Exercise 1. (Discussion of “Delta–Gamma–Method” for Value–at–Risk estimation)

Consider a European option on a non–dividend–paying stock. Denote the strike price by $X$, the price of the underlying by $S$, the time (measured in years) to expiry by $t$, the (annualized) volatility (assumed to be constant) by $\sigma$, and the risk–free rate of interest (assumed to be constant) by $r$.

Make the “usual assumptions” about the price process of the underlying (Geometric Brownian Motion), choose a time horizon $d$ and a level of confidence $c$, and carefully discuss the differences between

- the “true” Value–at–Risk (with time horizon $d$ and level of confidence $c$) of the option,
- the respective Value–at–Risk estimated by the Delta–approach,

(Recall that the differences of the logarithms of the underlying are normally distributed under the above assumption.)

Work out a numerical example for $X = 105$, $S_0 = 100$, $t = 1$, $\sigma = 0.25$, $r = 0.04$, $d = 0.01$ and $c = 0.99$.

Exercise 2. (Discussion of “Delta–Hedging”)

Recall the no–arbitrage argument (“Delta–Hedging”) leading to the Black–Scholes formula. Clearly, it is not possible in reality to perform perfect Delta–Hedging continuously (not to speak of transaction costs...).

Assume that there are no transaction costs (i.e., it is possible to buy or sell arbitrary quantities of the underlying at the current price $S$), but that it is only possible to buy or sell once a day. Consider the option from Exercise 1 and the numerical values for the parameters, and apply Monte–Carlo–simulation to find out the distribution of profits and losses of the option, which is Delta–Hedged on a daily basis (assuming that one year has 250 days). I.e.: Generate daily price paths for $S$ at random, compute the corresponding option delta for every day, and simulate the necessary transactions (buy or sell the underlying at the current price $S$, in order to maintain every day a position in the underlying equal to the option delta).