

Stochastic Analysis, WS18/19, Sheet 9

1. Apply Ito's formula to $X_t = B_t^4$ to calculate the fourth moment of the standard normal distribution.
2. Use Ito's formula to verify that (for a nice deterministic function $s \mapsto \sigma_s$)

$$X_t = \exp \int_0^t \sigma_s dB_s - 1/2 \int_0^t \sigma_s^2 ds \quad (1)$$

satisfies the SDE $dX_t/X_t = \sigma_t dB_t$. Why would one call X a 'stochastic exponential'?

3. An integration by parts formula: Use Ito's formula to show that

$$\int_0^t h(s) dB_s = h(t)B_t - \int_0^t h'(s)B_s ds.$$