Stochastic Analysis, WS18/19, Sheet 6

1. Assume that X is a continuous and adapted process. Show that

$$\tau := \inf\{t : X_t = 0\}$$

is a stopping time.

Hint: Given $t \geq 0$, consider the event

$$\left\{ \inf_{s \in \mathbb{Q}, s \leq t} |X_s| = 0 \right\}.$$

2. Show in detail that the space of continuous, L^2-bounded martingales equipped with

$$\|M\|_{*,2}^2 := \mathbb{E}\left[\sup_{t \leq T} M_t^2\right]$$

is complete.

3. Let $f : [0, T] \to \mathbb{R}$ be a continuous, deterministic function. Show that

$$\int_0^T f(t)\, dB_t$$

is normally distributed.