Math Finance (cont. time), SS20, Sheet 2

Throughout you can make the simplifying Assumption 1.11. (I.e. Ω is finite, etc.)

1. If $\mathcal{M} = \{\mathbb{Q}\}$ then every $\mathbb{Q}$-martingale M has the representation

$$M_t = M_0 + \sum_{k=1}^{t} H_k \cdot (X_k - X_{k-1})$$

   for some predictable process H.

2. Conversely, if every $\mathbb{Q}$-martingale M has the representation

$$M_t = M_0 + \sum_{k=1}^{t} H_k \cdot (X_k - X_{k-1})$$

   for some predictable process H, then $\mathcal{M} = \{\mathbb{Q}\}$.

3. Assume that the underlying model satisfies NA. Show that if a claim C is attainable through a strategy (a, H) then a is the unique arbitrage free price.

4. Construct a complete and a non-complete (but still arbitrage free) model in the case of $T = 2$ timesteps.