**Math Finance (cont. time), SS20, Sheet 3**

1. Assume that $B, B'$ are Brownian motions (potentially) on different probability spaces. Then $B$ is a version of $B'$.

2. Let $\Omega = \{-1, 0, 1\}$, $S_0 = 2$, $S_1(\omega) = S_0 + \omega$. Determine $M_e$ and the set of all arbitrage free prices of the derivative $f(S) = (S_1 - 1)_+$

3. Assume that $X, X'$ are continuous processes that are modifications of each other. Show that $X$ and $X'$ are indistinguishable.

4. Let $B = (B_t)_{t\geq 0}$ be a Brownian motion. Show that $B' = (1/aB_{a^2t})_{t\geq 0}$ is a also a Brownian motion for $a > 0$.

   Show also that $B' = (-B_t)_{t\geq 0}$ is a Brownian motion.

5. Let $B$ be a Brownian motion wrt. a filtration $(\mathcal{F}_t)_{t\geq 0}$. Show that $B$ is a martingale.