Abstract—This paper presents an optimization procedure conceived to design parallel mechanisms (PMs) with legs of constant and/or variable length connected, at their endpoints, to a fixed base and a movable platform through universal and spherical joints, respectively. In the proposed procedure, the first natural frequency of the mechanism is the objective function to be maximized. The optimization problem is formulated by using dimensionless variables in order to identify the optimal geometry independently of mechanism size, platform density, and leg cross-sectional area and material. As a case study, the procedure is employed to find the optimal geometry of a 2-DOF spherical PM to be used as an orienting device in future space missions.

Index Terms—Natural frequencies, optimization, parallel mechanisms (PMs).

I. INTRODUCTION

Parallel mechanisms (PMs) exhibit high payload to weight ratio, high accuracy, high structural rigidity, and high dynamic capabilities. In general, however, they suffer from complex forward kinematic analysis, small workspace, and low dexterity. The design of an effective mechanism amounts to finding the system geometry that best complies with given application requirements. In the literature, several approaches have been proposed to design PMs with maximum workspace, maximum dexterity, isotropic behaviors, optimal inertia, and desired stiffness characteristics [1]. These approaches consist in finding the mechanism geometry that maximizes or minimizes given performance indexes (or objective functions). With regard to system rigidity, optimization of the stiffness of PMs is studied in [2]–[4] by means of stiffness maps that are constructed in properly chosen workspace sections by a contour plot procedure. In particular, the contour maps of the components of the Cartesian stiffness matrix are employed in [2]; the contour maps of the smallest eigenvalue of the Cartesian stiffness matrix, the contour maps of the largest eigenvalue of the Cartesian stiffness matrix, the contour maps of the engineering (single-dimensional) stiffness index, and the contour maps of the general stiffness index are employed in [3]; the contour maps of the average, in Euclidean norm, of the eigenvalues of the Cartesian stiffness matrix are employed in [4]. Geometrical optimization of PMs is performed in [5] by maximizing, through the Monte Carlo method, an objective function that consists of the weighed sum of the minimum and mean values (in the exploitable workspace) of both the volume of the rigidity ellipsoid and the smallest eigenvalue of the stiffness matrix. Geometrical optimization of PMs is performed in [6] by maximizing, through genetic algorithms, an objective function that consists of the sum of the mean and standard deviation of the average value of the eigenvalues of the stiffness matrix.

The optimization of the mechanism geometry via objective functions based on either the coefficients or the eigenvalues of the Cartesian stiffness matrix may have no physical meaning. Indeed, in general, a mechanism has both rotational and translational stiffnesses, and they are usually coupled. That is, the coefficients of the Cartesian stiffness matrix are not homogeneous. This fact makes it difficult to relate one coefficient to the others and, from a physical standpoint, makes the associated eigenvalue problem non-well-posed (see Appendix A). To attempt handling this problem, matrix normalization techniques, similar to those proposed in [7] and [8], may be used. These techniques are based on pre- and postmultiplication of the nonhomogeneous matrix by scaling matrices. Nonetheless, the choice of scaling matrices is rather wide and very subjective. Thus, optimization via performance indexes extracted from normalized matrices is weak.

Flawless optimization of mechanism geometry can instead be attempted by adopting objective functions based on mechanism natural frequencies (for example, the first, i.e., the lowest, natural frequency). Natural frequencies are objective frame- and scale-invariant indexes of both dynamic and kinematic mechanism performance. Indeed, besides being physically meaningful

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measures of mechanism weight and stiffness, natural frequencies also assess how close a PM is to singular configurations (a PM singularity appears as a set of zero natural frequencies, the dimension of which is equal to the number of DOFs acquired by the PM at that singularity). Although already foreseen in [9], the optimization of mechanism geometry via objective functions based on mechanism natural frequencies has yet not been addressed in the literature. As a matter of fact, mechanism natural frequencies have only been employed, in practice, as performance indexes to analyze the nonlinearity of the dynamics of PMs throughout their workspace [10], [11], measure the close-ness to singularity of PMs [12], [13], and generate commands (via input shaping techniques) for the control of PMs [14].

In this context, this paper tackles for the first time the geometrical optimization of PMs through maximizing their first natural frequency in the mechanism workspace. In particular, the paper proposes an efficient method for optimizing the geometry of a broad class of PMs, which feature a fixed base and a movable platform connected to each other through both links (legs) of type US having constant length (P standing for actuated prismatic joint). Note that the well-known Stewart–Gough platforms belong to this class of PMs. The proposed method is efficient because the optimization problem is expressed in a suitable and dimensionless formulation that makes it possible to find the optimal mechanism geometry independently of mechanism size, platform density, and leg cross-sectional area and material.

In the paper, the method is first presented, and second, as a case study, the method is applied to find the optimal mechanism, among the class of 2-DOF spherical PMs described in [15], to be used as an orienting device in future space missions. Note that the optimization of this mechanism via the maximization of its first natural frequency is worthy since space applications require mechanisms featuring low mass and high stiffness. The optimization problem is solved using a clustering optimization algorithm.

II. CLASS OF PMs

The method proposed in this paper has been conceived for the optimization of nonactuated PMs having a fixed base (i.e., a fixed interface) and a movable platform connected to each other by legs of type US. In the literature [15], these mechanisms are often indicated with the short-name US-PMs. The method also applies to the optimization of the geometry of actuated mechanisms, which can be obtained by connecting the base and the platform of US-PMs via additional actuated legs of type UPS. Note that, in order for the resulting mechanism to be completely controllable, the number of legs of type UPS, which must be added, equals the number of DOFs of the US-PM. In the following, PMs with legs of types US and UPS will be briefly referred to as US/UPS-PMs.

A schematic of a US/UPS-PM with 3 DOF is depicted in Fig. 1. In this figure, points $B_i$ and $P_i$ indicate the centers of the universal and spherical joints, which connect the $i$th leg to the base and the platform, respectively. Legs $P_i B_1$, $P_5 B_5$, and $P_6 B_6$ are of type US, while legs $P_1 B_4$, $P_5 B_5$, and $P_6 B_6$ are of type UPS. Practical examples of US/UPS-PMs can be found in [15]–[19]. A class of 1-DOF US/UPS-PMs, in which the relative motion between the base and platform is helicoidal, is proposed in [16] (mechanisms belonging to this class have at least five legs of type US and only one leg of type UPS). A class of 2-DOF US/UPS-PMs, in which the relative motion between the base and platform is spherical, is proposed in [15] (mechanisms belonging to this class have at least four legs of type US and only two legs of type UPS). A class of 3-DOF US/UPS-PMs, in which the relative motion between the base and platform is spherical, is proposed in [17] (mechanisms belonging to this class have at least three legs of type US and only three legs of type UPS). A class of 6-DOF US/UPS-PMs, in which the relative motion between the base and platform is general, is proposed in [18] and [19] (mechanisms belonging to this class have legs of type UPS only).

III. SIMPLIFYING HYPOTHESES

The proposed geometrical optimization method relies on a simplified vibration model of US/UPS-PMs: 1) The legs of type US and UPS are modeled as rods, and the U and S joints are frictionless; 2) leg elasticity is linear; 3) all legs are made of the same material (i.e., they have the same Young’s modulus $E$); 4) all legs have the same cross-sectional area $A$; 5) the cross-sectional area $A$ and the lengths $l_{i(i)} = |P_i - B_i|$ of the legs remain nearly constant when loads are applied to the mechanism (i.e., leg deflections are in the small deformation regime); 6) the base and platform are much stiffer than the legs and can be considered as rigid bodies; and 7) legs have negligible mass and inertia compared with those of the platform. Thus, the vibration model of US/UPS-PMs consists of two rigid bodies (a fixed base and a movable platform), which are connected by massless translational springs (legs) by means of frictionless U and S joints. The movable platform is the only body, which is subjected to inertia forces. The stiffness of the $i$th translational spring is $k_{i(i)} = E A / l_{i(i)}$. Moreover, 8) the preload of the legs is considered to be negligible in the nominal pose of the mechanism; 9) buckling effects in the legs are not considered; and 10) member interference is not considered either.
The simplifying hypotheses 1)–8) are reasonable for PMs that must be efficient, stiff, lightweight, and easy to manufacture and assemble. Hypotheses 9) and 10) are introduced to enlarge the space of the optimal solutions (for a detailed explanation, refer to Section VI-F) and simplify the optimization procedure.

IV. DEFINITIONS

Considering the system depicted in Fig. 1, define the frame of reference $S_h$, which is fixed to the base and centered at point $O$, and the frame of reference $S_p$, which is fixed to the platform, centered at the platform center of mass $G$, and has the axes directed along the platform principal axes of inertia. Then, introduce the generalized coordinate array $q$, which defines uniquely the relative pose (position and orientation) of the platform and base. The array $q$ has six components; for example, the first three entries can be the components of vector $g = (G−O)$ relative to $S_h$, which defines the position of the platform point $G$ with respect to $S_h$, and the last three entries can be the $z$-$y$-$x$ Euler angles, which define the orientation of $S_p$ with respect to $S_h$. The components of $q$ are functions of a number of independent variables, which equals the number of the mechanism DOF. The 6-D space $Q$, which is spanned by $q$ (for all the admissible values of its components), is defined as the desired mechanism workspace.

Referring to the mechanism legs, the number of legs of type US is $m$; the sum of the number of legs of US and UPS types is $n$. The difference ($n−m$) is equal to the number of mechanism DOF (i.e., redundant actuation is not considered). For the $i$th leg, define the position vectors $r_i = (P_i − G)$ and $s_i = (B_i − O)$, and the leg vector $u_i = (P_i − B_i) = (r_i + g − s_i)$. Then, the leg lengths $l_{ij}$ follow as $l_{ij} = |u_i|$. Moreover, define the geometric parameter array $x$, which contains the components of $r_i$ with respect to $S_p$ and the components of $s_i$ with respect to $S_h$.

PMs with legs of types US and UPS can be grouped in different classes depending on the relative motion of the base and platform (e.g., the class of 1-DOF helicoidal US/UPS-PMs described in [16], the class of 2-DOF spherical US/UPS-PMs described in [15], the class of 3-DOF spherical US/UPS-PMs described in [17], and the class of 6-DOF US/UPS-PMs described in [18] and [19]). The motions of all the mechanisms belonging to a given class are parameterized by the same independent variables and generalized coordinate arrays $q$ having a similar structure; thus, they are characterized by workspaces $Q$ that belong to the same topology. Each mechanism belonging to a given class is uniquely characterized by a single value of the geometric parameter array $x$. All the admissible values that the array $x$ can attain within the class generate the space of admissible mechanism geometries $X$. That is, each class of mechanisms is uniquely characterized by the topology of $Q$ in terms of mechanism motion, and by the space $X$ in terms of mechanism geometry (a clarifying example is provided in Section VI-A).

V. GEOMETRICAL OPTIMIZATION METHOD

Given a class (i.e., given the topology of $Q$ and the space $X$) identifying the optimal mechanism geometry consists of finding the array $x_{opt}$, which belongs to $X$, that maximizes an objective function $F$. Here, $F$ is the minimum value attained by the mechanism first natural frequency in $Q$. In the following, the maximum of $F$ (in $X$) will be referred to as $\omega_{opt}$, i.e., $\omega_{opt} = F(x_{opt})$. In practice, optimal mechanism designs should guarantee that $\omega_{opt} \geq \omega_{des}$, where $\omega_{des}$ is a desired limiting frequency.

In the following sections, the mathematics of the optimization problem is discussed, and a proper formulation is proposed.

A. Natural Frequency Evaluation

According to the simplified vibration model introduced in Section III, the natural frequencies of the mechanism can be found by solving the following eigenvalue problem [20]:

$$\det(K - \omega^2M) = 0$$

where $K$ is the $(6 \times 6)$ Cartesian stiffness matrix [20], [21] of the mechanism, $\omega$ is one of the six unknown mechanism natural frequencies, and $M$ is the $(6 \times 6)$ inertial matrix [20] of the mechanism. The mechanism first natural frequency is the square root of the lowest eigenvalue of (1).

Conversely to [9] and [20], which suggest solving (1) via the equivalent nonsymmetric eigenvalue problem

$$\det(M^{-1}K - \omega^21) = 0 \quad (2a)$$

where $1$ is the $(6 \times 6)$ identity matrix, for computational reasons, in this paper, (1) is instead solved via the equivalent symmetric eigenvalue problem [22], [23]

$$\det(M^{-0.5}KM^{-0.5} - \omega^21) = 0. \quad (2b)$$

Note that the formulation of (2b) is computationally more efficient than that of (2a) since matrix $(M^{-0.5}KM^{-0.5})$ is symmetric and homogeneous (i.e., its components have the same dimensional unit [time$^{-2}$]), while matrix $(M^{-1}K)$ is nonsymmetric and nonhomogeneous (computational issues related to the solution of (2a) and (2b) are highlighted in Appendix B). Moreover, in this paper, the components of $M$ and $K$ are expressed with respect to the frame $S_p$. This choice makes the matrix $M$ diagonal and independent of $q$ and $x$ (recall that $S_p$ is the center of mass and principal axis frame of the movable platform). That is, in (2b), only the matrix $K$ depends on both $x$ and $q$.

From (2b), the mathematical expression of the objective function $F$ is

$$F = \min_{q \in Q}[\lambda_1(M^{-0.5}KM^{-0.5})]$$

where the operator $\lambda_1(A)$ is used to indicate the smallest eigenvalue of a matrix $A$. Correspondingly, the optimal mechanism geometry $x_{opt}$ is such that

$$\omega_{opt} = F(x_{opt}) = \max_{x \in X}[\{F(x) \geq \omega_{des}\}] \quad (4)$$

Coming to the practical expressions of $M$ and $K$, the inertial matrix of the mechanism (platform) can be written as [20]

$$M = \rho \Pi$$

(5)
where $\rho$ is the platform density and $\Pi$ is the diagonal matrix

$$\Pi = \text{diag}(V, V, I_1/\rho, I_2/\rho, I_3/\rho) \tag{6}$$

whose components are the platform volume $V$ and the platform principal moments of inertia $I_1$, $I_2$, and $I_3$ (which are evaluated with respect to $S_p$). Of course, $V$, $I_1$, $I_2$, and $I_3$ do not depend on $x$ and $q$.

Assuming that the legs of the mechanism are not preloaded, the Cartesian stiffness matrix of the mechanism is $[1]$, $[20]$, $[21]$

$$K = J^T \Sigma J \tag{7}$$

where $\Sigma$ is a $(n \times n)$ diagonal matrix containing the axial stiffness constants $k_i$ of $n$ legs ($i = 1, \ldots, n$), and $J$ is the inverse kinematic Jacobian matrix [1] of the mechanism, i.e., the $(n \times 6)$ matrix whose rows coincide with the $n$ Plücker coordinate arrays $a_i$ of the leg axes (i.e., the lines joining $P_i$ to $B_i$)

$$a_i = \frac{1}{l_i} \left[ u_i \ r_i \wedge u_i \right]. \tag{8}$$

Because the $(1 \times 6)$ arrays $a_i$ are functions of both $l_i$ and $u_i$, they depend on both $x$ and $q$. Providing that the legs are made of the same material and have the same cross-sectional area, (7) can be more conveniently rewritten as

$$K = EAH \tag{9}$$

where $E$ and $A$ are the Young’s modulus of the leg material and the leg cross-sectional area, respectively, and $H$ is the $(6 \times 6)$ matrix

$$H = \sum_{i=1}^{n} \frac{1}{l_i} a_i^T a_i. \tag{10}$$

Of course, matrices $H$ and $K$ depend on both $x$ and $q$.

Using the practical expressions, given in (5) and (9), the eigenvalue problem, which is defined by (2b), reduces to

$$\det(EA/\rho \Lambda - \omega^2 I) = 0 \tag{11}$$

where $\Lambda$ is the $(6 \times 6)$ symmetric matrix

$$\Lambda = \Pi^{-0.5} H \Pi^{-0.5} \tag{12}$$

which depends on both $x$ and $q$, i.e., $\Lambda = \Lambda(x, q)$. That is, the first natural frequency $\omega_1$ is

$$\omega_1^2 = \lambda_1(\Lambda) EA/\rho \tag{13}$$

and depends on both $x$ and $q$.

From (13), it is clear that the maximization of $F$ reduces to the problem of finding the optimal value $x_{opt}$ such that

$$\min_{q \in Q} \left[ \lambda_1(\Lambda(x_{opt}, q)) \right] = \max_{x \in X} \left[ \min_{q \in Q} \left[ \lambda_1(\Lambda(x, q)) \right] \right]. \tag{14}$$

### B. Scalability

Equation (13) highlights that the geometry $x_{opt}$, which maximizes $F$, is independent of platform density $\rho$, leg cross-sectional area $A$, and Young’s modulus $E$, of the material of the legs.

In the following, we show that the optimal geometry is also independent of the mechanism size. That is, mechanism geometries, each of which is optimal for a given mechanism size, are similar. The size of the mechanism can be described by a “characteristic length” $h$; referring to Fig. 1, $h = |q_{ref}|$, which is the relative distance of the base and platform in the reference pose $q_{ref}$ of the mechanism. Usually, $q_{ref}$ can be chosen as the pose corresponding to the centroid of the mechanism positional workspace.

Consider the dimensionless matrix $\Pi_x$

$$\Pi_x = \text{diag}(V^*, V^*, I^*_1, I^*_2, I^*_3) \tag{15}$$

which depends on the dimensionless volume $V^* = V/h^3$, the dimensionless principal moments of inertia $I^*_j = I_j/\rho h^3$ ($j = 1, 2, 3$), and the dimensionless matrix $H_x$

$$H_x = \sum_{i=1}^{n} \frac{1}{l^*_i} \left( a^{*}_i \right)^T a^{*}_i \tag{16}$$

where

$$a^{*}_i = \frac{1}{l^*_i} \left[ u^*_i \ r^*_i \wedge u^*_i \right] \tag{17}$$

which is function of the dimensionless leg lengths $l^*_i = l_i/h$ and the dimensionless vectors $r^*_i = r_i/h$ and $u^*_i = u_i/h$, with $g^* = g/h$ and $s^*_i = s_i/h$. Note that $l^*_i$, $g^*$, $s^*_i$, and $u^*_i$ depend on the dimensionless geometric parameter array $x^* = x/h$ and the dimensionless generalized coordinate array $q^*$, which is obtained from $q$ by dividing its first three entries by $h$ while maintaining the last three entries unaltered. Using (15) and (16), it is easy to show that the eigenvalue problem given by (11) can be rewritten as

$$\det(EA/\rho h^4 \Lambda_* - \omega^2 I) = 0 \tag{18}$$

where $\Lambda_*$ is the dimensionless symmetric matrix

$$\Lambda_* = \Pi^{-0.5} H_* \Pi^{-0.5} \tag{19}$$

which is function of both $x^*$ and $q^*$, i.e., $\Lambda_* = \Lambda_*(x^*, q^*)$. The equivalence of (18) and (2b) is explicitly demonstrated in Appendix B. That is, (13) can be rewritten as

$$\omega^2_1 = \frac{\lambda_1(\Lambda_*) EA}{\rho h^4}. \tag{20}$$

Equation (20) highlights that the optimal geometry $x_{opt}^*$, which maximizes $F$, is independent of platform density $\rho$, mechanism size $h$, leg cross-sectional area $A$, and Young’s modulus $E$, of the material of the legs. That is, the optimization problem only requires the a priori knowledge of the aspect ratios between the platform dimensions and the mechanism characteristic length $h$, and only involves dimensionless geometric variables as unknowns.

In fact, given the aspect ratios between the platform dimensions and the characteristic length $h$ of the mechanism, the maximization of $F$ reduces to two consecutive problems. For instance, upon definition of a new objective function $F^*$

$$F^* (x^*) = \min_{q \in Q} \lambda_1(\Lambda_*(x^*, q^*)) \tag{21}$$

...
the first problem consists of determining the geometry $x_{\text{opt}}^i$, such that
\[ F^+(x_{\text{opt}}^i) = \max_{x \in \mathcal{X}} F^+(x). \] (22)

The second problem then consists of choosing the platform density $\rho$, the mechanism size $h$, the leg cross-sectional area $A$, and the Young’s modulus $E$ of the material of the legs so that
\[ \omega_{\text{opt}}^2 = F^+(x_{\text{opt}}^i) \frac{EA}{\rho h^2} \geq \omega_{\text{des}}^2. \] (23)

VI. CASE STUDY: OPTIMIZATION OF A 2-DOF SPHERICAL MECHANISM FOR SPACE APPLICATIONS

The geometric optimization method described in Section V was used to design the mechanism, which has the optimal geometry among the class of 2-DOF spherical US/UPS-PMs described in [15]. Mechanisms belonging to this class are lightweight, compact, and stiff, and they can be used as orienting devices in future space missions [24]. Note that the geometrical optimization via (23) is well suited to mechanisms that must be employed in space applications. Indeed, for space mechanisms, mass should be minimized for reducing costs related to the launch phase and stiffness should be maximized in order to increase orientation precision. Moreover, mechanism natural frequencies should be as high as possible to decouple the vibrations of the launcher from those of the satellite (i.e., the mechanism platform) during the different phases of a space mission.

A. Description of the Mechanism

An example of US/UPS-PM belonging to the class described in [13] is depicted in Fig. 2. The system comprises a nonactuated spherical 2-DOF US-PM and two additional actuated telescopic legs (legs of type UPS).

The nonactuated 2-DOF US-PM constrains the platform to freely rotate about axes $k$ and $i$ only. Axes $k$ and $i$ intersect at $C$ (i.e., the center of spherical motion) but do not necessarily need to be orthogonal. The nonactuated 2-DOF US-PM features a fixed base and a moving platform connected to each other through $m$ legs of type US (in Fig. 2, $m = 6$); the legs of type US are $P_1B_1, P_2B_2, P_3B_3, CB_1, CB_2,$ and $CB_3$. These $m$ legs must fall into two categories.

1) Legs of type 1: $P_i$ is located at $C$ and $B_i$ is placed anywhere on the base but does not lie on $k$.

2) Legs of type 2: $P_i$ lies on $i$, while the $B_i$ lies on $k$.

The numbers $m_1$ and $m_2$ of the legs of type 1 and type 2, respectively, must be $m_1 \geq 1$ and $m_2 \geq 1$; the total number of legs $m$ (i.e., $m = m_1 + m_2$) must be $m \geq 4$. In Fig. 2, $m_1 = m_2 = 3$; legs $CB_1, CB_2,$ and $CB_3$ are of type 1, while legs $P_1B_1, P_2B_2,$ and $P_3B_3$ are of type 2. If $m > 4$, the system is overconstrained.

Overconstrained architectures allow augmentation of the system resilience and the system stiffness-to-weight, stiffness-to-encumbrance, strength-to-weight, and strength-to-encumbrance ratios and allow the reduction of system backlash. However, overconstrained architectures reduce the range of motion of the mechanism and render its assembly more complex.

The additional actuated legs are used to drive the relative rotations of the base and platform about the axes $k$ and $i$. These additional legs are two telescopic links of type UPS (in Fig. 2, legs $P_7B_7$ and $P_8B_8$), which are placed in parallel to the legs of type US of the nonactuated US-PM. Proper placement of the additional actuated legs allows the motion of the US/UPS-PM (US-PM + actuated legs) to be decoupled, i.e., each actuator is responsible for the relative rotation of the base and platform about one single axis only (referring to Fig. 2, leg $P_7B_7$ has point $P_7$ lying on $i$, and thus, it is responsible for the rotation about $k$ only, while leg $P_8B_8$ has point $B_8$ lying on $k$, and thus, it is responsible for the rotation about $i$ only). The US/UPS-PM has a total number of legs $n$ (legs of type 1 + legs of type 2 + legs of type UPS) that equals $n = m + 2$ (in Fig. 2, $n = 8$).

According to the previous description, for a general mechanism belonging to the class of spherical 2-DOF US/UPS-PM, the entries of the geometric parameter array $x$ are: the three coordinates of vector $s_i$, referred to $S_b$, for each of the $m_1$ legs of type 1; the two distances $p_i = |r_i|$ and $b_i = |s_i|$ for each of the $m_2$ legs of type 2; one distance $p_i$ and the three coordinates of $s_i$, referred to $S_b$, for one of the two legs of type UPS; and one distance $b_i$ and the three coordinates of $r_i$, referred to $S_b$, for the other leg of type UPS.

Regarding system mobility, the mechanism DOF consists of two relative rotations of the base and platform: a first rotation $\theta$ about axis $k$, which is fixed to the base, and a second rotation $\beta$ about axis $i$, which is fixed to the platform. The pose of the mechanism is uniquely described by the generalized coordinate array $q = [q_1, q_2, q_3, \theta, q_5, \beta]$, where $q_1$, $q_2$, $q_3$, and $q_5$ are known constants.

B. Optimization Procedure

For the sake of reducing the calculations, the optimization procedure, which is used to find the optimal geometry among the class of 2-DOF spherical US/UPS-PMs described in Section VI-A, considers 25 mechanism instances having axes...
k and i, which are orthogonal, motion $q_1^*$, which is such that $q_1^* = q_2^* = q_3^* = 0$ and $q_4^* = 1$, the motion variables $\beta$ and $\theta$, which vary in the ranges \([-\pi/3, \pi/3]\) and \([-\pi/2, \pi/2]\), respectively (i.e., the dimensionless workspace $Q^*$ is a rectangle whose edges are equal to $2\pi/3$ and $\pi$, respectively, and that lies on a planar slice of a 6-D space), numbers of legs $m_1$ and $m_2$, which are integer and vary independently between 2 and 6, and an ideal platform, which is characterized by $\Pi = 1$ and $G \subseteq C$ (this makes $\Lambda = \mathbf{H}_s$). Each mechanism instance is identified by the pair $(m_1, m_2)$, i.e., the mechanism instance $(3, 4)$ refers to 2-DOF spherical PMs with three legs of type 1 and four legs of type 2. For each of these 25 instances, mechanisms with and without actuation (i.e., with and without the two legs of type UPS), and with both general base and planar base (the plane being orthogonal to the axis k) are analyzed. Note that, in case of nonactuated mechanisms, the first natural frequency $\omega_1$ is defined as the smallest nonzero eigenvalue of $\Lambda$, (indeed, since in the absence of the two legs of type UPS, the US-PM can freely rotate about k and i, then two eigenvalues of $\Lambda$ are zero). The particular case in which the mechanism is nonactuated is considered in order to understand which leg type, between type 1 and type 2, has the major influence on the smallest value reached by the first natural frequency in the mechanism workspace. The particular case in which the base is planar is considered since manipulators of this kind are easier to manufacture and integrate with the environment. Note that the majority of existing PM-based machine tools, vehicle simulators, and space mechanisms have a planar base.

To summarize, the optimization procedure consists of solving 100 optimization problems. These problems can be subdivided in four groups.

1. Twenty five problems refer to nonactuated mechanisms having a general base. Each of these problems is used to find the optimal geometry of each of the US-PMs (with a general base), which can be obtained by a proper combination $(m_1, m_2)$, for $m_1, m_2 = 2, \ldots, 6$, of legs of type US. For these problems, the number of entries of the unknown array $x^*$ varies between 0 and 18 [for the mechanism of the instance $(6, 6)$].

2. Twenty five problems refer to actuated mechanisms having a general base. Each of these problems is used to find the optimal geometry of each of the US/UPs-PMs (with a general base and with two actuated legs of type UPS), which can be obtained by a proper combination $(m_1, m_2)$, for $m_1, m_2 = 2, \ldots, 6$, of legs of type US. For these problems, the number of entries of the unknown array $x^*$ varies between 0 and 18 [for the mechanism of the instance $(6, 6)$].

3. Twenty five problems refer to nonactuated mechanisms having a planar base. Each of these problems is used to find the optimal geometry of each of the US-PMs (with a planar base), which can be obtained by a proper combination $(m_1, m_2)$, for $m_1, m_2 = 2, \ldots, 6$, of legs of type US. For these problems, the number of entries of the unknown array $x^*$ varies between 0 and 18 [for the mechanism of the instance $(6, 6)$].

4. Twenty five problems refer to actuated mechanisms having a planar base. Each of these problems is used to find the optimal geometry of each of the US/UPs-PMs (with a planar base and with two actuated legs of type UPS), which can be obtained by a proper combination $(m_1, m_2)$, for $m_1, m_2 = 2, \ldots, 6$, of legs of type US. For these problems, the number of entries of the unknown array $x^*$ varies between 0 and 18 [for the mechanism of the instance $(6, 6)$].

For all these problems, each entry of $x^*$ is allowed to vary in the range $[-3, 3]$, i.e., the space of admissible geometries $X^*$ is a hypercube whose edges length is equal to 6 (refer to Section VI-D for motivations).

Once the aforementioned optimization problems are solved, analysis and comparison of the optimal mechanisms make it possible to identify the most convenient architecture and geometry for

1. nonactuated 2-DOF spherical US-PM with a general base;
2. actuated 2-DOF spherical US/UPs-PM with a general base;
3. nonactuated 2-DOF spherical US-PM with a planar base;
4. actuated 2-DOF spherical US/UPs-PM with a planar base.

C. Numerical Algorithm

We solved each of the 100 optimization problems numerically. The optimization is based on a simplified version of $F^*$: contrarily to (21), in which the minimum of $\lambda_1(\Lambda_s)$ is evaluated on the continuous space $Q^*$, the simplified function only evaluates the minimum of $\lambda_1(\Lambda_s)$ on a discrete subset of $Q^*$ consisting of 651 points obtained by quantizing both $\beta$ and $\theta$, in the ranges $[-\pi/3, \pi/3]$ and $[-\pi/2, \pi/2]$, respectively, with $\pi/30$ resolution. That is, the goal was to maximize the simplified objective function

$$F^*(x^*) = \min_{0 \leq i, j \leq 20} \lambda_1(\Lambda_s(x^*, q_{ij}^*))$$  \hspace{1cm} (24)$$

where $q_{ij}^* = [0, 0, 1, -\pi/2 + i\pi/30, 0, -\pi/3 + j\pi/30]$ is a constant array for each set $(i, j)$, $i$ and $j$ being integer numbers. The computation of the eigenvalues is carried out by using the GNU Linear Programming Kit (GLPK) [25].

Due to the high nonlinearity of the objective function and due to the size of the geometric parameter array $x^*$, clustering optimization, which is an efficient stochastic global optimization methodology, was employed to maximize the simplified objective function. Clustering methods are known to often outperform other types of stochastic optimization programs on moderate size problems (number of unknown variables ranging from 2 to 30) [26]–[29].

In the present study, we used a well-established, derivative-free implementation of the clustering method, the GLOBAL software [30]. A C++ version of the optimization software, recently developed by the authors [31], was used. This software is compiled under the Linux operating system with the GNU Compiler Collection (GCC) compiler [32]. GLOBAL was executed
ten times for each of the 100 optimization problems during the geometrical optimization of the spherical mechanism in order to improve the space exploration of admissible geometries $X^\ast$. The GLOBAL software is presented in Appendix C.

D. Space of Admissible Geometries

The efficacy and the efficiency of the numerical algorithm depend on the size of the space of admissible geometries $X^\ast$. Indeed, the larger $X^\ast$, the higher the probability of finding an optimal geometry, $x_{opt}^\ast$, for which $F^\ast(x_{opt}^\ast)$ is larger, and the larger is the time required for the computation of $x_{opt}^\ast$. In this section, the size of $X^\ast$ is discussed.

The following procedure is employed in order to determine the proper size of $X^\ast$ that allows finding $x_{opt}^\ast$ in a limited computational time. Constraining each of the entries of $x$ to lie within the same range $[-\nu, \nu]$, different optimizations of mechanisms belonging to the instance $(2, 2)$ are performed by increasing the value of the parameter $\nu$ from 1 to 20. Results are given in Fig. 3, which shows that the variation of $F^\ast(x_{opt}^\ast)$ is negligible for $\nu \geq 3$ (a variation by 0.6% results in case either $\nu = 3$ or $\nu = 20$ is selected).

Fig. 4 shows the optimal geometry, for $\nu = 3$, of a nonactuated 2-DOF spherical US-PM belonging to the instance $(2, 2)$, with legs of type 1 having the joint centers $B_i$ lying on a plane orthogonal to axis $k$. Referring to points $P$ and $B$, which are depicted in Fig. 4, a sensitivity analysis, which is performed by comparing the optimal geometries obtained for different values of $\nu$, shows that the optimal values of the coordinates of points $P$, which are referred to $S_p$, and $B$, which are referred to $S_p$, were always equal to the current magnitude of $\nu$ (i.e., the extreme value of the range imposed for the current optimization).

However, the influence of the magnitude of $\nu$ on the magnitude of the objective function $F^\ast(x_{opt}^\ast)$ is negligible for $\nu > 3$.

A sensitivity analysis of the values attained by different entries of $x_{opt}^\ast$, to variation of $\nu$ is also performed for other mechanism instances. For every mechanism instance, the variation of $F^\ast(x_{opt}^\ast)$ confirms to be negligible for $\nu \geq 3$. Then, the limiting value $\nu = 3$ is chosen for all the optimizations performed. With $\nu = 3$, each of the ten executions of GLOBAL for each of the 100 optimization problems takes between 10 and 60 seconds (depending on the specific optimization problem) on a 1.4-GHz Intel processor.

E. FEM Validation

The code used for the computation of the first natural frequencies of the mechanism during the optimization procedure was validated by means of the finite-element method (FEM) software, Nastran. In order to comply with the hypotheses presented in Section III, the legs were modeled, in the Nastran model, by means of bar elements. The legs were constrained to the base and the platform through multipoint constraint (MPC) relationships as both the base and platform were considered as perfectly rigid.

The platform inertia was modeled by means of a lumped inertia (a matrix of rank 6). Natural frequencies computed during the optimization procedures were compared to results of modal analyses performed using Nastran for different test case configurations, and the percentage numerical error was less than 0.02%. For instance, referring to the test case manipulator configuration described in Tables I and II (see Fig. 2), the numerical percentage error was 0.01% (see Table III).

F. Results and Discussion

The design of a space mechanism should take into account not only its performance in terms of high stiffness and low mass (and therefore, high natural frequencies) but also its complexity because reliability is a critical parameter for qualifying a system for space use. The influence of the number of legs on the performance of the mechanism can be analyzed by means of the results presented in Tables IV and V and Figs. 5 and 6. In particular, Table IV and Fig. 5 report the 25 optimal values of $F^\ast(x_{opt}^\ast)$ obtained through the solution of the 25 problems of group 1) defined in Section VI-B (i.e., obtained through the optimization of 25 mechanism instances, with a general base and without actuation). Table V and Fig. 6 report the 25 optimal values of $F^\ast(x_{opt}^\ast)$ obtained through the solution of the
25 problems of group 3) defined in Section VI-B (i.e., obtained through the optimization of 25 mechanism instances with planar base and without actuation). Each point reported in Figs. 5 and 6 and Tables IV and V corresponds to the minimum value of $F^*(x_{opt}^*)$ among the ten optimal values obtained from the ten executions of GLOBAL for the optimization of the same mechanism instance (see Section VI-D).

The following conclusions can be deduced from Figs. 5 and 6:

1) Legs of type 1 have a main influence on $F^*(x_{opt}^*)$, i.e., on the smallest value assumed by the mechanism first natural frequency

$$F^*(x_{opt}^*)$$
in the mechanism workspace; 2) the slope of $F^*(x_{opt}^*)$ decreases when the number of legs increases; and 3) there is a significant improvement in $F^*(x_{opt}^*)$ when the number of legs of type 2 is increased from 2 to 3. Note that the legs of type 1 have greater influence on $F^*(x_{opt}^*)$ than the legs of type 2. The justification relies on the fact that, for $m_2 > 2$, the vibration mode related to the first natural frequency is mainly translational, the translation being in a direction $t$ orthogonal to the platform rotation axis $k$, and the considered manipulator workspace $Q$ comprises a pose where the platform rotation axis $i$ is nearly orthogonal to the direction $t$ (whenever a pose exists within $Q$ such that $i$ can be orthogonal to $t$, then the smallest value assumed by the translational stiffness along $t$, for all the admissible manipulator poses, does not depend on $m_2$).

Comparison between Figs. 5 and 6 shows that, for every mechanism instance, the optimal value $F^*(x_{opt}^*)$ is greater for the optimal mechanism having a general base than for the optimal mechanism having a planar base. This result can be expected due to the reduction of the space dimension of admissible geometries $X^*$.

Analyzing the optimal geometries corresponding to Figs. 5 and 6, it can be found that the optimal mechanisms, which belong to the instances with large values of $m_2$, have legs of type 2 that overlap. As an example, Fig. 7 depicts the optimal geometry of a nonactuated US-PM belonging to the instance $(2, 5)$ and having a planar base. Fig. 7 shows that the optimal mechanism has four legs of type 2 that overlap in position “b” and one single leg of type 2 in position “a”.

Fig. 8 summarizes the results obtained by analyzing the optimal nonactuated US-PMs with a planar base, which belong to the five different mechanism instances $(2, 2)$, $(2, 3)$, $(2, 4)$, $(2, 5)$, and $(2, 6)$. The figure highlights that by increasing $m_2$ from 1 to 5, the optimal geometry of the mechanism does not change. Indeed, in the optimal mechanisms, the third leg, the fourth leg, and the fifth leg overlap the leg that lies in position “b”. Fig. 8 also shows that among the six legs of type 2 of the optimal mechanism, which belongs to the instance $(2, 6)$, one leg is in position “a”, four legs are overlapping in position “b”, and another leg is in a position different from both “a” and “b” (optimization results show that this leg is directed along axis $k$). Note that a number $b$ of overlapping legs, which have cross-sectional area $A$ and lie in position “b”, are equivalent to a unique leg, which has cross-sectional area $A_b = bA$ and lies in position “b”. Thus, in practice, the optimal mechanism of the instance $(2, 5)$ is equivalent to the optimal mechanism of the instance $(2, 2)$ with the legs lying on “b” replaced by a single leg whose cross-sectional area is four times that of the other legs.

Of course, the equivalent mechanism will have better performance than that of the optimal mechanism of the instance $(2, 2)$ having legs with equal cross-sectional area. This case shows that, aside from simplifying the optimization process, the use of hypothesis 10), which makes it possible for the legs to interfere (thus to overlap), also allows the optimization (although rough) of the cross-sectional area of single legs.

Figs. 5–8 concern nonactuated US-PMs. The optimization results of the actuated US/UPS-PMs (i.e., the results obtained through the solution of the optimization problems described in group 2) and group 4) defined in Section VI-B) are similar, and thus are not reported in the paper. The only point to be remarked upon is that, for every mechanism instance, the optimal value $F^*(x_{opt}^*)$ is smaller for the optimal mechanism with actuators (i.e., the US/UPS-PM) than for the optimal mechanism without actuators (i.e., the US-PM). This situation can be expected since the two additional actuated legs lock the mechanism DOFs, and thus introduce two additional vibration modes, which involve the rotations about axes $k$ and $i$. In particular, the smallest $F^*(x_{opt}^*)$, attained by the mechanism with actuators, indicates that, in order not to decrease the performance of the system, the legs of type UPS (the actuated legs) should have larger cross-sectional area and/or be made of materials with a larger Young’s modulus than those of the legs of type US.

It is also worth noting that, for each optimization, several local minima were found near the one that was claimed to be
the global optimum. From an engineering point of view, this result is relevant because it allows engineers to select the most appropriate configuration when considering additional criteria (practical constraints, maximum workspace, etc.).

As an example, Fig. 9 shows three suboptimal configurations of an actuated 2-DOF spherical US/UP-S-PM belonging to the instance (3, 3) with all the legs of type 1 and the actuated leg controlling the rotation about k, which have joint centers Bi lying on a plane orthogonal to axis k. The variation of \( F^*(x_{opt}) \) for these configurations is less than 0.8%; their performance, therefore, could be considered equivalent for many practical applications.

The remarkable importance of the performed optimizations and the conclusions that can be inferred from them (briefly highlighted in this section) relies on their general validity. As described in Section V, under the hypothesis presented in Section III, the optimal geometry of the mechanisms does not change when mechanism size, platform density, and leg cross-sectional area and material change. The results presented in this section concern dimensionless mechanisms. From these results, the practical performance of a real system with size \( h \), platform density \( \rho \), leg cross-sectional area \( A \), and material Young’s modulus \( E \) can be simply computed using (23).

VII. Conclusion

A method for optimizing the geometry of a broad class of PMs with legs of types US and UPS is analyzed in this paper. The optimization method is based on the maximization of the first natural frequency of the manipulator within the manipulator workspace and is formulated in a dimensionless framework. The method shows that 1) the optimal mechanism geometry does not change if platform density, mechanism size, and leg cross-sectional area and material change uniformly, and 2) the performance of the optimal mechanism can be scaled using simple analytical equations. Although the conservative hypotheses limit the applicability of the method to simplified models of real mechanisms, it is believed that the proposed method can be a powerful tool for engineers during feasibility studies of novel PMs.

The particular 2-DOF spherical mechanism, which was optimized following the proposed method, shows potential as an orienting device for future space applications, especially in its configuration having two legs of type 1, five legs of type 2 (four of them are overlapping so that they can be substituted by a single leg whose cross-sectional area is four times the area of the other legs), and a flat interface with the spacecraft (planar base).

APPENDIX A

This Appendix discusses the non-well-posedness of the eigenvalue problem for Cartesian stiffness matrices.

Consider a Cartesian stiffness matrix of this form

\[
K = \begin{bmatrix}
a & 0 & 0 & 0 & c \\
0 & a & 0 & 0 & 0 \\
0 & 0 & a & 0 & 0 \\
0 & 0 & 0 & b & 0 \\
c & 0 & 0 & 0 & b \\
\end{bmatrix}
\]

(A1)

where the dimensions of the coefficients \( a, b, \) and \( c \) are newtons per meter, newtons-meter, and newton, respectively. The eigenvalues \( \lambda_i \) of \( K \) are the solutions of the characteristic polynomial

\[
(a - \lambda)^2 (b - \lambda)^2 [\lambda^2 - \lambda (a + b) + (ab - c^2)]
\]

(A2)

which, referring to the coefficient \( a + b \), highlights that the evaluation of the eigenvalues of \( K \) requires the sum of quantities (i.e., \( a \) and \( b \)) with different dimensions: newtons per meter and newtons-meter. This is not physically admissible.

APPENDIX B

This Appendix presents the explicit expressions of the matrices \( A' = (M^{-1}K) \) and \( A = (M^{-0.5}KM^{-0.5}) \), and discusses computational issues related to the solution of the associated eigenvalue problems.

Using (5), (6), (9), and (10), \( A' \) and \( A \) are given in (A3) and (A4), shown at the bottom of the next page, where \( a_{(i)j} \) is the \( j \)th component of \( a_{(i)} \). In term of dimensionless variables, (A3) and (A4) can be rewritten, respectively, as

\[
A' = \frac{EA}{\rho h^3} A'_i
\]

(A5)

\[
A = \frac{EA}{\rho h^3} A
\]

(A6)

where we have (A7) and (A8), shown at the bottom of the next page, which show that \( A' \) is nonsymmetric and nonhomogeneous, while \( A \) is symmetric and homogeneous. Clearly, \( A' \) is nonhomogeneous since the coefficients of the \((3 \times 3)\) upper-right and bottom-left submatrices of \( A'_i \) are, respectively, directly and inversely proportional to the characteristic length \( h \).

Regarding the computation of the natural frequencies, the eigenvalue problem of \( A' \) should be well posed, even if \( A' \)
is nonhomogeneous. However, despite in theory it is possible to prove that all the eigenvalues of \( \Lambda_i \) are independent of \( h \), in practice, due to the finite precision arithmetic of calculators, the eigenvalues of \( \Lambda_i \) depend on \( h \) if the calculations are performed using rough numbering formats. That is, the computation of the natural frequencies via \( \Lambda_i \), rather than via \( \Lambda \), requires the use of more precise numbering formats (e.g., double instead of single precision), and thus more memory storage. Moreover, since the solution of the nonsymmetric eigenvalue problem is more difficult and slower than that of the symmetric problem [22], [23], the extraction of the natural frequencies from \( \Lambda_i \), rather than from \( \Lambda \), is computationally more intensive.

**APPENDIX C**

This Appendix presents the implementation of GLOBAL software for the optimization of the spherical mechanism. GLOBAL is designed to solve nonlinear bound-constrained optimization problems of the form

\[
\min f(x)
\]

subject to \( x_{i}^m \leq x_i \leq x_i^M \)

where \( f: \mathbb{R}^n \to \mathbb{R} \) is the objective function, and \( x_{i}^m, x_i^M, i = 1, \ldots, n \), are real numbers that define the variable bounds (in this study, the following parameters were used: \( x_i^m = -3, x_i^M = 3, i = 1, \ldots, n \)). The framework algorithm of GLOBAL ensures that the best local minimum found during the optimization is, with high probability, the global minimum. The idea behind clustering methods is to explore the possible local optimum points of the function and their basins of attractions. This is done by iteratively maintaining a set \( X_0 \) of points consisting of the found local optima, and a set \( X_1 \) of those points from

\[
A = \sum_{i=1}^{n} \frac{EA}{l_i} \begin{bmatrix}
  a_{i1}(1) / M & a_{i1}(12) / M & a_{i1}(13) / M & a_{i1}(14) / M & a_{i1}(15) / M & a_{i1}(16) / M \\
  a_{i2}(1) / M & a_{i2}(12) / M & a_{i2}(13) / M & a_{i2}(14) / M & a_{i2}(15) / M & a_{i2}(16) / M \\
  a_{i3}(1) / M & a_{i3}(12) / M & a_{i3}(13) / M & a_{i3}(14) / M & a_{i3}(15) / M & a_{i3}(16) / M \\
  a_{i4}(1) / M & a_{i4}(12) / M & a_{i4}(13) / M & a_{i4}(14) / M & a_{i4}(15) / M & a_{i4}(16) / M \\
  a_{i5}(1) / M & a_{i5}(12) / M & a_{i5}(13) / M & a_{i5}(14) / M & a_{i5}(15) / M & a_{i5}(16) / M \\
  a_{i6}(1) / M & a_{i6}(12) / M & a_{i6}(13) / M & a_{i6}(14) / M & a_{i6}(15) / M & a_{i6}(16) / M
\end{bmatrix}
\]

\[
A' = \sum_{i=1}^{n} \frac{EA}{l_i} \begin{bmatrix}
  a_{i1}^{1}(11) / N^V & a_{i1}^{1}(12) / N^V & a_{i1}^{1}(13) / N^V & a_{i1}^{1}(14) / N^V & a_{i1}^{1}(15) / N^V & a_{i1}^{1}(16) / N^V \\
  a_{i2}^{1}(11) / N^V & a_{i2}^{1}(12) / N^V & a_{i2}^{1}(13) / N^V & a_{i2}^{1}(14) / N^V & a_{i2}^{1}(15) / N^V & a_{i2}^{1}(16) / N^V \\
  a_{i3}^{1}(11) / N^V & a_{i3}^{1}(12) / N^V & a_{i3}^{1}(13) / N^V & a_{i3}^{1}(14) / N^V & a_{i3}^{1}(15) / N^V & a_{i3}^{1}(16) / N^V \\
  a_{i4}^{1}(11) / N^V & a_{i4}^{1}(12) / N^V & a_{i4}^{1}(13) / N^V & a_{i4}^{1}(14) / N^V & a_{i4}^{1}(15) / N^V & a_{i4}^{1}(16) / N^V \\
  a_{i5}^{1}(11) / N^V & a_{i5}^{1}(12) / N^V & a_{i5}^{1}(13) / N^V & a_{i5}^{1}(14) / N^V & a_{i5}^{1}(15) / N^V & a_{i5}^{1}(16) / N^V \\
  a_{i6}^{1}(11) / N^V & a_{i6}^{1}(12) / N^V & a_{i6}^{1}(13) / N^V & a_{i6}^{1}(14) / N^V & a_{i6}^{1}(15) / N^V & a_{i6}^{1}(16) / N^V
\end{bmatrix}
\]

\[
A'' = \sum_{i=1}^{n} \frac{1}{l_i} \begin{bmatrix}
  a_{i1}^{2}(1) / M & a_{i1}^{2}(12) / M & a_{i1}^{2}(13) / M & a_{i1}^{2}(14) / M & a_{i1}^{2}(15) / M & a_{i1}^{2}(16) / M \\
  a_{i2}^{2}(1) / M & a_{i2}^{2}(12) / M & a_{i2}^{2}(13) / M & a_{i2}^{2}(14) / M & a_{i2}^{2}(15) / M & a_{i2}^{2}(16) / M \\
  a_{i3}^{2}(1) / M & a_{i3}^{2}(12) / M & a_{i3}^{2}(13) / M & a_{i3}^{2}(14) / M & a_{i3}^{2}(15) / M & a_{i3}^{2}(16) / M \\
  a_{i4}^{2}(1) / M & a_{i4}^{2}(12) / M & a_{i4}^{2}(13) / M & a_{i4}^{2}(14) / M & a_{i4}^{2}(15) / M & a_{i4}^{2}(16) / M \\
  a_{i5}^{2}(1) / M & a_{i5}^{2}(12) / M & a_{i5}^{2}(13) / M & a_{i5}^{2}(14) / M & a_{i5}^{2}(15) / M & a_{i5}^{2}(16) / M \\
  a_{i6}^{2}(1) / M & a_{i6}^{2}(12) / M & a_{i6}^{2}(13) / M & a_{i6}^{2}(14) / M & a_{i6}^{2}(15) / M & a_{i6}^{2}(16) / M
\end{bmatrix}
\]
which a local optimization routine led to a local optimum. The elements of \( X_0 \) and \( X_1 \) are called seed points. At each iteration of the algorithm, a set of random sample points is generated. The sample points are then added to a cluster of points “growing around” the seed points. GLOBAL evokes the clusters by using the single linkage (nearest-neighbor) method. From the unclustered sample points, a local optimization algorithm is executed, resulting in new seed points. The core of GLOBAL is given by the following algorithm:

**Input:** \( f, x^m, x^M \).

**Step 0:** Let \( X_0 \) and \( X_1 \) be empty.

**Step 1:** Generate a number \( N_{\text{samp}} \) of sample points within the variable bound \( [x^m, x^M] \) with uniform distribution.

**Step 2:** Let \( A \) be the best \( 100 \cdot N_{\text{sel}} / N_{\text{samp}} \) percentage of all the sample points generated so far.

**Step 3:** Attempt to cluster each element of \( A \) from the seed points \( X_0 \cup X_1 \). If all points of \( A \) are clustered, then go to step 6. Otherwise, let \( B \) be the set of unclustered points.

**Step 4:** For each element \( x \) left in \( B \), run a local search from \( x \). Let \( x' \) be the local minimum point identified during the local search. If \( x' \) can be clustered to the current clusters, let \( X_1 = X_1 \cup \{ x' \} \), otherwise, let \( X_0 = X_0 \cup \{ x' \} \), and attempt to cluster the remaining elements in \( B \) to a new cluster around \( x' \).

**Step 5:** If a new local minimum is found at step 4, then go to step 1.

**Step 6:** Let \( x^* \) be the element of \( X_0 \) with the smallest function value \( f^* \). Return \((x^*, f^*)\) as the candidates for the global minimum point and the global minimum value, respectively. Note that the integer control parameters \( N_{\text{samp}} \) and \( N_{\text{sel}} \) must be set by the user; during the present study, we used the values \( N_{\text{samp}} = 300 \) and \( N_{\text{sel}} = 10 \), respectively.

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