Lecture 6

VAR - Market Risk
VAR - measure which summarizes into one number a class of risks

- Assumes that the portfolio is “frozen” over a time horizon
- Assumes that the risk profile of an institution is constant over the same risk horizon
- Assumes that the entire portfolio will be marked to market at the end of the holding horizon
Steps in computing VAR

- So VAR summarizes the expected maximum loss (worst case) over a target horizon within a given confidence level.
- Therefore, given the portfolio, the target horizon and the confidence level,
  1. Mark to market the current portfolio
  2. Define the risk factors impacting the portfolio’s value
Steps in computing VAR (cont’d)

• Measure the variability of risk factors
• Set the time horizon (holding period)
• Set the confidence level (e.g. 99%)
• Report the worst loss by processing the preceding information
Methods of obtaining VAR

• Analytic assumption on probability distribution (e.g. normal) - reading $x \sigma$ corresponding to tail end probability
  – for normal distr 1.96 sigma -> 95%
  – 2.33 sigma ->99%
  – Build loss histogram (empiric) and read off tail end corresponding to specific quantile (e.g. 99%)
VAR for general distributions

• \( W = W_0(1+R) \)
• Expected return and volatility of \( R \) are \( \mu \) & \( \sigma \)
• Relative VAR is defined as Dollar loss relative to the mean: \( \text{VAR(mean)} = E(W) - W^* \)
• \( \text{VAR(mean)} = -W_0(R^* - \mu) \)
• Absolute VAR: \( \text{VAR (zero)} = W_0 - W^* = -W_0R^* \)
General form - VAR derived from the probability distribution of the future portfolio value f(w)

• Given confidence level c
• find worst possible realization W*, such that
• \( P(W > W^*) = c = \int_{W^*}^{\infty} f(W) dW \)
• \( W^* \) is called the distribution‘s quantile

Or alternatively, the probability of a value lower than \( W^* \), \( p = P(w < W^*) \) is 1-c

\[
1 - c = \int_{-\infty}^{W^*} f(w) dw = P(w \leq W^*) = p
\]
VAR for parametric distributions

- Involves the estimation of parameters, such as the standard deviation of the distribution
- firstly need to „translate“ the general distribution \( f(w) \) into a standard normal \( \Phi(\varepsilon), \varepsilon(0,1) \).
- Associate \( W^* \) with the cutoff return \( R^* \):
  \[
  W^* = W_0(1+R^*)
  \]
VAR for parametric distributions (cont’d)

• In general $R^*<0$ so can be written as $-|R^*|$

Further, we can associate $R^*$ with a standard normal deviate $\alpha>0$ by setting 

$$\alpha = \frac{-R^* - \mu}{\sigma}$$

Equivalent to set:

$$1-c = \int_{-\infty}^{\infty} f(w)dw = \int_{-\infty}^{-|R^*|} f(r)dr = \int_{-\infty}^{-\alpha} \Phi(\varepsilon) d\varepsilon$$

In other words, finding VAR is equivalent to finding the deviate $\alpha$ such that the area to the left equates $1-c$ - possible by means of the standard cumulative normal distribution:

$$N(d) = \int_{-\infty}^{d} \Phi(\varepsilon) d\varepsilon$$
VAR for parametric distributions (cont‘d)

VAR(mean) = -W_0 (R^* - \mu) = W_0 \alpha \sigma \sqrt{\Delta t}
VAR(zero) = -W_0 R^* = W_0 (\alpha \sigma \sqrt{\Delta t} - \mu \Delta t)
VAR as a Risk Measure

- Desirable properties of a risk measure (also for capital adequacy purposes) - Artzner 1999:
  - **Monotonicity**: if \( W_1 < W_2, \rho(W_1) > \rho(W_2) \) or if a portfolio has systematically lower returns than another then its risk must be higher
  - **Translation invariance**: \( \rho(W+k) = \rho(W) - k \) or adding \( k \) cash to a portfolio should reduce the risk by \( k \)
VAR as a Risk Measure (cont’d)

• **Homogeneity**: $\rho(bW) = b\rho(W)$, or increasing the size of a portfolio by $b$ should scale its risk by the same factor (rules out liquidity effects of large portfolios - see LTCM)

• **Subadditivity**: $\rho(W_1 + W_2) < \rho(W_1) + \rho(W_2)$ or merging portfolios cannot increase risk
So how well does VAR fare against these criteria?

- One could concoct (skewed) trading strategies where quantile based VAR fails subadditivity.
- It is possible to show (homework for you!!!) that the shortfall measure
  \[ E(-X/X \leq -VAR) \]
  Expected loss conditional on exceeding VAR satisfies all 4 properties, or „is a coherent measure“ (Heath).
  Obviously, when VAR is normally distributed, it is subadditive (remember Markowitz?)
How do institutions use VAR?

• VAR can be used as a benchmark measure
  – across business fields (which operation is riskier?)
  – evolution in time
VAR as a potential loss measure
  - horizon defined by liquidation period
  - or time required to hedge
  - consistency with periodical (mostly daily) P&L reports
VAR as Equity Capital

• VAR as capital cushion for financial institutions (Basle committee)
• is to encompass all classes of risk: market, credit, operational, other (liquidity, business, etc.)
• choice of confidence level should reflect the degree of risk aversion and the cost of losing more than $\alpha\text{VAR}$
• choice of time horizon should reflect estimated time for corrective measures to become effective
Risk profile as a function of the bank's credit rating

- Convert EDF to credit rating

<table>
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<th>Desired Rating</th>
<th>1 year</th>
<th>10 years</th>
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<tr>
<td>Aaa</td>
<td>0.02%</td>
<td>1.49%</td>
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<tr>
<td>Aaa</td>
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<td>0.09%</td>
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<tr>
<td>Baa</td>
<td>0.17%</td>
<td>10.50%</td>
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<tr>
<td>Ba</td>
<td>0.77%</td>
<td>21.24%</td>
</tr>
<tr>
<td>Baa</td>
<td>2.32%</td>
<td>37.98%</td>
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</table>

(*) adapted from Moody's default rates between 1920 and 1998
Criteria for backtesting

• Backtesting involves systematic comparisons of VAR with the subsequently realized P&L in an attempt to detect biases in the reported VAR numbers
• longer holding horizons imply fewer independent observations
• Basel Committee recommends backtesting over 1 day horizons, although require a 10 day period for capital adequacy (presumably to cover for liquidity)
• Choice of confidence level is also important: too high CL reduces the number of events falling outside. In practice 95% & 99%
Internal Models for capital adequacy accepted by Basel Committee

- 99% confidence level - reflects tradeoff between the regulators‘ desire to ensure a safe and sound financial system and the adverse effects of a high confidence level on the banks ROE
- 10 day holding period - liquidity+ tradeoff between costs of constant monitoring and benefits of early detection of potential problems
- VAR multiplied by a „safety factor“ $SF \geq 3.0$
Why the „safety factor“?

• Presumably to account for model risk (e.g. understating risks due to short sample periods, unstable & unstationary correlations, approximations of probability distributions, etc.)

• Stahl (BAKred) derived $k=3$ analytically from Chebyshev‘s inequality (remember 2nd lecture?)
Sketch of Stahl’s derivation

• Chebyshev: for any random variable $x$ with finite variance, probability of falling outside a specific interval: 

$$P(|x-\mu| > r\sigma) \leq \frac{1}{r^2}$$

Assuming we know $\sigma$.

If the distribution is symmetrical, 

$$P((x-\mu) < -r\sigma) \leq \frac{1}{2r^2}$$

Set $1/2r^2=1\%$, then $r(99\%) = 7.071$, therefore 

$\text{VAR}(\text{max})=7.071\sigma$

Reported VAR of normal distr. @ 99% cl

is $2.326\sigma$

so if true distr. is misspecified, 

$k=\text{VAR} \text{max}/\text{VAR}=7.071/2.326=3$
Derivations of Portfolio VAR

• Analytical methods
  – delta normal method (variance / co-variance)
  – delta gamma approximations
  – delta, & full second order approximations

• Simulations
  – Historical
  – Monte Carlo
  – Scenario based
VAR as a function of risk factors

• From lectures 3 through 5 we learned that the value of any financial instrument can be modeled as a function of risk factors (e.g. \( P_{\text{zero}} = 100 \exp(-rT) \))

• VAR as total differential of pricing function:

\[
VAR(rf_j) = \sum_{j,i} \frac{\partial P(rf_j)}{\partial rf_i} \text{cov}(\Delta rf_j, \Delta rf_i) + \frac{1}{2!} \sum_{j,i} \frac{\partial^2 P(rf_j)}{\partial rf_j \partial rf_i} \text{cov}(Q_{nd \text{order}}) + \ldots
\]
Looks easier to handle than it really is

- If $rf_j$ are normally distributed, then VAR is normally distributed only if we account for the first order differentials
- VAR depends upon correlation estimates of rf.
- VAR is a forward looking risk measure, correlations (unless implied - see lecture 5 on options) are backward looking
- „Variance and correlations are great- the risk numbers derived upon them work almost all the times, except those very few times when they really matter“ (R. Gummerlock)
Let’s wrap some meat around the bone - VAR for a one year EUR interest rate swap

- Map cash flows to zero curve vertices ensuring zero NPV
- duration mapping
- linear interpolation
- sensitivity mapping
VAR for a simple interest rate swap (cont’d)

• Determine risk factor sensitivities (PV01)
• Establish correlations among the PV01s
• Imply distribution of returns and transform PV01s to returns (normal?)
• Plug in to VAR formula @ α confidence level off the implied distribution
Homework for next session

• Find the VAR of an asset swap (long corporate bond and short swap (pay fixed rate) - remember term structure of spreads!

• Find the VAR of a covered call options strategy (long stock, short call) - discuss „in the moneyness impacts“
So, what do we do?

• will follow next lecture