**Stochastic Analysis, WS18/19, Sheet 1**

1. Let $X$ be the simple symmetric random walk and $a, b > 0$. Show that
$$T_{a,b} := \inf\{t : X_t \in \{-a, b\}\}$$
is a finite stopping time and prove that $\lim X_n^{T_{a,b}} = X_{T_{a,b}}$ almost surely and in $L^2$. In particular we have $\mathbb{E}X_{T_{a,b}} = 0$. Can you use this to determine the distribution of $X_{T_{a,b}}$?

2. In the setup of the previous example, discuss what happens if we consider $a = 1, b = \infty$?

3. Let $T$ be a stopping time. Show that $\mathcal{F}_T$ is a $\sigma$-algebra. Assuming that $T \leq N \in \mathbb{N}$ and $A \in \mathcal{F}_T$, show that
$$T_A(\omega) := \begin{cases} T(\omega) & \text{if } \omega \in A \\ N & \text{else} \end{cases}$$
is a stopping time.

4. Let $T \leq N \in \mathbb{N}$ be a stopping time and $M$ a martingale. Show that $M_T = \mathbb{E}[M_N | \mathcal{F}_T]$.