Stochastic Analysis, WS18/19, Sheet 10

1. Let $(X_n)_{n\in\mathbb{N}}$ be i.i.d. with $\mathbb{P}(X_n = 2) = 1/3, \mathbb{P}(X_n = -1) = 2/3$ and set $S_n := X_0 + \ldots + X_n$, $\mathcal{F}_n = \sigma(X_0, \ldots, X_n)$. Let M be a bounded (say) $\mathcal{F}$-martingale. Show that there exists a unique adapted process H such that for all n
$$M_n = M_0 + (H \cdot S)_n.$$

2. Let $(X_n)_{n\in\mathbb{N}}$ be i.i.d. with $\mathbb{P}(X_n = 1) = \mathbb{P}(X_n = 0) = \mathbb{P}(X_n = -1) = 1/3$ and set $S_n := X_0 + \ldots + X_n$, $\mathcal{F}_n = \sigma(X_0, \ldots, X_n)$. Find a bounded $\mathcal{F}$-martingale M such that there exists no adapted process H satisfying
$$M_n = M_0 + (H \cdot S)_n.$$

3. Let B be Brownian motion and $f : \mathbb{R} \to \mathbb{R}$ a nice (bounded, smooth, etc.) function. From the martingale representation theorem we know that there exists a unique adapted (suitably bounded) H such that
$$f(B_T) = \mathbb{E}f(B_T) + \int_0^T H_t \, dB_t.$$
Can you determine H in terms of f?