

### Stochastic Analysis, WS18/19, Sheet 4

1. Let  $(B)$  be a Brownian motion,  $a > 0$ . Show that

$$X_t = 1/\sqrt{a}B_{at}, t \geq 0$$

is also a Brownian motion.

2. Let  $(B)$  be a Brownian motion. Show that

$$X_0 = 0 \quad \text{and} \quad X_t = tB_{1/t}, t > 0$$

is also a Brownian motion.

3. Let  $1 \leq p < q < r$ , let  $f : [0, T] \rightarrow \mathbb{R}$  be continuous, and let  $(\pi_n)$  be a sequence of partitions with mesh tending to 0. Suppose that  $Var_{q,(\pi_n)}(f) \in (0, \infty)$ . Show that  $Var_{p,(\pi_n)}(f) = \infty$ ,  $Var_{r,(\pi_n)}(f) \in (0, \infty) = 0$ .
4. (*continuation*) Show that there is a sequence of partitions  $(\tilde{\pi}_n)$  such that  $Var_{q,(\tilde{\pi}_n)}(f) = \infty$

Comment: I think that not all of these problems are easy. I am completely content if you do the first and the third problem.