Stochastic Analysis, WS18/19, Sheet 7

1. Let $f : [0, T] \to \mathbb{R}$ be a continuous, deterministic function. Show that

$$\int_0^T f(t)\, dB_t$$

is normally distributed.

2. Can you give an example of a local martingale M in discrete time which starts in $M_0 = 0$ and is not a martingale.

3. Show that $\int_0^T H_t\, dB_t = \int_0^T H'_t\, dB_t$ if the locally bounded processes H, H' are modifications of each other.

   If you feel more adventurous, try to show the following: Let $(M_t)_{t\in[0,T]}$ be a continuous martingale. Show that $\int_0^T H_t\, dM_t = \int_0^T H'_t\, dM_t$ if the locally bounded processes H, H' are modifications of each other.