Stochastic Analysis, WS18/19, Sheet 8

1. Let H be a Hilbertspace and assume that a sequence $f_1, f_2, \ldots \in H$ satisfies $\sup \|f_n\| < \infty$. Show that there exists $g_n \in \operatorname{conv}(f_n, f_{n+1}, \ldots), n \geq 1$ such that $(g_n)_n$ converges in $\|.\|$.

   Hint: You might take g_n to have (asymptotically) minimal norm in $\operatorname{conv}(f_n, f_{n+1}, \ldots)$. But there are also other (less elementary) approaches.

2. Show that every integrable process in discrete time is the sum of a martingale and a predictable process which starts in 0. What happens if the integrable process is a supermartingale?

3. Apply Ito's formula to B_t^2 and $\exp(B_t)$.