Stochastic Analysis, WS18/19, Sheet 5

Exercises related to the Ito-isometry...

1. Use the Ito-isometry to calculate the variance of

$$\int_0^t |B_s|^{1/2} \, dB_s.$$

2. Use the Ito-isometry to calculate the variance of

$$\int_0^t (B_s + s)^{1/2} \, dB_s.$$

3. The integral

$$I_1 := \int_0^t B_s \, ds$$

is an ordinary Riemann integral, but ${\cal I}_1$ is a random variable. Calculate it's mean and its variance.

4. Calculate the mean and the variance of

$$I_2 := \int_0^t B_s^2 \, ds$$

5. Recall and meditate upon your favourite version of the Doob-maximal inequality.