Mathematical Finance 1
Exercise sheet 10

1. Let $S = (S_t)_{t=0}^T$ be a submartingale and τ_1, τ_2 stopping times satisfying $\tau_1 \leq \tau_2 \leq T$. Show that $E[S_{\tau_1}] \leq E[S_{\tau_2}]$.

2. Use Jensen's inequality to prove the arithmetic-geometric mean inequality, i.e. that for positive real numbers $a_1, \ldots, a_n$ the following holds:

$$\sqrt[n]{a_1 a_2 \ldots a_n} \leq \frac{a_1 + a_2 + \ldots + a_n}{n}.$$

Hint: Consider the convex function $x \mapsto \exp(x)$ and a random variable taking the values $\lg(a_1), \ldots, \lg(a_n)$.