -b-c+d+e,1-b-c+e \end{matrix};1\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3263

Description: Transformation formula ([2], Appendix (III.33), $q\to 1$, reversed) in form of a rule. A q-analogue is HYPQ's T3263.

$$ {}_3F_2\left[\begin{matrix} a,b,c \\ d,e \end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix} 1-c,d,-a-b+d,1+a-e \\ 1+a-c,-a+d,-b+d,1-e \end{matrix}\right] {}_3F_2\left[\begin{matrix} 1+a-d,-c+e,a \\ 1+a+b-d,1+a-c \end{matrix};1\right] $$
$$ +\Gamma\left[\begin{matrix} 1-c,a+b-d,d,1+a-e,e,-a-b-c+d+e \\ a,b,1-b-c+d,1+a+b-d-e,-c+e,-a-b+d+e \end{matrix}\right] {}_3F_2\left[\begin{matrix} 1-b,-a-b-c+d+e,-b+d \\ 1-a-b+d,1-b-c+d \end{matrix};1\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3264

Description: Transformation formula ([2], Appendix (III.34), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T3264`.

$$
{}_3F_2\left[\begin{matrix}a,b,c\\d,e\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}1+b-e,1+c-e\\1-e,1+b+c-e\end{matrix}\right] {}_3F_2\left[\begin{matrix}-a+d,b,c\\d,1+b+c-e\end{matrix};1\right]
- \Gamma\left[\begin{matrix}d,1+a-e,1+b-e,1+c-e,-1+e\\a,b,c,1-e,1+d-e\end{matrix}\right] {}_3F_2\left[\begin{matrix}1+c-e,1+b-e,1+a-e\\1+d-e,2-e\end{matrix};1\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3267

Description: Transformation formula ([2], Ex. 3.6, $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T3267`.

$$
{}_3F_2\left[\begin{matrix}a,b,c\\d,e\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}d,1+a-e,1+b-e,1+c-e\\-a+d,1-e,1+a+b-e,1+a+c-e\end{matrix}\right] {}_3F_2\left[\begin{matrix}a,1+a-e,1+a+b+c-d-e\\1+a+b-e,1+a+c-e\end{matrix};1\right]
- \Gamma\left[\begin{matrix}d,1+a-e,1+b-e,1+c-e,-1+e\\a,b,c,1-e,1+d-e\end{matrix}\right] {}_3F_2\left[\begin{matrix}1+a-e,1+b-e,1+c-e\\2-e,1+d-e\end{matrix};1\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T3268

Description: Transformation formula ([2], Ex. 3.6, $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T3268`.

$$
{}_3F_2\left[\begin{matrix}a,b,c\\d,e\end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix}a-b,d,e,-a-b-c+d+e\\a,-b+d,-b+e,-a-c+d+e\end{matrix}\right] {}_3F_2\left[\begin{matrix}b,-a+d,-a+e\\1-a+b,-a-c+d+e\end{matrix};1\right]
+ \Gamma\left[\begin{matrix}-a+b,d,e,-a-b-c+d+e\\b,-a+d,-a+e,-b-c+d+e\end{matrix}\right] {}_3F_2\left[\begin{matrix}a,-b+d,-b+e\\-b-c+d+e,1+a-b\end{matrix};1\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4301

Description: Transformation formula ([7], (4.3.5.1)) in form of a rule. A q-analogue is HYPQ's `T4301`.

$$
{}_4F_3\left[\begin{matrix}a,b,c,-n\\e,f,1+a+b+c-e-f-n\end{matrix};1\right]
\longrightarrow \frac{(-a+e,-a+f)_n}{(e,f)_n} {}_4F_3\left[\begin{matrix}-n,a,1+a+c-e-f-n,1+a+b-e-f-n\\1+a+b+c-e-f-n,1+a-e-n,1+a-f-n\end{matrix};1\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4302

Description: Transformation formula ([2], Appendix (III.16), $q \uparrow 1$) in form of a rule. A q-analogue is HYPQ's `T4302`.

$$
{}_4F_3\left[\begin{matrix}a,b,c,-n\\e,f,1+a+b+c-e-f-n\end{matrix};1\right]
\longrightarrow \frac{(a,-a-b+e+f,-a-c+e+f)_n}{(e,f,-a-b-c+e+f)_n} {}_4F_3\left[\begin{matrix}-n,-a+e,-a+f,-a-b-c+e+f\\-a-b+e+f,-a-c+e+f,1-a-n\end{matrix};1\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4303

Description: Transformation formula ([7], (2.4.1.1)) in form of a rule. A q-analogue is HYPQ's `T4303`.

$$ {}_4F_3\left[\begin{matrix} a,b,c,-n \\ e,f,1+a+b+c-e-f-n \end{matrix};1\right] \longrightarrow \frac{(-a-b+e+f,-a-c+e+f)_n}{(-a+e+f,-a-b-c+e+f)_n} {}_7V_6(-1-a+e+f;-a+f,-a+e,b,c,-n;1) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4304

Description: Transformation formula ([7], (4.3.6.4)) in form of a rule. A q-analogue is HYPQ's `T4304`.

$$ {}_4F_3\left[\begin{matrix} a,b,c,-n \\ e,f,1+a+b+c-e-f-n \end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix} -b-c+e+f+n,-a-c+e+f+n, \\ -c+e+f+n,-b+e+f+n, \end{matrix}\right. $$
$$ \left.\begin{matrix} -a-b+e+f+n,e+f+n \\ -a+e+f+n,-a-b-c+e+f+n \end{matrix}\right] {}_7V_6(-1+e+f+n;a,b,c,e+n,f+n;1) $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4306

Description: Transformation formula ([2], Appendix (III.21), $q\uparrow 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T4306`.

$$ {}_4F_3\left[\begin{matrix} a,b,c,d \\ \frac{1}{2}+a+b,\frac{c+d}{2},\frac{1+c+d}{2} \end{matrix};1\right] \longrightarrow {}_3F_2\left[\begin{matrix} 2a,2b,c \\ \frac{1}{2}+a+b,c+d \end{matrix};1\right] $$

provided both series terminate.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4309

Description: Transformation formula ([2], Ex. 2.13(i), $q\to 1$) in form of a rule. A q-analogue is HYPQ's `T4309`.

$$ {}_4F_3\left[\begin{matrix} a,b,c,d \\ 1+a-b,1+a-c,1+a-d \end{matrix};1\right] \longrightarrow \Gamma\left[\begin{matrix} 3+2a-2b-2c-2d,2+2a-b-c-d \\ 3+3a-2b-2c-2d,2+a-b-c-d \end{matrix}\right] $$
$$ {}_7V_6(1+2a-b-c-d;\frac{a}{2},\frac{1}{2}+\frac{a}{2},1+a-c-d,1+a-b-d,1+a-b-c;1) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4310

Description: Transformation formula ([2], Ex. 2.13(ii), $q\to 1$) in form of a rule. A q-analogue is HYPQ's `T4310`.

$$ {}_4F_3\left[\begin{matrix} a,b,c,d \\ 1+a-b,1+a-c,1+a-d \end{matrix};-1\right] $$
$$ \longrightarrow \Gamma\left[\begin{matrix} 2+2a-b-c-d,1+\frac{a}{2} \\ 1+a,2+\frac{3a}{2}-b-c-d \end{matrix}\right] {}_6V_5(1+2a-b-c-d;\frac{a}{2},1+a-c-d,1+a-b-d,1+a-b-c;-1) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4312

Description: Transformation formula ([2], Ex. 3.4, $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T4312`.

$$ {}_4F_3\left[\begin{matrix} \frac{a}{2}, \frac{1}{2}+\frac{a}{2}, \frac{1}{2}-\frac{n}{2}, -\frac{n}{2} \\ \frac{d}{2}, \frac{1}{2}+\frac{d}{2}, \frac{1}{2}+b \end{matrix}; 1\right] \longrightarrow \frac{(d-a)_n}{(d)_n} {}_3F_2\left[\begin{matrix} a, b, -n \\ 1+a-d-n, 2b \end{matrix}; 2\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4313

Description: Transformation formula ([2], Ex. 8.15, $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T4313`.

$$ {}_4F_3\left[\begin{matrix} a, b, c, d \\ 1-a+b, 1-a+c, 1-a+d \end{matrix}; 1\right] \longrightarrow \Gamma\left[\begin{matrix} 1-d, a+b-d, a+c-d, 1+b+c-d \\ a-d, 1+b-d, 1+c-d, a+b+c-d \end{matrix}\right] $$

$$ \times {}_9F_8\left[\begin{matrix} b+c-d, 1+\frac{b}{2}+\frac{c}{2}-\frac{d}{2}, \frac{1}{2}-\frac{a}{2}+\frac{b}{2}+\frac{c}{2}-\frac{d}{2}, 1-\frac{a}{2}+\frac{b}{2}+\frac{c}{2}-\frac{d}{2}, a+b-d, a+c-d, a, b, c \\ \frac{b}{2}+\frac{c}{2}-\frac{d}{2}, \frac{1}{2}+\frac{a}{2}+\frac{b}{2}+\frac{c}{2}-\frac{d}{2}, \frac{a}{2}+\frac{b}{2}+\frac{c}{2}-\frac{d}{2}, 1-a+c, 1-a+b, 1-a+b+c-d, 1+c-d, 1+b-d \end{matrix}; 1\right] $$

provided at least one of a, b, c is a non-negative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4331

Description: Transformation formula ([1], Ex. 6, p. 97, reversed) in form of a rule. A q-analogue is HYPQ's `T5461`.

$$ {}_4V_3(a; b, c; z) \longrightarrow \frac{(1+z)}{(1-z)^{a+1}} {}_3F_2\left[\begin{matrix} \frac{1}{2}+\frac{a}{2}, 1+\frac{a}{2}, 1+a-b-c \\ 1+a-b, 1+a-c \end{matrix}; -\frac{4z}{(1-z)^2}\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4332

Description: Transformation formula ([1], 4.6(1)) in form of a rule. A q-analogue is HYPQ's `T10904`.

$$ {}_4F_3\left[\begin{matrix} b, x, y, -n \\ a-x, a-y, a+n \end{matrix}; 1\right] \longrightarrow \frac{(a, a-x-y)_n}{(a-x, a-y)_n} {}_5F_4\left[\begin{matrix} x, y, \frac{a}{2}-\frac{b}{2}, \frac{1}{2}+\frac{a}{2}-\frac{b}{2}, -n \\ a-b, \frac{a}{2}, \frac{1}{2}+\frac{a}{2}, 1-a-n+x+y \end{matrix}; 1\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4362

Description: Transformation formula ([7], (2.4.4.3), reversed) in form of a rule. A q-analogue is HYPQ's `T4362`.

$$ {}_4F_3\left[\begin{matrix} a, b, c, d \\ e, f, 1+a+b+c+d-e-f \end{matrix}; 1\right] $$

$$ \longrightarrow \Gamma\left[\begin{matrix} -a+e+f, -a-b-c+e+f, -a-b-d+e+f, -a-c-d+e+f \\ -a-b+e+f, -a-c+e+f, -a-d+e+f, -a-b-c-d+e+f \end{matrix}\right] $$

$$ {}_7V_6(-1-a+e+f; -a+f, -a+e, b, c, d; 1) $$

$$ -\Gamma\left[\begin{matrix} e, a+b+c+d-e-f, f, -a-b-c+e+f, -a-b-d+e+f, -a-c-d+e+f, -b-c-d+e+f \\ a, b, c, d, -a-b-c-d+e+f, -a-b-c-d+2e+f, -a-b-c-d+e+2f \end{matrix}\right] $$

$$ {}_4F_3\left[\begin{matrix} -a-b-c+e+f, -a-b-d+e+f, -a-c-d+e+f, -b-c-d+e+f \\ -a-b-c-d+2e+f, -a-b-c-d+e+2f, 1-a-b-c-d+e+f \end{matrix}; 1\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T4391

Description: Transformation formula ([2], (3.5.7), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T5465`.

$$
\begin{aligned}
&{}_4F_3\left[\begin{matrix} a, c, d, e \\ b, 2a, 1-2b+c+d+e \end{matrix}; 1\right] \\
&\longrightarrow \Gamma\left[\begin{matrix} b, \frac{1}{2}+b, b-\frac{c}{2}-\frac{d}{2}, \frac{1}{2}+b-\frac{c}{2}-\frac{d}{2}, b-\frac{c}{2}-\frac{e}{2}, \frac{1}{2}+b-\frac{c}{2}-\frac{e}{2}, b-\frac{d}{2}-\frac{e}{2}, \frac{1}{2}+b-\frac{d}{2}-\frac{e}{2} \\ b-\frac{c}{2}, \frac{1}{2}+b-\frac{c}{2}, b-\frac{d}{2}, \frac{1}{2}+b-\frac{d}{2}, b-\frac{e}{2}, \frac{1}{2}+b-\frac{e}{2}, b-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}, \frac{1}{2}+b-\frac{c}{2}-\frac{d}{2}-\frac{e}{2} \end{matrix}\right] \\
&\qquad {}_9V_8(-\frac{1}{2}+b; -a+b, \frac{c}{2}, \frac{1}{2}+\frac{c}{2}, \frac{d}{2}, \frac{1}{2}+\frac{d}{2}, \frac{e}{2}, \frac{1}{2}+\frac{e}{2}; 1) \\
&-\Gamma\left[\begin{matrix} \frac{1}{2}+a, b, b-\frac{c}{2}-\frac{d}{2}, \frac{1}{2}+b-\frac{c}{2}-\frac{d}{2}, a+2b-c-d-e, b-\frac{c}{2}-\frac{e}{2}, \frac{1}{2}+b-\frac{c}{2}-\frac{e}{2}, b-\frac{d}{2}-\frac{e}{2}, \frac{1}{2}+b-\frac{d}{2}-\frac{e}{2}, \\ \frac{c}{2}, \frac{1}{2}+\frac{c}{2}, \frac{d}{2}, \frac{1}{2}+\frac{d}{2}, 3b-c-d-e, b-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}, \frac{1}{2}+b-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}, a+b-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}, \frac{1}{2}+a+b-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}, \end{matrix}\right. \\
&\qquad \left.\begin{matrix} -b+\frac{c}{2}+\frac{d}{2}+\frac{e}{2}, \frac{1}{2}-b+\frac{c}{2}+\frac{d}{2}+\frac{e}{2} \\ \frac{e}{2}, \frac{1}{2}+\frac{e}{2} \end{matrix}\right] {}_4F_3\left[\begin{matrix} a+2b-c-d-e, 2b-c-d, 2b-c-e, 2b-d-e \\ 3b-c-d-e, 2a+2b-c-d-e, 1+2b-c-d-e \end{matrix}; 1\right]
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5401

Description: Transformation formula ([7], (2.4.3.4)) in form of a rule. A q-analogue is HYPQ's `T5401`.

$$
\begin{aligned}
&{}_5F_4\left[\begin{matrix} a, b, c, d, -n \\ 1+a-b, 1+a-c, 1+a-d, -2-2a+2b+2c+2d-n \end{matrix}; 1\right] \\
&\longrightarrow \frac{(2+a-b-c-d, 3+3a-2b-2c-2d)_n}{(2+2a-b-c-d, 3+2a-2b-2c-2d)_n} {}_9V_8(1+2a-b-c-d; 1+a-c-d, \\
&\qquad 1+a-b-d, 1+a-b-c, \frac{a}{2}, \frac{1}{2}+\frac{a}{2}, 3+3a-2b-2c-2d+n, -n; 1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5402

Description: Transformation formula ([2], Appendix (III.26), $q \uparrow 1$) in form of a rule. A q-analogue is HYPQ's `T5402`.

$$
\begin{aligned}
&{}_5F_4\left[\begin{matrix} -n, b, c, d, e \\ 1-b-n, 1-c-n, 1-d-n, -2+2b+2c+2d+e+2n \end{matrix}; 1\right] \\
&\longrightarrow \frac{(2-b-c-d-e-2n, 3-2b-2c-2d-3n)_n}{(2-b-c-d-2n, 3-2b-2c-2d-e-3n)_n} {}_9V_8(1-b-c-d-2n; 1-c-d-n, \\
&\qquad 1-b-d-n, 1-b-c-n, \frac{-n}{2}, \frac{1}{2}-\frac{n}{2}, e, 3-2b-2c-2d-e-3n; 1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5403

Description: Transformation formula ([1], 4.6(1), reversed) in form of a rule. A q-analogue is HYPQ's `T5403`.

$$
{}_5F_4\left[\begin{matrix} x, y, a, \frac{1}{2}+a, -n \\ 2a, b, \frac{1}{2}+b, 1-2b-n+x+y \end{matrix}; 1\right] \longrightarrow \frac{(2b-x, 2b-y)_n}{(2b, 2b-x-y)_n} {}_4F_3\left[\begin{matrix} -2a+2b, x, y, -n \\ 2b-x, 2b-y, 2b+n \end{matrix}; 1\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T5468

Description: Transformation formula ([2], Ex. 2.25, $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T5468`.

$$
\begin{aligned}
&{}_5F_4\left[\begin{matrix}a,b,c,d,e\\1+a-b,1+a-c,1+a-d,-2-2a+2b+2c+2d+e\end{matrix};1\right]\\
&+\Gamma\left[\begin{matrix}1+a-b,1+a-c,3+2a-2b-2c-2d,1+a-d,3+3a-2b-2c-2d-e,3+2a-b-2c-2d-e,\\a,b,c,d,4+3a-2b-2c-3d-e,4+3a-2b-3c-2d-e,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}3+2a-2b-c-2d-e,3+2a-2b-2c-d-e,-3-2a+2b+2c+2d+e\\4+3a-3b-2c-2d-e,3+2a-2b-2c-2d-e,e\end{matrix}\right]\\
&\qquad{}_5F_4\left[\begin{matrix}3+2a-2b-2c-2d,3+3a-2b-2c-2d-e,3+2a-b-2c-2d-e,\\4+2a-2b-2c-2d-e,4+3a-3b-2c-2d-e,\end{matrix}\right.\\
&\qquad\qquad\left.\begin{matrix}3+2a-2b-c-2d-e,3+2a-2b-2c-d-e\\4+3a-2b-3c-2d-e,4+3a-2b-2c-3d-e\end{matrix};1\right]\\
&\longrightarrow\Gamma\left[\begin{matrix}3+2a-2b-2c-2d,2+2a-b-c-d,3+3a-2b-2c-2d-e,2+a-b-c-d-e\\3+3a-2b-2c-2d,2+a-b-c-d,3+2a-2b-2c-2d-e,2+2a-b-c-d-e\end{matrix}\right]\\
&\quad{}_9V_8(1+2a-b-c-d;\frac{a}{2},\frac{1}{2}+\frac{a}{2},1+a-c-d,1+a-b-d,1+a-b-c,e,3+3a-2b-2c-2d-e;1)\\
&+\Gamma\left[\begin{matrix}1+a-b,1+a-c,3+2a-2b-2c-2d,1+a-d,6+4a-3b-3c-3d-2e,3+3a-2b-2c-2d-e,\\a,1+a-b-c,1+a-b-d,1+a-c-d,7+5a-4b-4c-4d-2e,\end{matrix}\right.\\
&\quad\left.\begin{matrix}3+2a-b-2c-2d-e,3+2a-2b-c-2d-e,3+2a-2b-2c-d-e,-2-a+b+c+d+e\\3+2a-2b-2c-2d-e,3+2a-b-c-2d-e,3+2a-b-2c-d-e,3+2a-2b-c-d-e,e\end{matrix}\right]\\
&\qquad{}_9V_8(5+4a-3b-3c-3d-2e;2+\frac{3a}{2}-b-c-d-e,\frac{5}{2}+\frac{3a}{2}-b-c-d-e,2+a-b-c-d,\\
&\qquad3+3a-2b-2c-2d-e,3+2a-b-2c-2d-e,3+2a-2b-c-2d-e,3+2a-2b-2c-d-e;1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T6501

Description: Transformation formula ([7], (2.4.3.3)) in form of a rule. A q-analogue is HYPQ's `T6501`.

$$
\begin{aligned}
&{}_6F_5\left[\begin{matrix}a,1+\frac{a}{2},b,c,d,-n\\\frac{a}{2},1+a-b,1+a-c,1+a-d,-1-2a+2b+2c+2d-n\end{matrix};1\right]\\
&\longrightarrow\frac{(2+3a-2b-2c-2d,1+a-b-c-d)_n}{(2+2a-2b-2c-2d,2+2a-b-c-d)_n}\\
&{}_9V_8(1+2a-b-c-d;1+a-c-d,1+a-b-d,1+a-b-c,1+\frac{a}{2},\frac{1}{2}+\frac{a}{2},2+3a-2b-2c-2d+n,-n;1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T6531

Description: Transformation formula ([7], (2.4.3.5)) in form of a rule. A q-analogue is HYPQ's `T7601`.

$$
{}_6F_5\left[\begin{matrix} a, 1+\frac{a}{2}, b, c, d, -n \\ \frac{a}{2}, 1+a-b, 1+a-c, 1+a-d, -2a+2b+2c+2d-n \end{matrix}; 1\right]
$$
$$
\longrightarrow \frac{(1+3a-2b-2c-2d+2n)}{(1+3a-2b-2c-2d)} \frac{(a-b-c-d, 1+3a-2b-2c-2d)_n}{(2+2a-b-c-d, 1+2a-2b-2c-2d)_n} {}_9V_8(1+2a-b-c-d;
$$
$$
1+a-c-d, 1+a-b-d, 1+a-b-c, \frac{1}{2}+\frac{a}{2}, 1+\frac{a}{2}, 1+3a-2b-2c-2d+n, -n; 1)
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T6532

Description: Transformation formula ([2], Ex. 2.13(ii), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T8706`.

$$
{}_6V_5(a; b, c, d, 1+2a-2b-c-d; -1) \longrightarrow \Gamma\left[\begin{matrix} 1+2b, 1+a-b \\ 1+b, 1+a \end{matrix}\right] {}_4F_3\left[\begin{matrix} 2b, -a+2b+c, -a+2b+d, 1+a-c-d \\ 1+a-c, 1+a-d, -a+2b+c+d \end{matrix}; -1\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T6533

Description: Transformation formula ([2], (3.10.4), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T10904`.

$$
{}_6V_5(a; b, x, y, -n; -1) \longrightarrow \frac{(1+a, 1+a-x-y)_n}{(1+a-x, 1+a-y)_n} {}_3F_2\left[\begin{matrix} -n, x, y \\ -a-n+x+y, 1+a-b \end{matrix}; 1\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T6534

Description: Transformation formula ([1], 4.4(2)) in form of a rule. A q-analogue is HYPQ's `T7701`.

$$
{}_6V_5(a; b, c, d, e; -1) \longrightarrow \Gamma\left[\begin{matrix} 1+a-d, 1+a-e \\ 1+a, 1+a-d-e \end{matrix}\right] {}_3F_2\left[\begin{matrix} 1+a-b-c, d, e \\ 1+a-b, 1+a-c \end{matrix}; 1\right]
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7631

Description: Transformation formula ([7], (2.4.1.1), reversed) in form of a rule. A q-analogue is HYPQ's `T8701`.

$$
{}_7V_6(a; b, c, d, e, f; 1) \longrightarrow \Gamma\left[\begin{matrix} 1+a-d, 1+a-e, 1+a-f, 1+a-d-e-f \\ 1+a, 1+a-d-e, 1+a-d-f, 1+a-e-f \end{matrix}\right]
$$
$$
{}_4F_3\left[\begin{matrix} 1+a-b-c, d, e, f \\ 1+a-b, 1+a-c, -a+d+e+f \end{matrix}; 1\right]
$$

provided the ${}_7F_6$ series converges and the ${}_4F_3$ series terminates.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7632

Description: Transformation formula ([7], (2.4.1.1), reversed) in form of a rule. A q-analogue is HYPQ's `T8702`.

$$ {}_7V_6(a;b,c,d,e,-n;1) \longrightarrow \frac{(1+a,1+a-d-e)_n}{(1+a-d,1+a-e)_n}\,{}_4F_3\left[\begin{matrix}1+a-b-c,d,e,-n\\1+a-b,1+a-c,-a+d+e-n\end{matrix};1\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7633

Description: Transformation formula ([7], (4.3.6.4), reversed) in form of a rule. A q-analogue is HYPQ's `T8703`.

$$ {}_7V_6(a;b,c,d,e,1+a-e+n;1) $$
$$ \longrightarrow \Gamma\left[\begin{matrix}1+a-d,1+a-c,1+a-b,1+a-b-c-d\\1+a-c-d,1+a-b-d,1+a-b-c,1+a\end{matrix}\right]{}_4F_3\left[\begin{matrix}b,c,d,-n\\1+a-e,-a+b+c+d,e-n\end{matrix};1\right] $$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7634

Description: Transformation formula ([1], 7.5.(1)) in form of a rule. A q-analogue is HYPQ's `T8704`.

$$ {}_7V_6(a;b,c,d,e,f;1) \longrightarrow \Gamma\left[\begin{matrix}1+a-e,1+a-f,2+2a-b-c-d,2+2a-b-c-d-e-f\\1+a,1+a-e-f,2+2a-b-c-d-e,2+2a-b-c-d-f\end{matrix}\right] $$
$$ {}_7V_6(1+2a-b-c-d;1+a-c-d,1+a-b-d,1+a-b-c,e,f;1) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7635

Description: Transformation formula ([1], 7.5.(2)) in form of a rule. A q-analogue is HYPQ's `T8705`.

$$ {}_7V_6(a;b,c,d,e,f;1) \longrightarrow \Gamma\left[\begin{matrix}1+a-c,1+a-d,1+a-e,1+a-f,3+3a-2b-c-d-e-f,\\1+a,b,2+2a-b-d-e-f,2+2a-b-c-e-f,2+2a-b-c-d-f,\end{matrix}\right. $$
$$ \left.\begin{matrix}2+2a-b-c-d-e-f\\2+2a-b-c-d-e\end{matrix}\right]{}_7V_6(2+3a-2b-c-d-e-f;1+a-b-c,1+a-b-d, $$
$$ 1+a-b-e,1+a-b-f,2+2a-b-c-d-e-f;1) $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7636

Description: Transformation formula ([2], (3.5.10), $q\to 1$) in form of a rule. A q-analogue is HYPQ's `T8710`.

$$ {}_7V_6(a;b,c,\frac{1}{2}+c,d,\frac{1}{2}+d;1) $$
$$ \longrightarrow \Gamma\left[\begin{matrix}1+2a-2b,1+2a-2c,1+2a-2d,1+2a-b-2c-2d\\1+2a,1+2a-2b-2c,1+2a-b-2d,1+2a-2c-2d\end{matrix}\right]{}_3F_2\left[\begin{matrix}2c,b,\frac{1}{2}+a-2d\\1+2a-b-2d,\frac{1}{2}+a\end{matrix};1\right] $$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7637

Description: Transformation formula ([2], Ex. 2.13(i), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T10902`.

$$
{}_7V_6(a; b, \frac{1}{2} + b, c, d, 1 + 2a - 2b - c - d; 1)
\longrightarrow \Gamma\begin{bmatrix} 1+a-2b, 1+2a-2b \\ 1+a, 1+2a-4b \end{bmatrix} {}_4F_3\begin{bmatrix} 2b, -a+2b+c, -a+2b+d, 1+a-c-d \\ 1+a-c, 1+a-d, -a+2b+c+d \end{bmatrix}; 1
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7691

Description: Transformation formula ([7], (2.4.4.3)) in form of a rule. A q-analogue is HYPQ's `T8761`.

$$
\begin{aligned}
&{}_7V_6(a; b, c, d, e, f; 1) \\
&\longrightarrow \Gamma\begin{bmatrix} 1+a-d, 1+a-e, 1+a-f, 1+a-d-e-f \\ 1+a, 1+a-d-e, 1+a-d-f, 1+a-e-f \end{bmatrix} {}_4F_3\begin{bmatrix} 1+a-b-c, d, e, f \\ 1+a-b, 1+a-c, -a+d+e+f \end{bmatrix}; 1 \\
&\quad + \Gamma\begin{bmatrix} 1+a-b, 1+a-c, 1+a-d, 1+a-e, 1+a-f, 2+2a-b-c-d-e-f, -1-a+d+e+f \\ 1+a, 1+a-b-c, d, e, f, 2+2a-b-d-e-f, 2+2a-c-d-e-f \end{bmatrix} \\
&\qquad {}_4F_3\begin{bmatrix} 1+a-d-e, 1+a-d-f, 1+a-e-f, 2+2a-b-c-d-e-f \\ 2+2a-b-d-e-f, 2+2a-c-d-e-f, 2+a-d-e-f \end{bmatrix}; 1
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7692

Description: Transformation formula ([7], (4.3.7.8)) in form of a rule. A q-analogue is HYPQ's `T8762`.

$$
\begin{aligned}
&{}_7V_6(a; b, c, d, e, f; 1) \\
&\longrightarrow -\Gamma\left[\begin{matrix} a-b, 1+a-c, 1+a-d, 1+a-e, 1+a-f, -a+b+d, -a+b+e, -a+b+f, -a+d+e+f, \\ 1+a, -a+b, 1+b-c, 1+b-d, 1+b-e, 1+b-f, d, e, f, 1+a-b-c, -2a+b+d+e+f, \end{matrix}\right. \\
&\qquad \left.\begin{matrix} a+1-d-e-f, 1-c, 1-a+2b \\ 1+2a-b-d-e-f \end{matrix}\right] {}_7V_6(-a+2b; b, -a+b+c, -a+b+d, -a+b+e, -a+b+f; 1) \\
&\quad + \Gamma\begin{bmatrix} 1+a-d, 1+a-e, 1+a-f, 1+a-d-e-f, 1-c, 1-c+e+f, -a+b+e, -a+b+f \\ 1+a, 1+a-d-e, 1+a-d-f, 1+a-e-f, 1-c+e, 1-c+f, -a+b+e+f \end{bmatrix} \\
&\quad {}_7V_6(-c+e+f; 1+a-b-c, 1+a-c-d, -a+e+f, e, f; 1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7693

Description: Transformation formula ([2], Ex. 2.15, $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T8764`.

$$
\begin{aligned}
&{}_7V_6(2a; a+b, a+c, a+d, a+e, a+f; 1)\\
&\longrightarrow -\Gamma\left[\begin{matrix} a-b, 1+2b, -b+c, b+c, 1-a-d, 1+a-d, 1-a-e, 1+a-e, 1-a-f, 1+a-f \\ 1+2a, -a+b, -a+c, a+c, 1-b-d, 1+b-d, 1-b-e, 1+b-e, 1-b-f, 1+b-f \end{matrix}\right]\\
&\qquad {}_7V_6(2b; a+b, b+c, b+d, b+e, b+f; 1)\\
&\quad -\Gamma\left[\begin{matrix} a-c, b-c, b+c, 1+2c, 1-a-d, 1+a-d, 1-a-e, 1+a-e, 1-a-f, 1+a-f \\ 1+2a, -a+b, a+b, -a+c, 1-c-d, 1+c-d, 1-c-e, 1+c-e, 1-c-f, 1+c-f \end{matrix}\right]\\
&\qquad {}_7V_6(2c; a+c, b+c, c+d, c+e, c+f; 1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7694

Description: Transformation formula ([7], (4.3.7.8), reversed) in form of a rule. A q-analogue is HYPQ's `T8763`.

$$
\begin{aligned}
&{}_7V_6(a; b, c, d, e, f; 1)\\
&\longrightarrow \Gamma\left[\begin{matrix} 1+a-b, 1-d, 1+a-e, -a+c+e, 1+a-f, 1+a-b-e-f, -a+c+f, 1-d+e+f \\ 1+a, -a+c, 1+a-b-e, 1-d+e, 1+a-b-f, 1+a-e-f, 1-d+f, -a+c+e+f \end{matrix}\right]\\
&\qquad {}_7V_6(-d+e+f; 1+a-b-d, -a+e+f, 1+a-c-d, e, f; 1)\\
&\quad +\Gamma\left[\begin{matrix} 1+a-b, 1+a-c, 1-d, 1+a-d, 1+a-e, -a+c+e, 1+a-f, 3+2a-2b-d-e-f, \\ 1+a, b, 1-b+c, 1+a-c-d, 2+a-b-d-e, e, 2+a-b-d-f, \end{matrix}\right.\\
&\qquad \left.\begin{matrix} 2+2a-b-c-d-e-f, -a+c+f, -1-a+b+e+f \\ 2+2a-b-c-e-f, 2+2a-b-d-e-f, f, -1-2a+b+c+e+f \end{matrix}\right]\\
&\qquad {}_7V_6(2+2a-2b-d-e-f; 1+a-b-d, 1-b, 2+2a-b-c-d-e-f, 1+a-b-f, 1+a-b-e; 1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T7740

Description: Transformation formula ([2], (5.6.1); Appendix (III.38), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T8810`.

$$
\begin{aligned}
&{}_7H_7\left[\begin{matrix} 1+\frac{a}{2}, b, c, d, e, f, g \\ \frac{a}{2}, 1+a-b, 1+a-c, 1+a-d, 1+a-e, 1+a-f, 1+a-g \end{matrix}; 1\right]\\
&\longrightarrow \Gamma\left[\begin{matrix} 1-b, 1+a-b, 1-c, 1+a-c, 1-d, 1+a-d, 1-e, 1+a-e, 1-f, 1+a-f, 1-a+2f, \\ 1-a, 1+a, 1+a-b-f, 1+a-c-f, 1+a-d-f, 1+a-e-f, 1-b+f, 1-c+f, 1-d+f, \end{matrix}\right.\\
&\qquad \left.\begin{matrix} -f+g, -a+f+g \\ 1-e+f, -a+g, g \end{matrix}\right] {}_7V_6(-a+2f; -a+b+f, -a+c+f, -a+d+f, -a+e+f, -a+f+g; 1)\\
&\quad +\Gamma\left[\begin{matrix} 1-b, 1+a-b, 1-c, 1+a-c, 1-d, 1+a-d, 1-e, 1+a-e, 1-g, 1+a-g, f-g, \\ 1-a, 1+a, -a+f, f, 1+a-b-g, 1+a-c-g, 1+a-d-g, 1+a-e-g, 1-b+g, 1-c+g, \end{matrix}\right.\\
&\qquad \left.\begin{matrix} -a+f+g, 1-a+2g \\ 1-d+g, 1-e+g \end{matrix}\right] {}_7V_6(-a+2g; -a+b+g, -a+c+g, -a+d+g, -a+e+g, -a+f+g; 1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8731

Description: Transformation formula ([6], (7.7), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T10906`.

$$
\begin{aligned}
&{}_8V_7(2a+n; c, d, e, \frac{1}{2}+a+n, 1+4a-c-d-e+n, -n; -1)\\
&\quad\longrightarrow \frac{(1+2a-c, 1+2a-d, 1+2a-e, 1+2a-c-d-e)_n}{(1+2a, 1+2a-c-d, 1+2a-c-e, 1+2a-d-e)_n}\\
&\quad {}_{11}V_{10}(a; -n, \frac{c}{2}, \frac{1}{2}+\frac{c}{2}, \frac{d}{2}, \frac{1}{2}+\frac{d}{2}, \frac{e}{2}, \frac{1}{2}+\frac{e}{2}, \frac{1}{2}+2a-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}+\frac{n}{2}, 1+2a-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}+\frac{n}{2}; 1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T8732

Description: Transformation formula ([6], (7.8), $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T10907`.

$$
\begin{aligned}
&{}_8V_7(2a-e; \frac{1}{2}+a-e, c, d, e, 1+4a-c-d-e+n, -n; -1)\\
&\quad\longrightarrow \frac{(1+2a-c, 1+2a-d, 1+2a-e, 1+2a-c-d-e)_n}{(1+2a, 1+2a-c-d, 1+2a-c-e, 1+2a-d-e)_n}\\
&\quad {}_{11}V_{10}(a; e, \frac{c}{2}, \frac{1}{2}+\frac{c}{2}, \frac{d}{2}, \frac{1}{2}+\frac{d}{2}, \frac{1}{2}+2a-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}+\frac{n}{2}, 1+2a-\frac{c}{2}-\frac{d}{2}-\frac{e}{2}+\frac{n}{2}, \frac{1}{2}-\frac{n}{2}, \frac{-n}{2}; 1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9831

Description: Transformation formula ([7], (2.4.4.1)) in form of a rule. A q-analogue is HYPQ's `T10901`.

$$
\begin{aligned}
&{}_9V_8(a; b, c, d, e, f, 2+3a-b-c-d-e-f+n, -n; 1)\\
&\longrightarrow \frac{(1+a, 1+a-e-f, 2+2a-b-c-d-e, 2+2a-b-c-d-f)_n}{(1+a-e, 1+a-f, 2+2a-b-c-d-e-f, 2+2a-b-c-d)_n}\\
&\quad {}_9V_8(1+2a-b-c-d; 1+a-c-d, 1+a-b-d, 1+a-b-c, e, f, 2+3a-b-c-d-e-f+n, -n; 1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9832

Description: Transformation formula ([7], (2.4.3.4), reversed) in form of a rule. A q-analogue is HYPQ's `T121101`.

$$
\begin{aligned}
&{}_9V_8(a; b, c, d, \frac{1}{2}+a-\frac{b+c+d}{2}, 1+a-\frac{b+c+d}{2}, b+c+d+n, -n; 1)\\
&\longrightarrow \frac{(1+a, -1-2a+2b+2c+2d)_n}{(-a+b+c+d, b+c+d)_n}\,{}_5F_4\left[\begin{matrix}1+2a-b-c-d, 1+a-c-d, 1+a-b-d, 1+a-b-c, -n\\ 1+a-b, 1+a-c, 1+a-d, 2+2a-2b-2c-2d-n\end{matrix}; 1\right]
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9833

Description: Transformation formula ([2], Appendix (III.26), $q \uparrow 1$, reversed) in form of a rule. A q-analogue is HYPQ's T121102.

$$ {}_9V_8\left(\frac{-1+b+c+d-n}{2};b,c,d,\frac{-n}{2},\frac{1}{2}-\frac{n}{2},e,b+c+d-e;1\right) \longrightarrow \frac{(\frac{1}{2}+\frac{b}{2}+\frac{c}{2}+\frac{d}{2}-\frac{n}{2},b+c+d-e)_n}{(\frac{1}{2}+\frac{b}{2}+\frac{c}{2}+\frac{d}{2}-e-\frac{n}{2},b+c+d)_n} {}_5F_4\left[\begin{matrix}-n,\frac{1}{2}+\frac{b}{2}-\frac{c}{2}-\frac{d}{2}-\frac{n}{2},\frac{1}{2}-\frac{b}{2}+\frac{c}{2}-\frac{d}{2}-\frac{n}{2},\frac{1}{2}-\frac{b}{2}-\frac{c}{2}+\frac{d}{2}-\frac{n}{2},e\\ \frac{1}{2}-\frac{b}{2}+\frac{c}{2}+\frac{d}{2}-\frac{n}{2},\frac{1}{2}+\frac{b}{2}-\frac{c}{2}+\frac{d}{2}-\frac{n}{2},\frac{1}{2}+\frac{b}{2}+\frac{c}{2}-\frac{d}{2}-\frac{n}{2},1-b-c-d+e-n\end{matrix};1\right] $$

where n is a nonnegative integer.

See also: S2103, S3201, TListe, TransListe, Ers, PosListe.

T9834

Description: Transformation formula ([7], (2.4.3.5), reversed) in form of a rule. A q-analogue is HYPQ's T121103.

$$ {}_9V_8\left(a;b,c,d,1+a-\frac{b+c+d}{2},\frac{3}{2}+a-\frac{b+c+d}{2},-2+b+c+d+n,-n;1\right) \longrightarrow \frac{(-2+b+c+d)}{(-2+b+c+d+2n)}\frac{(1+a,-3-2a+2b+2c+2d)_n}{(-2-a+b+c+d,-2+b+c+d)_n} {}_6F_5\left[\begin{matrix}1+2a-b-c-d,\frac{3}{2}+a-\frac{b}{2}-\frac{c}{2}-\frac{d}{2},1+a-c-d,1+a-b-d,1+a-b-c,-n\\ \frac{1}{2}+a-\frac{b}{2}-\frac{c}{2}-\frac{d}{2},1+a-b,1+a-c,1+a-d,4+2a-2b-2c-2d-n\end{matrix};1\right] $$

where n is a nonnegative integer.

See also: S2103, S3201, TListe, TransListe, Ers, PosListe.

T9835

Description: Transformation formula ([1], 7.6(1)) in form of a rule. A q-analogue is HYPQ's T10903.

$$ {}_9V_8(a;b,c,d,e,f,2+3a-b-c-d-e-f+n,-n;1) \longrightarrow \frac{(1+a,b,1+a-c-e,1+a-d-e,1+a-e-f,-1-2a+b+c+d+f-n)_n}{(1+a-c,1+a-d,1+a-e,b-e,1+a-f,-1-2a+b+c+d+e+f-n)_n} {}_9V_8(-b+e-n;e,1+a-b-c,1+a-b-d,1+a-b-f,-1-2a+c+d+e+f-n,-a+e-n,-n;1) $$

where n is a nonnegative integer.

See also: S2103, S3201, TListe, TransListe, Ers, PosListe.

T9836

Description: Transformation formula ([2], Ex. 3.21(iii), $q \to 1$) in form of a rule. A q-analogue is HYPQ's T10905.

$$ {}_9F_8\left[\begin{matrix}a,1+\frac{a}{2},b,c,a-b-c,-A+C-n,-B-C-n,-A+B-n,-n\\ \frac{a}{2},1+a-b,1+a-c,1+b+c,-C-n,-A+B+C-n,-B-n,-A-n\end{matrix};1\right] \longrightarrow \frac{(1+c,1+A-B,1+A-C,1+B+C,1+a,1+b,1+a-b-c)_n}{(1+b+c,1+A,1+B,1+A-B-C,1+C,1+a-b,1+a-c)_n} {}_9F_8\left[\begin{matrix}A,1+\frac{A}{2},B,C,A-B-C,-a+c-n,-b-c-n,-a+b-n,-n\\ \frac{A}{2},1+A-B,1+A-C,1+B+C,-c-n,-a+b+c-n,-b-n,-a-n\end{matrix};1\right] $$

where n is a nonnegative integer.

See also: S2103, S3201, TListe, TransListe, Ers, PosListe.

T9837

Description: Transformation formula ([7], (2.4.3.3), reversed) in form of a rule. A q-analogue is HYPQ's `T121104`.

$$
{}_9V_8\Big(a;c,d,1+2a-b-c-d,1+\frac{b}{2},\frac{1}{2}+\frac{b}{2},2a-b+n,-n;1\Big)
\longrightarrow \frac{(1+a,2a-2b)_n}{(a-b,2a-b)_n}\,{}_6F_5\left[\begin{matrix}b,1+\frac{b}{2},-a+b+c,-a+b+d,1+a-c-d,-n\\ \frac{b}{2},1+a-c,1+a-d,-a+b+c+d,1-2a+2b-n\end{matrix};1\right]
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9838

Description: Transformation formula ([2], Ex. 8.15, $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T121107`.

$$
{}_9F_8\left[\begin{matrix}d,1+\frac{d}{2},\frac{1}{2}-\frac{a}{2}+\frac{d}{2},1-\frac{a}{2}+\frac{d}{2},a-c+d,a-b+d,a,b,c\\ \frac{d}{2},\frac{1}{2}+\frac{a}{2}+\frac{d}{2},\frac{a}{2}+\frac{d}{2},1-a+c,1-a+b,1-a+d,1-b+d,1-c+d\end{matrix};1\right]
\longrightarrow \Gamma\left[\begin{matrix}a-b-c+d,1-c+d,1-b+d,a+d\\ 1-b-c+d,a-c+d,a-b+d,1+d\end{matrix}\right]{}_4F_3\left[\begin{matrix}a,b,c,b+c-d\\ 1-a+b,1-a+c,1-a+b+c-d\end{matrix};1\right]
$$

provided at least one of a,b,c is a non-negative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9891

Description: Transformation formula ([2], Appendix (III.39), $q \uparrow 1$) in form of a rule. A q-analogue is HYPQ's `T10961`.

$$
\begin{aligned}
&{}_9V_8(a;b,c,d,e,f,g,2+3a-b-c-d-e-f-g;1)\\
&\longrightarrow \Gamma\left[\begin{matrix}2+2a-c-d-e,-1-2a+b+c+d+e,1+a-f,1+a-g,\\ 1+a,-a+b,2+2a-c-d-e-f,2+2a-c-d-e-g,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}-1-2a+b+c+d+e+f+g,-a+b+f,-a+b+g,2+2a-c-d-e-f-g\\ -a+b+f+g,-1-2a+b+c+d+e+f,-1-2a+b+c+d+e+g,1+a-f-g\end{matrix}\right]\\
&\qquad{}_9V_8(1+2a-c-d-e;b,1+a-d-e,1+a-c-e,1+a-c-d,f,g,2+3a-b-c-d-e-f-g;1)\\
&\quad-\Gamma\left[\begin{matrix}1-a+2b,a-b,1+a-c,1+a-d,1+a-e,1+a-f,1+a-g,-1-2a+b+c+d+e+f+g,\\ 1+a,-a+b,c,d,e,f,g,2+3a-b-c-d-e-f-g,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}-a+b+c,-a+b+d,-a+b+e,-a+b+f,-a+b+g,2+2a-c-d-e-f-g\\ 1+b-c,1+b-d,1+b-e,1+b-f,1+b-g,-1-3a+2b+c+d+e+f+g\end{matrix}\right]\\
&\qquad{}_9V_8(-a+2b;b,-a+b+c,-a+b+d,-a+b+e,-a+b+f,-a+b+g,2+2a-c-d-e-f-g;1)\\
&\quad+\Gamma\left[\begin{matrix}-2a+2b+c+d+e,1+2a-b-c-d-e,1+a-c,1+a-d,1+a-e,1+a-f,\\ 1+a,-a+b,f,g,2+3a-b-c-d-e-f-g,1+b-f,1+b-g,\end{matrix}\right.\\
&\qquad\begin{matrix}1+a-g,-1-2a+b+c+d+e+f+g,-a+b+c,-a+b+d,-a+b+e,\\ -1-3a+2b+c+d+e+f+g,1+a-d-e,1+a-c-e,1+a-c-d,\end{matrix}\\
&\qquad\left.\begin{matrix}-a+b+f,-a+b+g,2+2a-c-d-e-f-g\\ -a+b+d+e,-a+b+c+e,-a+b+c+d\end{matrix}\right]{}_9V_8(-1-2a+2b+c+d+e;b,-a+b+c,-a+b+d,\\
&\qquad\quad -a+b+e,-1-2a+b+c+d+e+f,-1-2a+b+c+d+e+g,1+a-f-g;1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9892

Description: Transformation formula ([2], (3.5.7), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T10962`.

$$
\begin{aligned}
&{}_9V_8(a; b, c, \frac{1}{2}+c, d, \frac{1}{2}+d, e, \frac{1}{2}+e; 1)\\
&\quad \longrightarrow \Gamma\left[\begin{matrix}\frac{1}{2}+a-c, 1+a-c, \frac{1}{2}+a-d, 1+a-d, \frac{1}{2}+a-e, 1+a-e, \frac{1}{2}+a-c-d-e, 1+a-c-d-e\\ \frac{1}{2}+a, 1+a, \frac{1}{2}+a-c-d, 1+a-c-d, \frac{1}{2}+a-c-e, 1+a-c-e, \frac{1}{2}+a-d-e, 1+a-d-e\end{matrix}\right]\\
&\qquad\qquad {}_4F_3\left[\begin{matrix}\frac{1}{2}+a-b, 2c, 2d, 2e\\ \frac{1}{2}+a, 1+2a-2b, -2a+2c+2d+2e\end{matrix}; 1\right]\\
&\qquad + \Gamma\left[\begin{matrix}1+a-b, \frac{1}{2}+a-c, 1+a-c, \frac{1}{2}+a-d, 1+a-d, \frac{3}{2}+3a-b-2c-2d-2e,\\ 1+a, c, \frac{1}{2}+c, d, \frac{1}{2}+d, \frac{3}{2}+3a-2c-2d-2e, 1+2a-b-c-d-e,\end{matrix}\right.\\
&\qquad\qquad \left.\begin{matrix}\frac{1}{2}+a-e, 1+a-e, -\frac{1}{2}-a+c+d+e, -a+c+d+e\\ \frac{3}{2}+2a-b-c-d-e, e, \frac{1}{2}+e\end{matrix}\right]\\
&\qquad\qquad {}_4F_3\left[\begin{matrix}\frac{3}{2}+3a-b-2c-2d-2e, 1+2a-2c-2d, 1+2a-2c-2e, 1+2a-2d-2e\\ \frac{3}{2}+3a-2c-2d-2e, 2+4a-2b-2c-2d-2e, 2+2a-2c-2d-2e\end{matrix}; 1\right]
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9893

Description: Transformation formula ([2], Ex. 2.30, $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T10963`.

$$
\begin{aligned}
&{}_9V_8(a; b, c, d, e, f, g, 2+3a-b-c-d-e-f-g; 1)\\
&\quad + \Gamma\left[\begin{matrix}a-b, 1-a+2b, 1+a-c, -a+b+c, 1+a-d, -a+b+d, 1+a-e, -a+b+e, 1+a-f,\\ 1+a, -a+b, 1+b-c, c, 1+b-d, d, 1+b-e, e, 1+b-f, f, 1+b-g,\end{matrix}\right.\\
&\qquad \left.\begin{matrix}-a+b+f, 1+a-g, 2+2a-c-d-e-f-g, -a+b+g, -1-2a+b+c+d+e+f+g\\ 2+3a-b-c-d-e-f-g, g, -1-3a+2b+c+d+e+f+g\end{matrix}\right]\\
&\quad {}_9V_8(-a+2b; b, -a+b+c, -a+b+d, -a+b+e, -a+b+f, -a+b+g, 2+2a-c-d-e-f-g; 1)\\
&\longrightarrow \Gamma\left[\begin{matrix}1+a-c, -a+b+c, 1+a-d, -a+b+d, 1+a-e, -a+b+e, 1+a-f, -a+b+f,\\ 1+a, -a+b, 1+b-g, 2+2a-c-d-e-g, 2+2a-c-d-f-g, 2+2a-c-e-f-g,\end{matrix}\right.\\
&\qquad\qquad 3+3a-c-d-e-f-2g, 2+2a-c-d-e-f-g,\\
&\qquad 2+2a-d-e-f-g, g, -1-2a+b+c+d+e+g, -1-2a+b+c+d+f+g,\\
&\qquad\qquad \left.\begin{matrix}-1-2a+b+c+d+e+f+g, -2-3a+b+c+d+e+f+2g\\ -1-2a+b+c+e+f+g, -1-2a+b+d+e+f+g\end{matrix}\right]\\
&\qquad {}_9V_8(2+3a-c-d-e-f-2g; b, 1+a-c-g, 1+a-d-g, 1+a-e-g, 1+a-f-g,\\
&\qquad\qquad 2+2a-c-d-e-f-g, 2+3a-b-c-d-e-f-g; 1)\\
&\quad + \Gamma\left[\begin{matrix}1+a-c, -a+b+c, 1+a-d, -a+b+d, 1+a-e, -a+b+e, 1+a-f, -a+b+f,\\ 1+a, -a+b, 1+a-c-g, 1+a-d-g, 1+a-e-g, 1+a-f-g, 2+3a-b-c-d-e-f-g,\end{matrix}\right.\\
&\qquad \left.\begin{matrix}2+3a-b-c-d-e-f-2g, 1+a-g, -a+b+g, -1-3a+2b+c+d+e+f+2g\\ -a+b+c+g, -a+b+d+g, -a+b+e+g, -a+b+f+g, -1-3a+2b+c+d+e+f+g\end{matrix}\right]\\
&\quad {}_9V_8(-2-3a+2b+c+d+e+f+2g; b, -1-2a+b+d+e+f+g, -1-2a+b+c+e+f+g,\\
&\qquad\qquad -1-2a+b+c+d+f+g, -1-2a+b+c+d+e+g, -a+b+g, g; 1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9894

Description: Transformation formula ([2], Ex. 2.25, $q \to 1$, reversed) in form of a rule. A q-analogue is HYPQ's `T121161`.

$$
\begin{aligned}
&{}_9V_8(a; b, \tfrac{1}{2}+b, c, d, 1+2a-2b-c-d, e, 1+2a-2b-e; 1)\\
&+\Gamma\left[\begin{matrix}1+a-2b, 1+2a-2b, 1+a-c, 1+a-d, -a+2b+c+d, 3+3a-4b-2e, 1+a-e, 1+a-2b+c-e,\\ 1+a, 2b, c, 1+2a-2b-c-d, d, 3+4a-6b-2e, 1+a-2b-e,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}2+3a-4b-c-d-e, 1+a-2b+d-e, -1-a+2b+e\\ 2+2a-2b-c-e, 2+2a-2b-d-e, 1+c+d-e, e\end{matrix}\right]\\
&{}_9V_8(2+3a-4b-2e; 1+a-b-e, \tfrac{3}{2}+a-b-e, 1+a-2b, 1+2a-2b-e, 1+a-2b+c-e, 1+a-2b+d-e,\\
&\qquad 2+3a-4b-c-d-e; 1)\\
&\longrightarrow \Gamma\left[\begin{matrix}1+a-2b, 1+2a-2b, 1+a-e, 1+2a-4b-e\\ 1+a, 1+2a-4b, 1+a-2b-e, 1+2a-2b-e\end{matrix}\right]\\
&\qquad {}_5F_4\left[\begin{matrix}2b, -a+2b+c, -a+2b+d, 1+a-c-d, e\\ 1+a-c, 1+a-d, -a+2b+c+d, -2a+4b+e\end{matrix}; 1\right]\\
&\quad +\Gamma\left[\begin{matrix}1+a-2b, 1+2a-2b, 1+a-c, 1+a-d, -a+2b+c+d, 1+a-e, 1+a-2b+c-e,\\ 1+a, 2b, -a+2b+c, 1+a-c-d, -a+2b+d, 1+a-2b-e,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}2+3a-4b-c-d-e, 1+a-2b+d-e, -1-2a+4b+e\\ 2+3a-4b-c-e, 2+3a-4b-d-e, 1+a-2b+c+d-e, e\end{matrix}\right]\\
&\qquad {}_5F_4\left[\begin{matrix}1+2a-4b, 1+2a-2b-e, 1+a-2b+c-e, 1+a-2b+d-e, 2+3a-4b-c-d-e\\ 2+2a-4b-e, 2+3a-4b-c-e, 2+3a-4b-d-e, 1+a-2b+c+d-e\end{matrix}; 1\right]
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T9940

Description: Transformation formula ([2], (5.6.3); Appendix (III.40), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T101010`.

$$
\begin{aligned}
&{}_9H_9\left[\begin{matrix}1+\frac{a}{2}, b, c, d, e, f, g, h, k\\ \frac{a}{2}, 1+a-b, 1+a-c, 1+a-d, 1+a-e, 1+a-f, 1+a-g, 1+a-h, 1+a-k\end{matrix}; 1\right]\\
&\longrightarrow \Gamma\left[\begin{matrix}1-b, 1+a-b, 1-c, 1+a-c, 1-d, 1+a-d, 1-e, 1+a-e, 1-f, 1+a-f, 1-g, 1+a-g,\\ 1-a, 1+a, 1+a-b-g, 1+a-c-g, 1+a-d-g, 1+a-e-g, 1+a-f-g,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}1-a+2g, -g+h, -a+g+h, -g+k, -a+g+k\\ 1-b+g, 1-c+g, 1-d+g, 1-e+g, 1-f+g, h, -a+h, k, -a+k\end{matrix}\right]\\
&\quad\times {}_9V_8(-a+2g; -a+b+g, -a+c+g, -a+d+g, -a+e+g, -a+f+g, -a+g+h, -a+g+k; 1)\\
&\quad+\Gamma\left[\begin{matrix}1-b, 1+a-b, 1-c, 1+a-c, 1-d, 1+a-d, 1-e, 1+a-e, 1-f, 1+a-f, 1-h, 1+a-h,\\ 1-a, 1+a, g, -a+g, 1+a-b-h, 1+a-c-h, 1+a-d-h, 1+a-e-h, 1+a-f-h,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}g-h, -a+g+h, 1-a+2h, -h+k, -a+h+k\\ 1-b+h, 1-c+h, 1-d+h, 1-e+h, 1-f+h, k, -a+k\end{matrix}\right]\\
&\quad\times {}_9V_8(-a+2h; -a+b+h, -a+c+h, -a+d+h, -a+e+h, -a+f+h, -a+g+h, -a+h+k; 1)\\
&\quad+\Gamma\left[\begin{matrix}1-b, 1+a-b, 1-c, 1+a-c, 1-d, 1+a-d, 1-e, 1+a-e, 1-f, 1+a-f, 1-k, 1+a-k,\\ 1-a, 1+a, g, -a+g, h, -a+h, 1+a-b-k, 1+a-c-k, 1+a-d-k, 1+a-e-k, 1+a-f-k,\end{matrix}\right.\\
&\qquad\left.\begin{matrix}g-k, h-k, -a+g+k, -a+h+k, 1-a+2k\\ 1-b+k, 1-c+k, 1-d+k, 1-e+k, 1-f+k\end{matrix}\right]\\
&\quad\times {}_9V_8(-a+2k; -a+b+k, -a+c+k, -a+d+k, -a+e+k, -a+f+k, -a+h+k, -a+g+k; 1)
\end{aligned}
$$

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T111031

Description: Transformation formula ([6], (7.8), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T121105`.

$$
\begin{aligned}
{}_{11}V_{10}(a;c,\frac{1}{2}+c,d,\frac{1}{2}+d,e,\frac{1}{2}+e,\frac{1}{2}+2a-c-d-e+\frac{n}{2},1+2a-c-d-e+\frac{n}{2},-n;1)\\
\longrightarrow \frac{(1+2a,1+2a-2c-2d,1+2a-2c-2e,1+2a-2d-2e)_n}{(1+2a-2c,1+2a-2d,1+2a-2e,1+2a-2c-2d-2e)_n}\\
{}_8V_7(2a+n;2c,2d,2e,\frac{1}{2}+a+n,1+4a-2c-2d-2e+n,-n;-1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

T111032

Description: Transformation formula ([6], (7.7), $q \to 1$) in form of a rule. A q-analogue is HYPQ's `T121106`.

$$
\begin{aligned}
{}_{11}V_{10}(a;2e,c,\frac{1}{2}+c,d,\frac{1}{2}+d,\frac{1}{2}+2a-c-d-e+\frac{n}{2},1+2a-c-d-e+\frac{n}{2},\frac{1}{2}-\frac{n}{2},\frac{-n}{2};1)\\
\longrightarrow \frac{(1+2a,1+2a-2c-2d,1+2a-2c-2e,1+2a-2d-2e)_n}{(1+2a-2c,1+2a-2d,1+2a-2e,1+2a-2c-2d-2e)_n}\\
{}_8V_7(2a-2e;\frac{1}{2}+a-2e,2c,2d,2e,1+4a-2c-2d-2e+n,-n;-1)
\end{aligned}
$$

where n is a nonnegative integer.

See also: `S2103, S3201, TListe, TransListe, Ers, PosListe.`

TeX

Description: Switch that changes the output of TeXForm to be usable with Plain-TeX and LaTeX. By default the output of TeXForm is usable with AMS-TeX.

Usage: `TeX.`

Example(s):

```
In[1]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised basic hypergeometric series.

In[2]:= TeXForm[F[{a,b},{c},z]]

Out[2]//TeXForm=
  {} _{2} F _{1} \!\left [ \matrix { a, b}\\ { c}\endmatrix ; {\displaystyle
   z}\right ]
```

```
In[3]:= TeX

In[4]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with Plain-TeX and LaTeX.
TeXForm uses V[] for very well-poised basic hypergeometric series.

In[5]:= TeXForm[F[{a,b},{c},z]]

Out[5]//TeXForm=
  {} _{2} F _{1} \!\left [ \matrix { a, b\cr c} ; {\displaystyle z} \right ]
```

See also: `AmSTeX`, `AmSLaTeX`, `LaTeX`, `TeXMat`, `TeXFV`.

TeXFV

Description: Switch that toggles between writing very well-poised hypergeometric series in terms of `V` and in terms of `F`, respectively, when written in TeXForm. By default very well-poised hypergeometric series are written in terms of `V`.

Usage: `TeXFV`.

Example(s):

```
In[1]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised basic hypergeometric series.

In[2]:= F[{a,1+a/2,b,c},{a/2,a+1-b,a+1-b},z]

                          a
             a, 1 + -, b, c
                          2
Out[2]=  F [                          ; z ]
        4 3  a
             -, 1 + a - b, 1 + a - b
             2

In[3]:= Reset

Out[0]= 0

In[1]:= hypAttributes
```

```
Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised basic hypergeometric series.

In[2]:= F[{a,1+a/2,b,c},{a/2,a+1-b,a+1-c},z]

                        a
        [     a, 1 + -, b, c         ]
                        2
Out[2]=  F  [                       ; z ]
        4 3 [ a
            [ -, 1 + a - b, 1 + a - c   ]
              2

In[3]:= TeXForm[%]

Out[3]//TeXForm= {} _{4} V _{3} ({\displaystyle a; b, c}; {\displaystyle z})

In[4]:= TeXFV

In[5]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses F[] for very well-poised basic hypergeometric series.

In[6]:= TeXForm[%2]

Out[6]//TeXForm=
  {} _{4} F _{3} \!\left [ \matrix { a, 1 + {a\over 2}, b, c}\\ { {a\over 2},
   1 + a - b, 1 + a - c}\endmatrix ; {\displaystyle z}\right ]

In[7]:= TeXFV

In[8]:= hypAttributes

Automatic evaluation of p and F is inactive.
Automatic cancelling in F is active.
The output of TeXForm can be used with AmS-TeX.
TeXForm uses V[] for very well-poised basic hypergeometric series.
```

See also: F, V, hypAttributes.

TeXMat

Description: Function that writes (to be precise: appends) an expression `Expr` in `InputForm` to a file `[name].m` and the `TeXForm` of `Expr` to the file `[name].tex`. The expressions are numbered automatically. The number can be reset by `SchreibeZahl`. The string `comment` is optional. It allows to place the comment `comment` above the expression and the number in each of the two files.

Usage: `TeXMat[Expr,name,comment]`.

Example(s):

```
In[1]:= TeXMat[p[a/3,2*n],filename]

In[2]:= TeXMat[F[{a,b},{c},z],filename,"A hypergeometric _2F_1 series"]

In[3]:= !type filename.m
A[1]:=
p[a/3, 2*n]
"A hypergeometric _2F_1 series"
A[2]:=
F[{a, b}, {c}, z]

In[3]:= !type filename.tex
A[1]:=
({ \textstyle {a\over 3}}) _{2\,n}
"A hypergeometric _2F_1 series"
A[2]:=
{} _{2} F _{1} \!\left [ \matrix { a, b}\\ { c}\endmatrix ; {\displaystyle
  z}\right ]
```

See also: `AmSTeX`, `AmSLaTeX`, `LaTeX`, `TeX`, `TeXFV`, `SchreibeZahl`.

Tgl2103

Description: Transformation formula ([7], (1.3.15)) in form of an equation. It is the same transformation as that in `T2103`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl2104

Description: Transformation formula ([2], Appendix (III.4), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T2104`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl2106

Description: Transformation formula ([2], Appendix (III.6), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T2106`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl2107

Description: Transformation formula ([2], Appendix (III.7), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T2107`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2110

Description: Transformation formula ([6], (3.2)) in form of an equation. It is the same transformation as that in `T2110`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2112

Description: Transformation formula ([6], (5.10)) in form of an equation. It is the same transformation as that in `T2112`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2131

Description: Transformation formula ([2], Appendix (III.6), reversed, $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T2131`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2132

Description: Transformation formula ([7], (2.5.7)) in form of an equation. It is the same transformation as that in `T2132`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2133

Description: Transformation formula ([6], (5.12)) in form of an equation. It is the same transformation as that in `T2133`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2134

Description: Transformation formula ([1], Ex. 4.(iii), p. 97, reversed) in form of an equation. It is the same transformation as that in `T2134`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2135

Description: Transformation formula ([1], Ex. 4.(iii), p. 97) in form of an equation. It is the same transformation as that in `T2135`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2136

Description: Transformation formula ([6], (5.10), reversed) in form of an equation. It is the same transformation as that in `T2136`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2137

Description: Transformation formula ([6], (5.12), reversed) in form of an equation. It is the same transformation as that in `T2137`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2138

Description: Transformation formula ([6], (3.31), reversed) in form of an equation. It is the same transformation as that in `T2138`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2139

Description: Transformation formula ([6], (3.31)) in form of an equation. It is the same transformation as that in `T2139`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2140

Description: Transformation formula ([6], (3.2), reversed) in form of an equation. It is the same transformation as that in `T2140`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2141

Description: Transformation formula ([2], (3.4.8), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T2141`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2163

Description: Transformation formula ([2], Ex 3.8, $q \to 1$; [7], pp. 36/37) in form of an equation. It is the same transformation as that in `T2163`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2191

Description: Transformation formula ([7], (1.8.10)) in form of an equation. It is the same transformation as that in `T2191`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl2192

Description: Transformation formula ([2], Ex 3.8, $q \to 1$, reversed; [7], pp. 36/37) in form of an equation. It is the same transformation as that in `T2192`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3204

Description: Transformation formula ([1], Ex. 7, p. 98) in form of an equation. It is the same transformation as that in `T3204`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3205

Description: Transformation formula ([7], (2.3.3.7)) in form of an equation. It is the same transformation as that in `T3205`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3206

Description: Transformation formula ([1], Ex. 7, p. 98, terminating form) in form of an equation. It is the same transformation as that in `T3206`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3207

Description: Transformation formula (Thomae [1879] in [2] (3.1.1)) in form of an equation. It is the same transformation as that in `T3207`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3217

Description: Transformation formula ([1] 4.4(2), reversed) in form of an equation. It is the same transformation as that in `T3217`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3231

Description: Transformation formula ([2], Appendix (III.21), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T3231`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3232

Description: Transformation formula ([1], Ex. 4.(iv), p.97, reversed, first form) in form of an equation. It is the same transformation as that in `T3232`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3233

Description: Transformation formula ([1], Ex. 4.(iv), p.97, reversed, second form) in form of an equation. It is the same transformation as that in `T3233`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3234

Description: Transformation formula ([7], (2.5.7), reversed) in form of an equation. It is the same transformation as that in `T3234`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3235

Description: Transformation formula ([2], Ex 3.4, $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T3235`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3236

Description: Transformation formula ([2], (3.4.8), $q \to 1$) in form of an equation. It is the same transformation as that in `T3236`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3237

Description: Transformation formula ([2], (3.10.4), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T3237`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3238

Description: Transformation formula ([1], Ex. 6, p. 97) in form of an equation. It is the same transformation as that in `T3238`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3239

Description: Transformation formula ([1], Ex. 4.(iv), p. 97) in form of an equation. It is the same transformation as that in `T3239`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3240

Description: Transformation formula ([2], (3.5.10), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T3240`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl3261

Description: Transformation formula ([2], (III.33), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T3261`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3262

Description: Transformation formula ([7], (4.3.4.2)) in form of an equation. It is the same transformation as that in `T3262`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3263

Description: Transformation formula ([2], Appendix (III.33), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T3263`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3264

Description: Transformation formula ([2], Appendix (III.34), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T3264`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3267

Description: Transformation formula ([2], Ex. 3.6, $q \to 1$) in form of an equation. It is the same transformation as that in `T3267`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl3268

Description: Transformation formula ([2], Ex. 3.6, $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T3268`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4301

Description: Transformation formula ([7], (4.3.5.1)) in form of an equation. It is the same transformation as that in `T4301`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4302

Description: Transformation formula ([2], Appendix (III.16), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T4302`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4303

Description: Transformation formula ([7], (2.4.1.1)) in form of an equation. It is the same transformation as that in `T4303`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4304

Description: Transformation formula ([7], (4.3.6.4)) in form of an equation. It is the same transformation as that in `T4304`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4306

Description: Transformation formula ([2], Appendix (III.21), $q \uparrow 1$, reversed) in form of an equation. It is the same transformation as that in `T3231`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4309

Description: Transformation formula ([2], Ex. 2.13(i), $q \to 1$) in form of an equation. It is the same transformation as that in `T4309`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4310

Description: Transformation formula ([2], Ex. 2.13(ii), $q \to 1$) in form of an equation. It is the same transformation as that in `T4310`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4312

Description: Transformation formula ([2], Ex. 3.4, $q \to 1$) in form of an equation. It is the same transformation as that in `T4312`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4313

Description: Transformation formula ([2], Ex. 8.15, $q \to 1$) in form of an equation. It is the same transformation as that in `T4313`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4331

Description: Transformation formula ([1], Ex. 6, p. 97, reversed) in form of an equation. It is the same transformation as that in `T4331`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4332

Description: Transformation formula ([1], 4.6(1)) in form of an equation. It is the same transformation as that in `T4332`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4362

Description: Transformation formula ([7], (2.4.4.3), reversed) in form of an equation. It is the same transformation as that in `T4362`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl4391

Description: Transformation formula ([2], (3.5.7), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T4391`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5401

Description: Transformation formula ([7], (2.4.3.4)) in form of an equation. It is the same transformation as that in `T5401`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5402

Description: Transformation formula ([2], Appendix (III.26), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T5402`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5403

Description: Transformation formula ([1], 4.6(1), reversed) in form of an equation. It is the same transformation as that in `T5403`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl5468

Description: Transformation formula ([2], Ex. 2.25, $q \to 1$) in form of an equation. It is the same transformation as that in `T5468`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl6501

Description: Transformation formula ([7], (2.4.3.3)) in form of an equation. It is the same transformation as that in `T6501`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl6531

Description: Transformation formula ([7], (2.4.3.5)) in form of an equation. It is the same transformation as that in `T6531`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl6532

Description: Transformation formula ([2], Ex. 2.13(ii), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T6532`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl6533

Description: Transformation formula ([2], (3.10.4), $q \to 1$) in form of an equation. It is the same transformation as that in `T6533`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl6534

Description: Transformation formula ([1], 4.4(2)) in form of an equation. It is the same transformation as that in `T6534`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl7631

Description: Transformation formula ([7], (2.4.1.1), reversed) in form of an equation. It is the same transformation as that in `T7631`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl7632

Description: Transformation formula ([7], (2.4.1.1), reversed) in form of an equation. It is the same transformation as that in `T7632`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl7633

Description: Transformation formula ([7], (4.3.6.4), reversed) in form of an equation. It is the same transformation as that in `T7633`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl7634

Description: Transformation formula ([1], 7.5.(1)) in form of an equation. It is the same transformation as that in `T7634`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl7635

Description: Transformation formula ([1], 7.5.(2)) in form of an equation. It is the same transformation as that in `T7635`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7636

Description: Transformation formula ([2], (3.5.10), $q \to 1$) in form of an equation. It is the same transformation as that in `T7636`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7637

Description: Transformation formula ([2], Ex. 2.13(i), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T7637`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7691

Description: Transformation formula ([7], (2.4.4.3)) in form of an equation. It is the same transformation as that in `T7691`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7692

Description: Transformation formula ([7], (4.3.7.8)) in form of an equation. It is the same transformation as that in `T7692`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7693

Description: Transformation formula ([2], Ex. 2.15, $q \to 1$) in form of an equation. It is the same transformation as that in `T7693`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl7694

Description: Transformation formula ([7], (4.3.7.8), reversed) in form of an equation. It is the same transformation as that in `T7694`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8731

Description: Transformation formula ([6], (7.7), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T8731`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl8732

Description: Transformation formula ([6], (7.8), $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T8732`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9831

Description: Transformation formula ([7], (2.4.4.1)) in form of an equation. It is the same transformation as that in `T9831`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9832

Description: Transformation formula ([7], (2.4.3.4), reversed) in form of an equation. It is the same transformation as that in `T9832`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9833

Description: Transformation formula ([2], Appendix (III.26), $q \uparrow 1$, reversed) in form of an equation. It is the same transformation as that in `T9833`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9834

Description: Transformation formula ([7], (2.4.3.5), reversed) in form of an equation. It is the same transformation as that in `T9834`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9835

Description: Transformation formula ([1], 7.6(1)) in form of an equation. It is the same transformation as that in `T9835`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9836

Description: Transformation formula ([2], Ex. 3.21(iii), $q \to 1$) in form of an equation. It is the same transformation as that in `T9836`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9837

Description: Transformation formula ([7], (2.4.3.3), reversed) in form of an equation. It is the same transformation as that in `T9837`.

See also: `Sgl2101`, `TransListe$gl`, `Gleichung`.

Tgl9838

Description: Transformation formula ([2], Ex. 8.15, $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T9838`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl9891

Description: Transformation formula ([2], Appendix (III.39), $q \uparrow 1$) in form of an equation. It is the same transformation as that in `T9891`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl9892

Description: Transformation formula ([2], (3.5.7), $q \to 1$) in form of an equation. It is the same transformation as that in `T9892`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl9893

Description: Transformation formula ([2], Ex. 2.30, $q \to 1$) in form of an equation. It is the same transformation as that in `T9893`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl9894

Description: Transformation formula ([2], Ex. 2.25, $q \to 1$, reversed) in form of an equation. It is the same transformation as that in `T9894`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl111031

Description: Transformation formula ([6], (7.8), $q \to 1$) in form of an equation. It is the same transformation as that in `T111031`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tgl111032

Description: Transformation formula ([6], (7.7), $q \to 1$) in form of an equation. It is the same transformation as that in `T111032`.

See also: `Sgl2101, TransListe$gl, Gleichung.`

Tli2103

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (1.3.15)) in form of a rule. The transformation is the same as that in `T2103`.

See also: `Tli2107, Tli2104, TransListe$gl, Gleichung.`

Tli2104

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.4), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T2104`.

Example(s):

Here we demonstrate the iterated application of `Tli`-rules. We continue the `Tli2107`-example. For information of how to read the resulting listing confer `Tli2107`.

```
In[4]:= %2/.Tli2104

                  -m, -n
Out[4]= {2,  F [            ; z ], {{{{
            2 1    a

       m         -m, 1 - a - m     1 - z
      z    F  [                 ; -(-----)  ] (a + n)
           2 1   1 - a - m - n       z                m

>          ----------------------------------------}}, T2107}, T2104},
                               (a)
                                  m

       n         -n, 1 - a - n     1 - z
      z    F  [                 ; -(-----)  ] (a + n)
           2 1   1 - a - m - n       z                m

>      {{{{----------------------------------------}}, T2107},
                               (a)
                                  m

>       {FPerm[2, 1, u], T2104}},

       m         -m, 1 - a - m     1 - z
      z    F  [                 ; -(-----)  ] (a + m)
           2 1   1 - a - m - n       z                n

>      {{{----------------------------------------},
                               (a)
                                  n

>        {FPerm[2, 1, u], T2107}}, {FPerm[2, 1, u], T2104}},
```

```
              n      -n, 1 - a - n    1 - z
            z   F                  ; -(-----)  (a + m)
                2 1  1 - a - m - n      z            n
⟩      {{{----------------------------------------},
                                (a)
                                   n

⟩        {FPerm[2, 1, u], T2107}}, T2104}}
```

```
In[5]:= %1/.FPerm[2,1,u]/.T2107/.FPerm[2,1,u]/.T2104
Is n a nonnegative integer?
[y|n]:    y

              m      -m, 1 - a - m    1 - z
            z   F                  ; -(-----)  (a + m)
                2 1  1 - a - m - n      z            n
Out[5]= -----------------------------------------
                                (a)
                                   n
```

See also: `Tli2107`, `TransListe`, `Gleichung`.

Tli2106

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.6), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T2106`.

See also: `Tli2107`, `Tli2104`, `TransListe`, `Gleichung`.

Tli2107

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.7), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T2107`.

Example(s):

```
In[1]:= F[{-m,-n},{a},z]

              -m, -n
Out[1]=  F          ; z
         2 1    a
```

```
In[2]:= %/.Tli2107
Is m a nonnegative integer?
[y|n]:    y
Is n a nonnegative integer?
[y|n]:    y

                                              -m, -n
                                        F                 ; 1 - z   (a + n)
                                        2 1  1 - a - m - n                   m
                       -m, -n
Out[2]= {1,  F               ; z  , {{{-----------------------------------}},
             2 1    a                                    (a)
                                                            m

                        -n, -m
                  F                 ; 1 - z   (a + m)
                  2 1  1 - a - m - n                 n
〉     T2107}, {{------------------------------------},
                                  (a)
                                     n

〉     {FPerm[2, 1, u], T2107}}}
```

The first entry in this list counts the number of iterations of `Tli`-rules (cf. `Tli2104`), the second entry displays the original expression to which the `Tli`-rules are applied. The subsequent entries of the list always display an expression together with the sequence of rules that have to be applied to obtain this expression from the original expression. For instance, the above list says that the number of iterations is 1, the original expression is ${}_2F_1\left[\begin{matrix}-m,-n\\ a\end{matrix};z\right]$, and (if the parameters of the original series are permuted) by the application of `T2101` two different expressions can be obtained from the original series. The second of them is obtained by first permuting the upper parameters by `FPerm[2,1,u]` and then applying `T2101`.

```
In[3]:= %1/.FPerm[2,1,u]/.T2107
Is n a nonnegative integer?
[y|n]:    y

                -n, -m
          F                 ; 1 - z   (a + m)
          2 1  1 - a - m - n                 n
Out[3]= ------------------------------------
                          (a)
                             n
```

For an example of how to iterate `Tli`-rules see `Tli2104`.

See also: `Tli2104, TransListe, Gleichung.`

Tli2110

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (3.2)) in form of a rule. The transformation is the same as that in `T2110`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2112

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (5.10)) in form of a rule. The transformation is the same as that in `T2112`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2131

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.6), reversed, $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T2131`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2132

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.5.7)) in form of a rule. The transformation is the same as that in `T2132`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2133

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (5.12)) in form of a rule. The transformation is the same as that in `T2133`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2134

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 4.(iii), p. 97, reversed) in form of a rule. The transformation is the same as that in `T2134`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2135

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 4.(iii), p. 97) in form of a rule. The transformation is the same as that in `T2135`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2136

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (5.10), reversed) in form of a rule. The transformation is the same as that in `T2136`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli2137

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (5.12), reversed) in form of a rule. The transformation is the same as that in `T2137`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli2138

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (3.31), reversed) in form of a rule. The transformation is the same as that in `T2138`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli2139

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (3.31)) in form of a rule. The transformation is the same as that in `T2139`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli2140

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (3.2), reversed) in form of a rule. The transformation is the same as that in `T2140`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli2141

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.4.8), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T2141`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli2163

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex 3.8, $q \to 1$; [7], pp. 36/37) in form of a rule. The transformation is the same as that in `T2163`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli2191

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (1.8.10)) in form of a rule. The transformation is the same as that in `T2191`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli2192

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex 3.8, $q \to 1$, reversed; [7], pp. 36/37) in form of a rule. The transformation is the same as that in `T2192`.

See also: `Tli2107, Tli2104, TransListe, Gleichung`.

Tli3204

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 7, p. 98) in form of a rule. The transformation is the same as that in `T3204`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3205

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.3.3.7)) in form of a rule. The transformation is the same as that in `T3205`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3206

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 7, p. 98, terminating form) in form of a rule. The transformation is the same as that in `T3206`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3207

Description: Rule that gives a list of all possible outcomes under application of the transformation formula (Thomae [1879] in [2] (3.1.1)) in form of a rule. The transformation is the same as that in `T3207`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3217

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1] 4.4(2), reversed) in form of a rule. The transformation is the same as that in `T3217`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3231

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.21), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T3231`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3232

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 4.(iv), p.97, reversed, first form) in form of a rule. The transformation is the same as that in `T3232`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3233

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 4.(iv), p.97, reversed, second form) in form of a rule. The transformation is the same as that in `T3233`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3234

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.5.7), reversed) in form of a rule. The transformation is the same as that in `T3234`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3235

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex 3.4, $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T3235`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3236

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.4.8), $q \to 1$) in form of a rule. The transformation is the same as that in `T3236`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3237

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.10.4), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T3237`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3238

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 6, p. 97) in form of a rule. The transformation is the same as that in `T3238`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3239

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 4.(iv), p. 97) in form of a rule. The transformation is the same as that in `T3239`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3240

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.5.10), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T3240`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3261

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (III.33), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T3261`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3262

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (4.3.4.2)) in form of a rule. The transformation is the same as that in `T3262`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3263

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.33), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T3263`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3264

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.34), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T3264`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3267

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 3.6, $q \to 1$) in form of a rule. The transformation is the same as that in `T3267`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli3268

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 3.6, $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T3268`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4301

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (4.3.5.1)) in form of a rule. The transformation is the same as that in `T4301`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4302

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.16), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T4302`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4303

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.1.1)) in form of a rule. The transformation is the same as that in `T4303`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4304

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (4.3.6.4)) in form of a rule. The transformation is the same as that in `T4304`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4306

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.21), $q \uparrow 1$, reversed) in form of a rule. The transformation is the same as that in `T3231`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4309

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.13(i), $q \to 1$) in form of a rule. The transformation is the same as that in `T4309`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4310

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.13(ii), $q \to 1$) in form of a rule. The transformation is the same as that in `T4310`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4312

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 3.4, $q \to 1$) in form of a rule. The transformation is the same as that in `T4312`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4313

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 8.15, $q \to 1$) in form of a rule. The transformation is the same as that in `T4313`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4331

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], Ex. 6, p. 97, reversed) in form of a rule. The transformation is the same as that in `T4331`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4332

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], 4.6(1)) in form of a rule. The transformation is the same as that in `T4332`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4362

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.4.3), reversed) in form of a rule. The transformation is the same as that in `T4362`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli4391

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.5.7), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T4391`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli5401

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.3.4)) in form of a rule. The transformation is the same as that in `T5401`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli5402

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.26), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T5402`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli5403

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], 4.6(1), reversed) in form of a rule. The transformation is the same as that in `T5403`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli5468

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.25, $q \to 1$) in form of a rule. The transformation is the same as that in `T5468`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli6501

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.3.3)) in form of a rule. The transformation is the same as that in `T6501`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli6531

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.3.5)) in form of a rule. The transformation is the same as that in `T6531`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli6532

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.13(ii), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T6532`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli6533

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.10.4), $q \to 1$) in form of a rule. The transformation is the same as that in `T6533`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli6534

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], 4.4(2)) in form of a rule. The transformation is the same as that in `T6534`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7631

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.1.1), reversed) in form of a rule. The transformation is the same as that in `T7631`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7632

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.1.1), reversed) in form of a rule. The transformation is the same as that in `T7632`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7633

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (4.3.6.4), reversed) in form of a rule. The transformation is the same as that in `T7633`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7634

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], 7.5.(1)) in form of a rule. The transformation is the same as that in `T7634`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7635

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], 7.5.(2)) in form of a rule. The transformation is the same as that in `T7635`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7636

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.5.10), $q \to 1$) in form of a rule. The transformation is the same as that in `T7636`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7637

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.13(i), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T7637`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7691

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.4.3)) in form of a rule. The transformation is the same as that in `T7691`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7692

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (4.3.7.8)) in form of a rule. The transformation is the same as that in `T7692`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7693

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.15, $q \to 1$) in form of a rule. The transformation is the same as that in `T7693`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli7694

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (4.3.7.8), reversed) in form of a rule. The transformation is the same as that in `T7694`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli8731

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (7.7), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T8731`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli8732

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (7.8), $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T8732`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9831

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.4.1)) in form of a rule. The transformation is the same as that in `T9831`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9832

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.3.4), reversed) in form of a rule. The transformation is the same as that in `T9832`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9833

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.26), $q \uparrow 1$, reversed) in form of a rule. The transformation is the same as that in `T9833`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9834

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.3.5), reversed) in form of a rule. The transformation is the same as that in `T9834`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9835

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([1], 7.6(1)) in form of a rule. The transformation is the same as that in `T9835`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9836

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 3.21(iii), $q \to 1$) in form of a rule. The transformation is the same as that in `T9836`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9837

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([7], (2.4.3.3), reversed) in form of a rule. The transformation is the same as that in `T9837`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9837

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 8.15, $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T9838`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9891

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Appendix (III.39), $q \uparrow 1$) in form of a rule. The transformation is the same as that in `T9891`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9892

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], (3.5.7), $q \to 1$) in form of a rule. The transformation is the same as that in `T9892`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9893

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.30, $q \to 1$) in form of a rule. The transformation is the same as that in `T9893`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli9894

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([2], Ex. 2.25, $q \to 1$, reversed) in form of a rule. The transformation is the same as that in `T9894`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli111031

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (7.8), $q \to 1$) in form of a rule. The transformation is the same as that in `T111031`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

Tli111032

Description: Rule that gives a list of all possible outcomes under application of the transformation formula ([6], (7.7), $q \to 1$) in form of a rule. The transformation is the same as that in `T111032`.

See also: `Tli2107, Tli2104, TransListe, Gleichung.`

TListe

Description: Rule that gives for a hypergeometric series a list of applicable transformation formulas. Each entry of this list has the format {`ArgumentPermutations,T`⟨`number`⟩}, where `ArgumentPermutations` is a sequence of reorderings of the parameters of the hypergeometric series (given in terms of `FPerm` and `FTausche`) and `T`⟨`number`⟩ is the name of the transformation in form of a rule which can be applied subsequently. You should be aware that `TListe` automatically applies `FOrdne` before checking which transformation could be applied.

Important Note: If the value returned by `TListe` is the empty set this does *not* mean that no transformation can be applied. You always must remember that the list of transformations included in this package is a list of *basic* transformations. There are numerous special cases of these transformations which are not contained in this list as a separate transformation. The examples below should illustrate these remarks.

Usage: `Expr/.TListe.`

Example(s):

```
In[1]:= F[{a,b},{c},z]
```

```
Out[1]=  F  [  a, b       ]
        2 1[        ; z  ]
           [   c         ]


In[2]:= %/.TListe
Is -a a nonnegative integer?
[y|n]:    n
Is -b a nonnegative integer?
[y|n]:    n
Be sure to apply "FOrdne" before using the following information!

Out[2]= {{T2103}, {T2104}, {T2191}}

In[3]:= F[{-n,2*a,2*b},{-2*n,1/2+a+b},1]

           [ -n, 2 a, 2 b        ]
           [                     ]
Out[3]=  F [       1         ; 1 ]
        3 2[-2 n, - + a + b      ]
           [       2             ]

In[4]:= %/.TListe
Is -2*b a nonnegative integer?
[y|n]:    n
Is -2*a a nonnegative integer?
[y|n]:    n
Is n a nonnegative integer?
[y|n]:    y
Be sure to apply "FOrdne" before using the following information!

Out[4]= {{T3204}, {T3205}, {T3206}, {T3207}, {T3217}, {T3261}, {T3262},

>    {FPerm[2,3,1,u], FTausche[1,2,l], T3231}}

In[5]:= %3/.FOrdne/.FPerm[2,3,1,u]/.FTausche[1,2,l]/.T3231
Is -2*a a nonnegative integer?
[y|n]:    n
Is -2*b a nonnegative integer?
[y|n]:    n
Is n a nonnegative integer?
[y|n]:    y

           [       a, b, -n, -n            ]
           [                               ]
Out[5]=  F [ 1                1 - 2 n; 1   ]
        4 3[ - + a + b, -n, -------        ]
           [ 2                  2          ]
```

Now we consider two examples illustrating the note above. Though none of the implemented transformations can be applied, both series can be transformed, the first by a limiting case of Whipple's transformation, the second by a number of specialized very well-poised ${}_7\phi_6$ transformations. These facts are also observed by using this package.

```
In[6]:= F[{a,1+a/2,b,c,d,e},{a/2,1+a-b,1+a-c,1+a-d,1+a-e},-1]

                          a
                   a, 1 + -, b, c, d, e
                          2
Out[6]=  F                                             ; -1
        6 5 a
            -, 1 + a - b, 1 + a - c, 1 + a - d, 1 + a - e
            2

In[7]:= %/.TListe

Out[7]= {}

In[8]:= Tgl7632
Do you want to set values for the equation?  [y|n]:  n

                             a
                      a, 1 + -, b, c, d, e, -n
                             2
Out[8]=  F                                                       ; 1    ==
        7 6 a
            -, 1 + a - b, 1 + a - c, 1 + a - d, 1 + a - e, 1 + a + n
            2

                                                  (1 + a, 1 + a - d - e)
                1 + a - b - c, d, e, -n                                n
>      F                                   ; 1    ----------------------
      4 3  1 + a - b, 1 + a - c, -a + d + e - n    (1 + a - d, 1 + a - e)
                                                                       n

In[9]:= Limes[%,n->Infinity]

                          a
                   a, 1 + -, b, c, d, e
                          2
Out[9]=  F                                             ; -1    ==
        6 5 a
            -, 1 + a - b, 1 + a - c, 1 + a - d, 1 + a - e
            2
```

```
        [ 1 + a - b - c, d, e           ]
       F[                          ; 1  ] Γ(1 + a - d) Γ(1 + a - e)
    3 2 [ 1 + a - b, 1 + a - c          ]
⟩      -----------------------------------------------------
                    Γ(1 + a) Γ(1 + a - d - e)

In[10]:= F[{1+a/2,1,c,d,e,f},{a/2,1+a-c,1+a-d,1+a-e,1+a-f},1]

                                 a
                             1 + -, 1, c, d, e, f
                                 2
Out[10]=  F [                                                ; 1 ]
         6 5 a
             -, 1 + a - c, 1 + a - d, 1 + a - e, 1 + a - f
             2

In[11]:= %/.TListe

Out[11]= {}

In[12]:= %%/.FEinf
Add the parameter:  a

                                    a
                             a, 1 + -, 1, c, d, e, f
                                    2
Out[12]=  F [                                                   ; 1 ]
         7 6    a
             a, -, 1 + a - c, 1 + a - d, 1 + a - e, 1 + a - f
                2

In[13]:= %/.TListe
Is -f a nonnegative integer?
[y|n]:    n
Is -1 - a + e + f a nonnegative integer?
[y|n]:    n
Is -1 - a + d + f a nonnegative integer?
[y|n]:    n
Is -1 - a + d + e a nonnegative integer?
[y|n]:    n
Is -1 - a + c + f a nonnegative integer?
[y|n]:    n
Is -1 - a + c + e a nonnegative integer?
[y|n]:    n
Is -1 - a + c + d a nonnegative integer?
[y|n]:    n
Is -a + f a nonnegative integer?
[y|n]:    n
```

```
Is -a + e a nonnegative integer?
[y|n]:    n
Is -a + d a nonnegative integer?
[y|n]:    n
Is -a + c a nonnegative integer?
[y|n]:    n

Be sure to apply "FOrdne" before using the following information!

Out[13]= {{T7631}, {T7634}, {T7635}, {T7691}, {T7692}}
```

See also: `SListe`, `FPerm`, `FTausche`, `TransListe`.

---

trans

Description: $(a)_n \to (-1)^n\,(1-n-a)_n$.

Usage: `Expr/.trans`.

Example(s):

```
In[1]:= p[a,n]

Out[1]= (a)
           n

In[2]:= %/.trans

            n
Out[2]= (-1)  (1 - a - n)
                         n
```

See also: `Ers`, `PosListe`, `ManipulationsListe`.

TransListe

Description: List of all transformation formulas.

Usage: `TransListe`.

See also: `TransListe$gl`, `TListe`.

TransListe$gl

Description: List of all transformation formulas.

Usage: `TransListe$gl`.

See also: `TransListe`.

V

Description: `V[a,List,z]` is the very well-poised hypergeometric series.

Usage: `V[a,List,z]`.

Example(s):

```
In[1]:= V[a,{b,c},z]

                        a
             a, 1 + -, b, c
                        2
Out[1]=  F                            ; z
        4 3  a
             -, 1 + a - b, 1 + a - c
             2
```

See also: `F, TeXFV, P, FFormat`.

ZB

Description: Rule that looks for a recurrence relation for a sum or hypergeometric series using Zeilberger's algorithm [10, 11, 12].
Here a call is made to the function `Zb` of the *Mathematica* implementation of Gosper's and Zeilberger's algorithms written by Peter Paule and Markus Schorn. The current version 1.1 or updates can be received via e-mail request to `peter.paule@risc.uni-linz.ac.at`. This implemenation provides the user with the objects

`Zb`, `Gosper`, `RunMode`, `FileName`, `SolAmount`, `Fnk`, `GoRat`, `GoSol`, `Cert`, `DegBound`, `System`, `SystemDimension`.

Also within the package HYP, all these objects work as described in the documentation of this implementation. Therefore the user is referred to this documentation and the description [5] in order to learn about the various features of these objects.
The package HYP provides two additional objects, `ZB` and `GOSPER`. The rule `ZB` allows to apply Zeilberger's algorithm directly to an expression containing a `SUM` or a hypergeometric series.

Usage: `Expr/.ZB[recvar, order]`.

Example(s):

```
In[1]:= SUM[Binomial[N,k]*Binomial[M,L-k],{k,0,Infinity}]

          ∞
         ___(          ) (   )
         \  (    M     ) ( N )
Out[1]=   > (          ) (   )
         /  ( -k + L ) ( k )
         ‾‾‾(          ) (   )
         k=0

In[2]:= %/.ZB[L,1]

    Peter Paule and Markus Schorn's implementation of the
             Zeilberger algorithm.  (Version 1.1)
```

```
Out[2]= {(-L + M + N) SUM[L] + (-1 - L) SUM[1 + L] == 0}

In[3]:= %1/.SUMF

          [  -L, -N         ]
         F[                 ] (1 - L + M)
        2 1[ 1 - L + M   ; 1 ]             L

Out[3]= --------------------------------
                      (1)
                         L

In[4]:= %/.ZB[L,1]

    Peter Paule and Markus Schorn's implementation of the
             Zeilberger algorithm.  (Version 1.1)
Is N a nonnegative integer?
[y|n]:    n

Out[4]= {(-L + M + N) SUM[L] + (-1 - L) SUM[1 + L] == 0}
```

See also: GOSPER.

---

zerl

Description: $(a)_n \to (a)_m(a+m)_{n-m}$,
$\Gamma(a) \to \Gamma(a+m)/(a)_m$.

The parameter m has to be entered on request.

Usage: Expr/.zerl.

Example(s):

```
In[1]:= p[a,2*n]

Out[1]= (a)
           2 n

In[2]:= %/.zerl
bottom-split by:  m+n

Out[2]= (a)      (a + m + n)
           m + n            -m + n
```

See also: Ers, PosListe, ManipulationsListe.

---

zus1

Description: $(a)_n(a+n)_m \to (a)_{n+m}$,
$(a)_n/\Gamma(a+n) \to 1/\Gamma(a)$.

Usage: `Expr/.zus1`.

Example(s):

```
In[1]:= p[a,2*n]*p[a+2*n,m-n]

Out[1]= (a)     (a + 2 n)
           2 n           m - n

In[2]:= %/.zus1

Out[2]= (a)
           m + n

In[3]:= p[a,2*n]/GAMMA[a+2*n]

          (a)
             2 n
Out[3]= ----------
        Γ(a + 2 n)

In[4]:= %/.zus1

          1
Out[4]= ----
        Γ(a)
```

See also: `zus2, zus3, erw1, erw2, Ers, PosListe, ManipulationsListe`.

---

zus2

Description: $(a)_n/(a)_m \to (a+m)_{n-m}$,
$\Gamma(a)\,(a)_m \to \Gamma(a+m)$.

Usage: `Expr/.zus2`.

Example(s):

```
In[1]:= p[a,m]/p[a,n]*p[b,m+n]

        (a)   (b)
           m     m + n
Out[1]= -------------
            (a)
               n

In[2]:= %/.zus2
```

```
Out[2]= (b)       (a + n)
           m + n         m - n

In[3]:= p[a,m]*GAMMA[a]*p[b,m+n]

Out[3]= Γ(a) (a)   (b)
                m     m + n

In[4]:= %/.zus2

Out[4]= Γ(a + m) (b)
                    m + n
```

See also: `zus1`, `zus3`, `erw1`, `erw2`, `Ers`, `PosListe`, `ManipulationsListe`.

---

zus3

Description: $(a)_n/(b)_m \to (a)_{n-m}$,

provided $a+n=b+m$, and

$$\Gamma(a+n)/\Gamma(a) \to (a)_n.$$

Usage: `Expr/.zus3`.

Example(s):

```
In[1]:= p[a+m,n]/p[a+n,m]

        (a + m)
               n
Out[1]= --------
        (a + n)
               m

In[2]:= %/.zus3

Out[2]= (a + m)
               -m + n

In[3]:= GAMMA[a]/GAMMA[a+n]

          Γ(a)
Out[3]= --------
        Γ(a + n)

In[4]:= %/.zus3

Out[4]= (a + n)
               -n

In[5]:= %/.neg1
```

```
         1
Out[5]= ----
        (a)
           n
```

See also: `zus1, zus2, erw1, erw2, Ers, PosListe, ManipulationsListe.`

References

1. W. N. Bailey, *Generalized hypergeometric series*, Cambridge University Press, Cambridge, 1935.
2. G. Gasper and M. Rahman, *Basic hypergeometric series*, Encyclopedia of Mathematics And Its Applications 35, Cambridge University Press, Cambridge, 1990.
3. R. W. Gosper, *Decision procedure for indefinite hypergeometric summation*, Proc. Nat. Acad. Sci. USA **75** (1978), 40–42.
4. C. Krattenthaler, *HYP and HYPQ — Mathematica packages for the manipulation of binomial sums and hypergeometric series, respectively q-binomial sums and basic hypergeometric series.*
5. P. Paule and M. Schorn, *A Mathematica version of Zeilberger's algorithm for proving binomial coefficient identities*, J. Symbolic Comput. (to appear).
6. M. Rahman and A. Verma, *Quadratic transformation formulas for basic hypergeometric series*, Trans. Amer. Math. Soc. **335** (1993), 277–302.
7. L. J. Slater, *Generalized hypergeometric functions*, Cambridge University Press, Cambridge, 1966.
8. A. Verma and V. K. Jain, *Some summation formulae for nonterminating basic hypergeometric series*, SIAM J. Math. Anal. **16** (1985), 647–655.
9. S. Wolfram, *MATHEMATICA — A system for doing mathematics by computer*, second edition, Addison–Wesley, New York, 1991.
10. D. Zeilberger, *A fast algorithm for proving terminating hypergeometric identities*, Discrete Math. **80** (1990), 207–211.
11. D. Zeilberger, *A holonomic systems approach to special functiions identities*, J. Comput. Appl. Math. **32** (1990), 321–368.
12. D. Zeilberger, *The method of creative telescoping*, J. Symbolic Comput. **11** (1991), 195–204.