FRAs (Swaplets) and Interest Rate Futures contracts

Forward rate agreements - exchanges of fixed rate for floating rate at a pre-determined time in the future

Obviously swaps are nothing else but series of standardized FRAs

Interest rate futures are also contracts on interest rates (typically interest rate instruments such as liquid bonds) - main difference is daily margining

Hence convexity effects

Path dependency, remember?
Pricing of a simple swap

Corp enters a $100 MM swap with dealer
Receives quarterly fixed rate and pays 3 months LIBOR
Trade occurs on Feb 14th, 1994 - effective date is Feb 16th, 1994 - maturity date March 20th, 1996

Fixed and floating rates are set based on actual / 360 day convention

Floating rate is set upon expiry of march IMM eurodollar contract

Applicable payment period

3/14/94  3/16/94  6/15/94
## Swap Pricing Guide

<table>
<thead>
<tr>
<th>Reset date</th>
<th>Payment date</th>
<th>Day count</th>
<th>Futures prices at expiry</th>
<th>Impl Futures rates</th>
<th>Discount Factor</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>14. Mrz 94</td>
<td>15. Jun 94</td>
<td>91</td>
<td>96,35</td>
<td>3,35%</td>
<td>0,99740</td>
<td>0,07758</td>
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<td>21. Sep 94</td>
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<td>0,98828</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>5,61%</td>
<td>0,90660</td>
<td>0,22917</td>
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</tbody>
</table>

**Total Discount Factor:** 0,01534

**Total R:** 4,63%
Discount factors and implied forward rates

Discount factors

\[
DF_j = \left( \prod_{k=1}^{j} (1 + IFR_k \frac{\text{days}_k}{360}) \right)^{\frac{1}{i}}
\]

Example \(0.9777 = \{(1 + 0.0335 \frac{28}{360})(1 + 0.0365 \frac{91}{360})(1 + 0.0398 \frac{98}{360})\}^{\frac{1}{i}}\)

NPV (fixed) = NPV (floating) therefore at market swap

\[P \star DF_j = P \star IFR_j = \star(1 + R)^n\]
Yield Curve composition

Many ways to build a yield curve

Determine implied zero rates from available prices of most liquid instruments in each market ("bootstrapping")

Need to convert among different types of compounding conventions

interpolate where no available prices exist (or lack of liquidity)

Objective is to obtain a smooth curve (among zero rates) as close as possible to the implied zero rates found from bootstrapping bonds (in Euroland the swap market is most liquid, but what about the term structure of spreads???)
From zero curve to implied forward curve

Solve for $r_{12}$

$$(1 + r_2 t_2)^2 = (1 + r_{01} t_{01})(1 + r_{12} t_{12})$$

Forward curve reflects market participants’ expectations of future rates

Normal yield curve (also forward curve) upward sloping due to liquidity premium

But if investors expect future rates to decrease one can expect inverted curves

In reality investors „agree“ on an IFC and reflect this into pricing of bonds
Next lecture

Spreads - reflective of liquidity, default and credit migration

credit derivative products

options
  - interest rates
  - equities

hybrid products

Risk - a historical perspective
Spreads - difficult to analyze and cumbersome to handle

Term structure (spread = f(tenor))
Correlations among vertices points: within the risk-free curve, within the risky curve and among the 2 curves.
Spreads - difficult to analyze and cumbersome to handle

Term structure (spread = f(tenor))
Correlations among vertice points: within the risk free curve, within the risky curve and among the 2 curves
Spreads reflect a multitude of economic effects

• Credit default probability
• Credit migration probability
• Market liquidity of issue
• Correlations and interdependencies among all these
What are Credit Derivatives and what are they used for?

• Until mid '90s risk management paradigm has been:
  • Market Risk - managed via hedges and portfolio diversification
  • Credit Risk Management - only via diversification (in absence of hedging means and instruments)

• Credit Derivatives emerged as instruments geared to:
  - hedge credit exposures
  - attain synthetic portfolio concentration effects (or diversification)
  - transfer chunks of credit risk among institutions without altering portfolio composition
Credit Options

Right but not obligation to buy (call) or sell (put) the specified credit risk at the strike price at option’s expiry (European) or until option’s expiry (American)

Underlying may be:

risky asset (bond, loan, portfolio)
spread over a referenced risk free asset

Payoff may be triggered by:

- decline in asset value below threshold
- change in spread
Credit Option on underlying asset

Binary Option - seller pays a fixed amount if and when default occurs, else no payments

\[ P[V(t);$100MM] = \begin{cases} $420 & \text{if } V(t)<$100MM; \\ 0 & \text{if } V(t)>$100MM \end{cases} \]

Credit Put Option (if asset declines in value)

\[ P[D(t);K]=\max[0;K-D(t)], \text{ where:} \]

\[ K=F[\exp-(r+\text{spread})(T-t)] \]
\[ D(t)=\text{market value of asset at time } t, \text{ the option's maturity} \]
\[ F=\text{face value of zero coupon debt instrument} \]
\[ r=\text{risk free rate} \]
\[ T=\text{bond's maturity and } t=\text{option's maturity} \]
Credit spread Options

Call option on the level of the credit spread over a referenced benchmark risk free bond (U.S. T bond)

spread widens, referenced asset declines in value

\[ C[\text{spread}(t);K] = (\text{spread} - K) \times \text{notional amount} \times \text{risk factor} \]

\( \text{spread}(t) \) = spread of financial asset over the riskless rate at option’s maturity
\( K \) = the specified strike spread for the financial asset over the riskless rate
risk factor = sensitivity of underlying financial asset to changes in interest rate - Duration & Convexity
Duration and Convexity as primitive measures of interest rate risk

\[ D_{eff} = \left[ \frac{\partial PV}{\partial r} \right] / PV \]

\[ C = \left[ \frac{\partial^2 PV}{\partial r^2} \right] / PV^2 \]

\[ PV(0.0001) = PV(0) + \frac{P'(0) \times (0.0001)}{1!} + \frac{P''(0) \times 0.0001^2}{2!} \]
So what is the risk factor?

Asset = BB Niagara Mohawk Power 7.75% due 2008

P (9/98) = $104.77, YTM = 7.08%

\[ D_{\text{mod}} = \frac{C / y^2 \left[ 1 - 1/(1 + y)^n \right] + n \left( 100 \frac{C}{y} \right) / (1 + y)^{n+1}}{P} \]

C = semi annual coupons

y = YTM/2

n = number of semiannual periods

P = Bond’s current price
So what is the risk factor? (cont’d)

\[
Conv = 2Cy^3 \times \left[ 1 \div \frac{1}{(1 + y)^n} \right] \div 2Cn / y^2 (1 + y)^{n+1} + \\
+ \left[ n(n + 1) \times (100 \div Cy) \right] / (1 + y)^{n+2}
\]

\[D_{\text{mod}} = 13.925 \text{ annualized} = 6.96 \; ; \; Conv = 26162.77\]

Conv annualized for $100 bond = 26162.77(2^2 \times 104.77) = 62.427$

Risk Fact = \(- (6.96) \times (0.01) + 1/2 \times (62.427) \times (0.01)^2 = 6.65\%\)
Why are Duration & Convexity „primitive“ risk measures?

• They only apply to sensitivities to yield to maturity as a proxy of the yield curve

• They neglect non parallel shifts of the yield curve (tilts, rotations)

• They are limited to first and second order effects

• They neglect implied correlations among first and second order

• They disregard correlations among yield curve vertices and for spread curves between spread points and the term structure of risk free rates

• But they are intuitive and easy to use in practice
Credit swaps

Credit default swap - primary purpose is to hedge credit exposure to a referenced asset or issuer.

Two types of credit default swaps:

1. Swap purchaser pays fee in return for credit insurance
   
   „Credit Insurance“

2. Swap buyer gives up returns to a credit risky asset in exchange for a fixed payment
   
   „Total Return Swap“
Credit Insurance Swaps

Cash Payment upon default

Swap premium is like an insurance premium
Credit event: foregone coupon payment, bankruptcy, debt rescheduling, foreclosure, downgrade, etc.
Credit default swaps as an exchange of payments

• Credit protection buyer keeps asset on balance sheet but receives a known payment on scheduled payment dates for the referenced asset.

• In return, pays to credit protection seller on each cash flow date the total return from the referenced asset.

• If total return <0, then credit protection buyer receives payment from credit protection seller equal to the amount of negative return on referenced asset + floating rate under the swap agreement.
Total Return Credit Swaps

• Includes all cash flows from referenced asset plus any appreciation / depreciation of asset value

• Swap purchaser pays floating rate plus any depreciation of referenced asset to swap seller

• Underlying asset typically determined by total return receiver

• Effectively total return payer neutralizes her position and earns LIBOR + spread (by transferring all economic exposure of asset basket to total return seller)
Total Return Credit Swaps - Mechanics

Dealer: all cash flows net out to spread over LIBOR that he receives from investor
Investor: receives total return on desired asset in a convenient format
Credit Forwards

Purchase of credit spread forward contracts over pre-specified time periods

If spread increases over specified strike price, buyer receives payment from seller, vice versa for reverse case

\[ \text{Paym't @ maturity} = [\text{credit spread at maturity} - \text{contracted credit spread}] \times \text{risk factor} \times \text{notional amount} \]

As opposed to options, credit forward participants share upside and downside risk, hence no option premium

In addition to hedging financing risk, credit forwards are used to forecast future default premiums: Zero Yield curve for commercial loans - zero yield curve for risk free instruments \( \Rightarrow \) zero credit spread curve \( \Rightarrow \) forward credit spreads (see last lecture on forward curves)
Credit linked notes

CLNs are hybrid instruments which combine the elements of a debt instrument with either an embedded credit option or credit swap.

Essentially a synthetic high yield bond, loan participation interest or credit investment.

Typically maturing btw. 3 months to 3 years, privately placed.

[Diagram showing coupon rates and credit ratings]
Credit linked notes - mechanics

• Investor buys note & simultaneously sells to note’s issuer a binary put option on the credit rating of a chosen (say BBB) borrower and receives 25 b.p. premium.

• Notional amount of option = face value of the note ($10 mm)

• Binary payout is set at 100 b.p. (difference btw. Average BBB and BB commercial lending rates)

• If option expires out of the money, option premium provides incremental coupon income up to 7.25%

• If credit rating drops to BB then short put expires in the money and note buyer receives 6%.
Credit linked notes combining options and forward contracts

<table>
<thead>
<tr>
<th>Maturity value of note</th>
<th>Note Value</th>
<th>Par Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>Short call on ref. Loan spread @ 300bp</td>
<td>Note Value</td>
</tr>
<tr>
<td></td>
<td>Long put on ref. Loan spread @ 100 bp</td>
<td>Par Value</td>
</tr>
<tr>
<td></td>
<td>Long credit spread forward @ curr loan spread (200 bp)</td>
<td>Par Value</td>
</tr>
</tbody>
</table>

CLN with an embedded forward, an embedded short credit call & an embedded long credit put

<table>
<thead>
<tr>
<th>Loan credit spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
</tr>
<tr>
<td>200</td>
</tr>
<tr>
<td>300</td>
</tr>
</tbody>
</table>

12345
continued from

5th Lecture
of
10th November 2003
Special purpose vehicles (SPVs)

Reasons why fund managers (other financial institutions) may be reluctant to enter into off balance sheet (like swaps, forwards or options) agreements:

• these contracts may not be approved vehicles by the entity‘s constituent board

• prospectus limitations

• Disclosure and liability issues

• Risk management issues
Special purpose Vehicles - SPVs (cont‘d)

Securities issued by SPs to investors, the returns of which are tied to the credit characteristics of an underlying pool of junk bonds, bank loans, emerging markets debt securities, etc.

All designed to provide the efficiencies of financial engineering avoiding the hassle of entering into derivative contracts

Trust is set up for each transaction

Investor records a privately placed issue as an asset on her balance sheet (144 A in the U.S.)

SPV records the security as an outstanding debt obligation on its balance sheet

Proceeds are used by investor to purchase U.S. T notes, which the trust holds on the asset side of balance sheet
Special purpose Vehicles SPV’s (contd)

Interest on T Notes is passed through to investment fund as coupon on trust certificates

Trust enters into total return swap with outside dealer (same who established the trust)

Dealer pools together basket of credit risky assets and enters in total return swap with the trust (pays total return on basket to the trust & receives LIBOR plus spread)

Trust passes through to certificate holders the return on basket

This structure insulates investment fund from derivatives transactions
Special Purpose Vehicles - Example

SPV with a leveraged credit swap

- Investor
  - 144 A securities
  - $20 mm
  - Call option
- SPV
  - Loan price change
    - LIBOR + 250 bp
  - LIBOR + 100 bp
  - Interest income
  - $20 mm
- U.S. T Bonds
- Dealer
  - $100 mm
  - LIBOR + 100 bp
- Capital Markets
  - $100 mm
  - LIBOR
- Referenced Loans
  - Total return
  - $100 mm
Here‘s the deal!

• Investor purchases $20 mm (2 year maturity) of 144 a placed certificates (notes) from the SPV (trust)
• In practice SPV will also sell some equity to (hedge fund, estate, group of HNWI, Mafia shark???) - so that assets will get consolidated on equity investor‘s balance sheet and not (our) investor!
• Trust buys $20 mm U.S. 2 years T Notes yielding 6.00%
• Trust enters into a 2 year swap with a dealer, dealer paying the trust the total return on $100 mm basket of loans (agency rated) (that is average interest income on basket swapped over LIBOR - this case 250 bp + price appreciation / depreciation of basket) while SPV pays LIBOR + 100 bp
• Note: the notional value of swap doesn‘t have to equal amount of sold trust notes (certificates)
Deal’s economics hinge upon investor’s ability to achieve leveraged returns, although her own investment in SPV is not leveraged...

**Dealer:** - receives $100 mm cash from capital markets financed by LIBOR
  - Uses proceeds to buy loan basket at par
  - hedges perfectly total return on basket by passing on to SPV LIBOR+250 - coupon plus price change
  - Pays SPV LIBOR + 100 (thus covering funding cost of LIBOR)

  - And is left with 100 b.p on $100mm notional secure profit ($1mm) - not bad for a hard day’s work!
Deal’s economics hinge upon investor’s ability to achieve leveraged returns, although her own investment in SPV is not leveraged (cont’d)

**Investor:** - receives the return on all SPV’s assets and contractual obligations (incl. Net income on swap of 150 bp plus any increase / decrease in value of the loans in basket + interest income earned on the collateral)

- assuming that over the 2 years there is no change in the basket’s value, then Profit = 6%*$20 mm (return on treasury notes) + 150 bp on $100mm = **$1.2mm + $1.5mm = $2.7mm** per year for 2 years!

**Power of leverage:** commitment of $20mm of capital against return on $100 mm (the 150 bp) - economically equivalent with 750 bp on $20 mm + income on the T Note
Deal‘s economics hinge upon investor‘s ability to achieve leveraged returns, although her own investment in SPV is not leveraged (cont‘d)

So investor‘s RoR in SPV is 7.50% higher than the comparable return on the T Note, far exceeding the credit spread if it would purchase the basket of loans outright

The $20mm T Notes serve as collateral for the SPV‘s side of the swap
If basket declines in value, this will pay for the decline <= trust‘s position is „first loss position“ - first $20mm loss on basket of loans comes at the SPV‘s expense and the certificate‘s holder.

If loss =$20mm, dealer liquidates T Note collateral and terminates the swap; SPV pays out any accrued interest earned on the T Notes and renders the certificates „worthless“

The remaining $80mm „second loss position“ is retained by the dealer because it still owns the basket of loans
The economics are highly attractive but they come at some costs (and risks)

Intense documentation:

- SPV (trust) document
- master swap agreement and supplementary schedule
- swap confirmations
- note indenture (for the SPV note)
- private placement memorandum (144 A)

- this is how lawyers and investment bankers make tons of money which they spend in St. Tropez

Risk managers come in to estimate the basket’s default probability and the likelihood to depreciate in value beyond the $20mm
Collateralized Bond and Loan Obligations (CBOs & CLOs)

CBOs - securities issued backed by pools of (typically) high yield bonds
CLOs - same for pools of commercial loans

used by banks to remove „unattractive“ bonds or loans out of their balance sheet
CLOs and CBOs operating mechanics

Security traunches divided by credit rating (similar to mortgage pools)

First traunche, highest secured debt issued against best quality loans in the pool - lower return and lower volatility than the more junior traunches and than the security pool as a whole

Occasinally credit enhancement facilities are added (such as LCs) to make them more attractive to investors (buys them investment grade rating from rating agencies)
Construction of a CLO Trust (Example)

- **CLO Trust**
  - Term bank loans
    - $800mm, 3 yr, BB & B
    - 9.0% coupon
  - US T Note
    - $100mm, 3yr
    - 6.0% coupon
  - Insurance Company (AA)
    - „first loss“
    - $50mm credit def. Put option
  - Covers first $75mm defaults of any loans
  - Mkt val: $750mm
    - 40 bp=$3mm
  - Traunche A (senior)
    - $100mm fv
    - 7.5% coupon
    - AAA rated
    - Principal Protected
  - Traunche B (mezzanine)
    - $600mm, 8.25% coupon
    - A rated
    - First loss protected
  - Traunche C (subordinated)
    - $200mm, 8.75% coupon
    - B+ rated
    - Equity Traunche
    - $800mm, 3 yr, BB & B
    - 9.0% coupon
CLO construction - economics

• The 3 year T Note matures at the same time as Traunche A securities become due and payable, so the T Note proceeds are used to pay Tr. A investors
• Why is yield on Tr. A higher than risk free rate? - because investors have a claim on pass through income (a portion albeit) from term loans
• Traunche B has higher rating than underlying loans because it has first loss protection from put option, but only for $75mm - so slightly lower credit rating than traunche A
• Traunche C (equity) doesn’t get paid until investors from A&B get full money. - just like equity holders in a company
• How does the trust pay the 40 bp ($3mm) and the premium on the credit default option? - from the spreads
Cash Flows for the CLO Trust

Inflows:
- Income from bank loans, 9% on $800mm $72000000
- Income from T Note, 6% on $100mm 6000000

Total $78000000

Outflows:
- Coupon on Traunche A, 7.5% on $100mm $7500000
- Coupon on Traunche B, 8.25% on $600mm 49500000
- Coupon on Traunche C, 8.75% on $200mm 17500000

Total $74500000

Net annual Trust Income $3500000