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KLAUS SCHMIDT AND ANATOLY VERSHIK

Abstract. We give three corrections to the paper [K. Schmidt and A.M. Vershik, Algebraic polymorphisms, *Ergod. Th. & Dynam. Sys.* **28** (2008), 633–642.]

(1) The statement in the penultimate paragraph on [1, p. 634] has to be corrected as follows: If π_1 is an injection then P is (the graph of) an endomorphism, and if π_2 is an injection then P is (the graph of) an exomorphism.

In other words, the symbols π_1 and π_2 should be interchanged.

(2) Corollary 1.7 in [1] is incorrect as stated. The correct statement should be:

Corollary. *Let $\mathsf{P} \subset G \times G$ be a correspondence and let $H \subset G$ be a closed normal subgroup. We denote by $K_{\mathsf{P}^n}^{(i)}$, $i = 1, 2$, the closed normal subgroups of G associated with the correspondence P^n, $n \geq 2$, in (1.4) by (1.9). The sequences of subgroups $(K_{\mathsf{P}^n}^{(i)}, n \geq 1)$ are nondecreasing, and we write $H_0^{(i)} = \overline{\bigcup_{n\geq 1} K_{\mathsf{P}^n}^{(i)}}$ for the closure of $\bigcup_{n\geq 1} K_{\mathsf{P}^n}^{(i)}$. Then the following holds.*

1. *$H_0^{(2)}$ is smallest invariant subgroup of Π_{P};*
2. *$H_0^{(1)}$ is the smallest co-invariant subgroup of Π_{P}.*

Proof. By definition, $\eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} K_{\mathsf{P}^n}^{(2)}) = K_{\mathsf{P}^{n+1}}^{(2)}$ for all $n \geq 1$.

If a closed normal subgroup $H \subset G$ is invariant under Π_{P} then [1, Theorem 1.6(1)] shows that $K_{\mathsf{P}^2}^{(2)} = \eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} K_{\mathsf{P}}^{(2)}) \subset \eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} H) \subset H$. Hence $K_{\mathsf{P}^3}^{(2)} = \eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} K_{\mathsf{P}^2}^{(2)}) \subset \eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} H) \subset H$ and, by induction, $K_{\mathsf{P}^n}^{(2)} \subset H_0^{(2)} \subset H$ for every $n \geq 1$.

In order to verify that $H_0^{(2)}$ is invariant under Π_{P} we note that $K_{\mathsf{P}^{n+1}}^{(2)} = \eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} K_{\mathsf{P}^n}^{(2)}) \subset H_0^{(2)}$ for every $n \geq 1$, and by letting $n \to \infty$ we see that $\eta_{\mathsf{P}}(K_{\mathsf{P}}^{(1)} H_0^{(2)}) \subset H_0^{(2)}$. According to [1, Theorem 1.6(1)] this proves that $H_0^{(2)}$ is invariant.

The proof of the second assertion is analogous. □

(3) The third correction concerns the semigroup $\mathcal{P}_f(\mathbb{T}^m)$ of all finite-to-one correspondences of $\mathbb{T}^m$. Denote by $\mathcal{L}$ the semigroup of all finite index subgroups of $\mathbb{Z}^m$ with intersection as composition (and not, as stated wrongly in [1, p. 637], with addition). We consider the semigroup

$$\mathcal{M} = \{(Q, \Lambda) | Q \in \mathrm{GL}(m, \mathbb{Q}), \Lambda \in \mathcal{L}, \Lambda \subset \Lambda_Q := \mathbb{Z}^m \cap Q\mathbb{Z}^m\}, \tag{1}$$

with composition

$$(Q, \Lambda) \cdot (Q', \Lambda') = (QQ', \Lambda \cap Q\Lambda'), \tag{2}$$

where we again have replaced addition by intersection.

This correction does not affect any of the results or proofs in that section.

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References

[1] K. Schmidt and A.M. Vershik, *Algebraic polymorphisms*, Ergod. Th. & Dynam. Sys. **28** (2008), 633–642.

Klaus Schmidt: Mathematics Institute, University of Vienna, Nordbergstrasse 15, A-1090 Vienna, Austria
and
Erwin Schrödinger Institute for Mathematical Physics, Boltzmanngasse 9, A-1090 Vienna, Austria
E-mail address: klaus.schmidt@univie.ac.at

Anatoly Vershik: Steklov Institute of Mathematics at St. Petersburg, 27 Fontanka, St.Petersburg 191011, Russia
E-mail address: vershik@pdmi.ras.ru