

Math Finance (cont. time), SS19, Sheet 5

1. Assume that H is simple and consider $I_u := \int_0^u H_t dB_t$. Show that the quadratic variation of I satisfies

$$\langle I, I \rangle_t = \int_0^t H_u^2 du.$$

2. Let $f : [0, T] \rightarrow \mathbb{R}$ be a continuous, deterministic function. Show that

$$\int_0^T f(t) dB_t$$

is normally distributed.

Hint: Maybe you want to use (without proof) the following: If a sequence of Gaussian RV converges in probability, then the limit is again Gaussian. (And expectation and variance converge to the expectation and variance of the limiting Gaussian.)

3. Apply Ito's formula to $X_t := f(t, B_t) := \exp(\sigma B_t - \sigma^2 t + \mu t)$, where $\mu, \sigma \in \mathbb{R}$. Note that

$$dX_t = X_t(\mu dt + \sigma dB_t).$$