

**Math Finance (cont. time), SS20 , Sheet 1**

1. Recall the concept of conditional expectation and its most important properties.
2. Show that if  $H$  is a bounded trading strategy and  $X$  is a martingale, then the process  $((H \cdot X)_t)_{t=0}^T$  is a martingale as well.
3. Show that  $X$  is a martingale iff  $\mathbb{E}(H \cdot X)_T = 0$  for all bounded trading strategies  $H$ .
4. Show that if  $X$  is a martingale, then  $X$  satisfies NA. More generally, if there exists an equivalent martingale measure for  $X$ , then  $X$  satisfies NA.
5. Construct a model which admits an arbitrage opportunity and a model which satisfies NA.