

Mathematical Finance 1

Exercise sheet 5

Please prepare the exercises of Sheet 3 and 4 which have not been treated in the exercise class last time.

1. Consider the following one-period models ($t \in \{0, 1\}$) for a financial market :
- (A) $\Omega = \{\omega_1, \omega_2\}$, and

$$S_0^1 = 5, \quad S_1^1(\omega_1) = 6, \quad S_1^1(\omega_2) = 4.9.$$

- (B) $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and

$$S_0^1 = 5, \quad S_1^1(\omega_1) = 6, \quad S_1^1(\omega_2) = 4.9, \quad S_1^1(\omega_3) = 3.$$

- (C) $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and

$$\begin{pmatrix} S_0^1 \\ S_0^2 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix}, \quad \begin{pmatrix} S_1^1(\omega_1) \\ S_1^2(\omega_1) \end{pmatrix} = \begin{pmatrix} 6 \\ 12 \end{pmatrix}, \quad \begin{pmatrix} S_1^1(\omega_2) \\ S_1^2(\omega_2) \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}, \quad \begin{pmatrix} S_1^1(\omega_3) \\ S_1^2(\omega_3) \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}.$$

In every model we assume that each ω_i has strict positive probability.

- 1) Which of the models satisfies (NA)? Describe for those models the set of equivalent martingale measures and for the others an arbitrage opportunity.
- 2) Which of the models satisfying (NA) is complete? Find for the non-complete models a claim which is not replicable.