

## Der Leitzins und die Zinsen auf dem Girokonto

Konversatorium "Berufsbild Mathematik (Finanzmathematik in der Praxis)"



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Wien, am 06. Juni 2018

# Definitions

## Account types

- » A **savings account** is a deposit account held at a retail bank that pays interest but cannot be used directly as money in the narrow sense of a medium of exchange. A **time deposit** is a bank deposit account that has a **specified** date of **maturity** and usually earns a fixed interest. Money cannot be withdrawn before the maturity date (unless a penalty is paid). When the time deposit expires, it can be withdrawn or it can be renewed for another term (subject to new interest rates).
- » The opposite, sometimes known as a **sight deposit** or "on call" deposit, can be withdrawn at any time, without any notice or penalty, e.g. a **current account** held at a bank. In economic terms, the funds held in a sight deposit are regarded as liquid funds and in accounting terms they are **considered as cash**.

## Liquidity

- » Market **liquidity** is a market's feature whereby an individual or firm can quickly purchase or sell an asset without causing a drastic change in the asset's price. Money, or cash, is the most liquid asset, because it can be "sold" for goods and services instantly with no loss of value.
- » In the case of sight deposits the bank faces **funding liquidity risk**, the risk that liabilities cannot be met when they fall due, i.e. when the customer wants to withdraw their funds.
- » As sight deposits do not have a contractual maturity, the bank needs to dispose of a clear understanding of their **duration** level within the banking books.

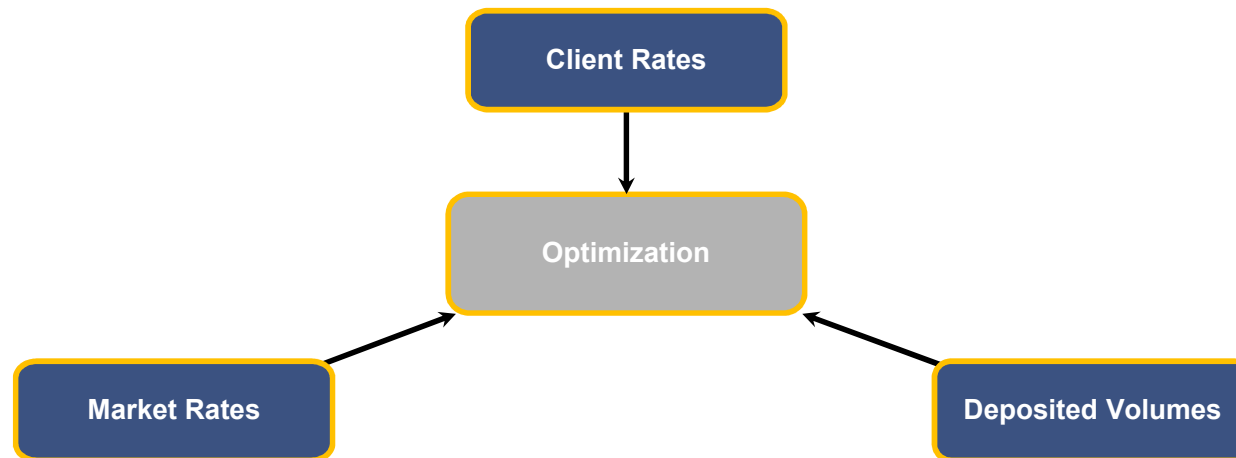
## Replication

- » In such models one attempts to find the **optimal portfolio** of investments (replication portfolio), which, if being refinanced by a particular product to be analyzed, best fits a set of well defined interest rate risk and/or liquidity risk criteria.

# Function of Interest Rate Replication Models

- » Bank pays **Client Rate** for **deposits of undetermined maturity** (current and savings accounts)
- » Bank **invests the deposits** into **maturity buckets** between 3M and 10Y with **fixed bucket weights in monthly time steps**
- » IR replication model optimizes the **investment weights** of the buckets aiming to
  1. generate **stable margins** between **investment rate** and **client rate**
  2. provide **sufficient liquidity** for buffering volume changes **by means of maturing investments**

**Note:** Liquidation of non-maturing investments would create IR risk



IR replication models mitigate IR and liquidity risk by optimization of the investment portfolio

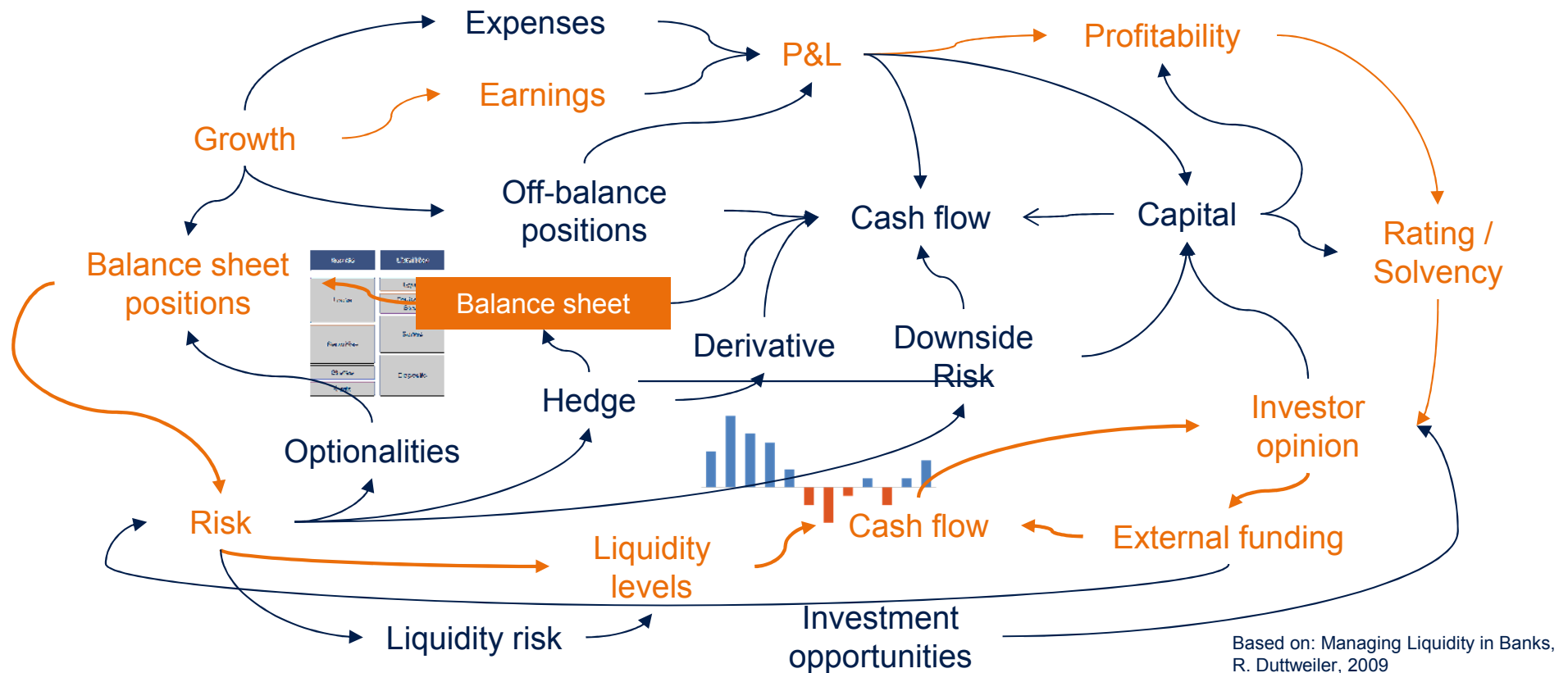
# Function of the Interest Rate Replication Model

- » The IR replication model determines **optimal** rolling **investment portfolios** for **deposits with undetermined maturity** such as current and savings accounts
- » The optimization **output** are **investment weights** for a set of **maturity buckets** between 3 months and 10 years
- » Weights are determined separately for the **savings, commercial, and current accounts**
- » The optimization aims to select investment weights that
  1. generate **stable margins** between investment and client rate
  2. provide **sufficient liquidity** to buffer volume changes **by means of maturing investments**
- » Technically, the optimization
  - › **minimizes the variance** of the mean **margin** on a 10 year forward looking horizon over a set of **10,000 hypothetical scenarios**
  - › Imposes **constraints** on the maturing volume **to provide liquidity for coverage of volume changes**
- » The model does **not optimize** the **expected margin**

The IR replication model mitigates IR and liquidity risk by optimization of the investment portfolio

# Challenge: Complex network of dependencies have to be understood and managed comprehensively

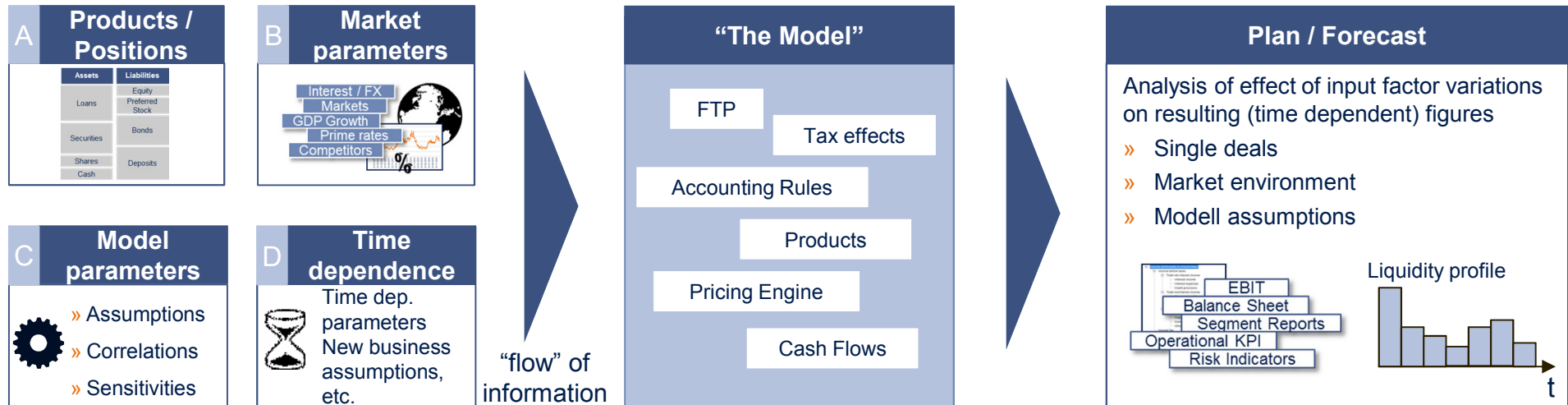
Sample (sub-)path for the effect from interest rate replication model on the balance sheet composition:



▶ Analysis paths is key → Systems and processes to analyze new pathways in a short time frame



# The development of a scenario model should be focussed on the essential value drivers initially neglecting driver interactions



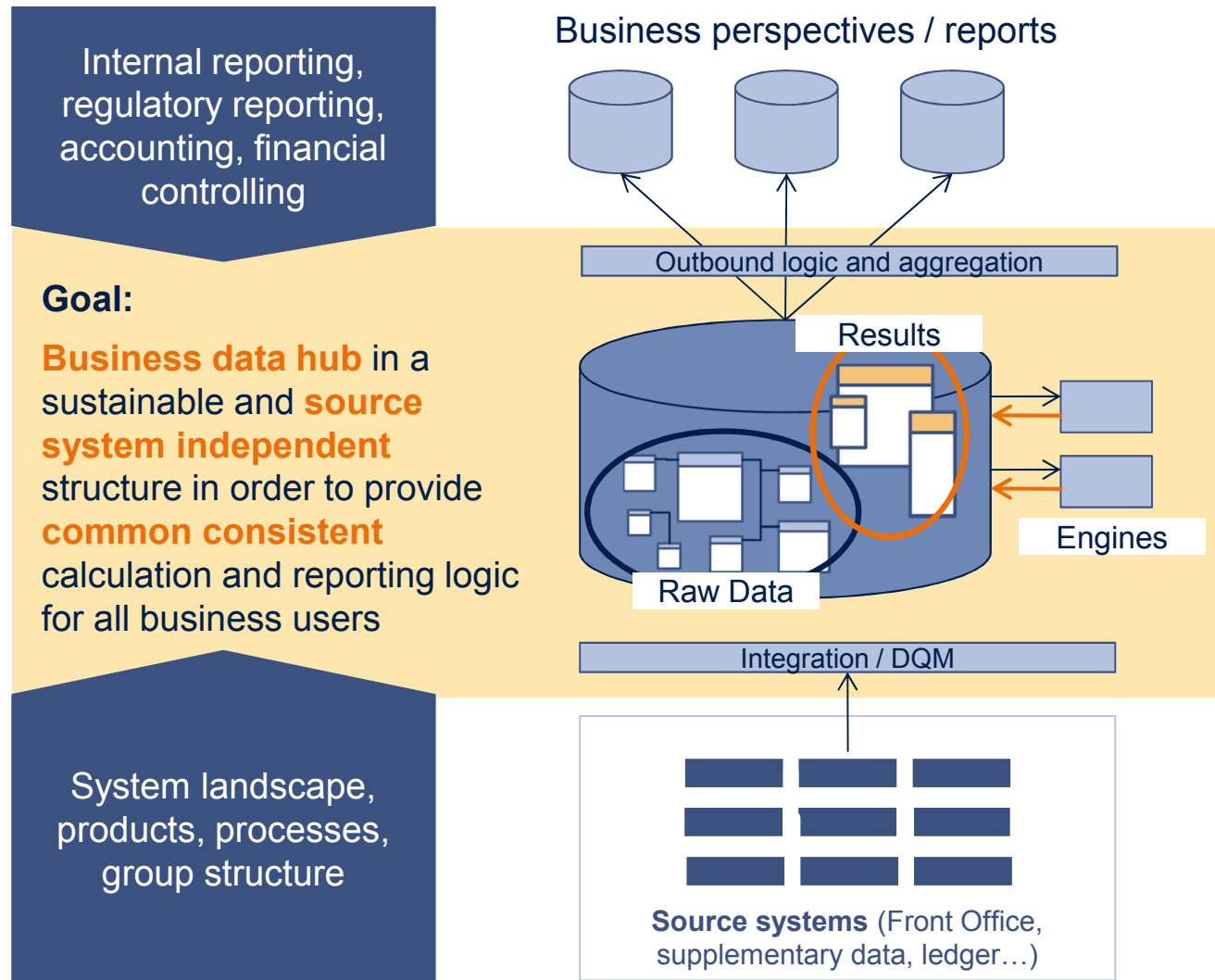
Product A	Coupling A+B	Coupling A+C
	Business Segment B	Coupling B+C
		Portfolio Desk C

**The development of a scenario model should consider materiality:**

- » Focus on primary and direct effects (*diagonal*)
- » Secondary effects and interactions/correlations of value drivers should be considered subsequently (*secondary diagonals*)

In its core, simple scenario driven model mechanics, however handling of complex correlations between model parameters needed

# Implementation: comprehensive bank management by a central data hub including raw data, calculation engines and result data aggregation



## Vision

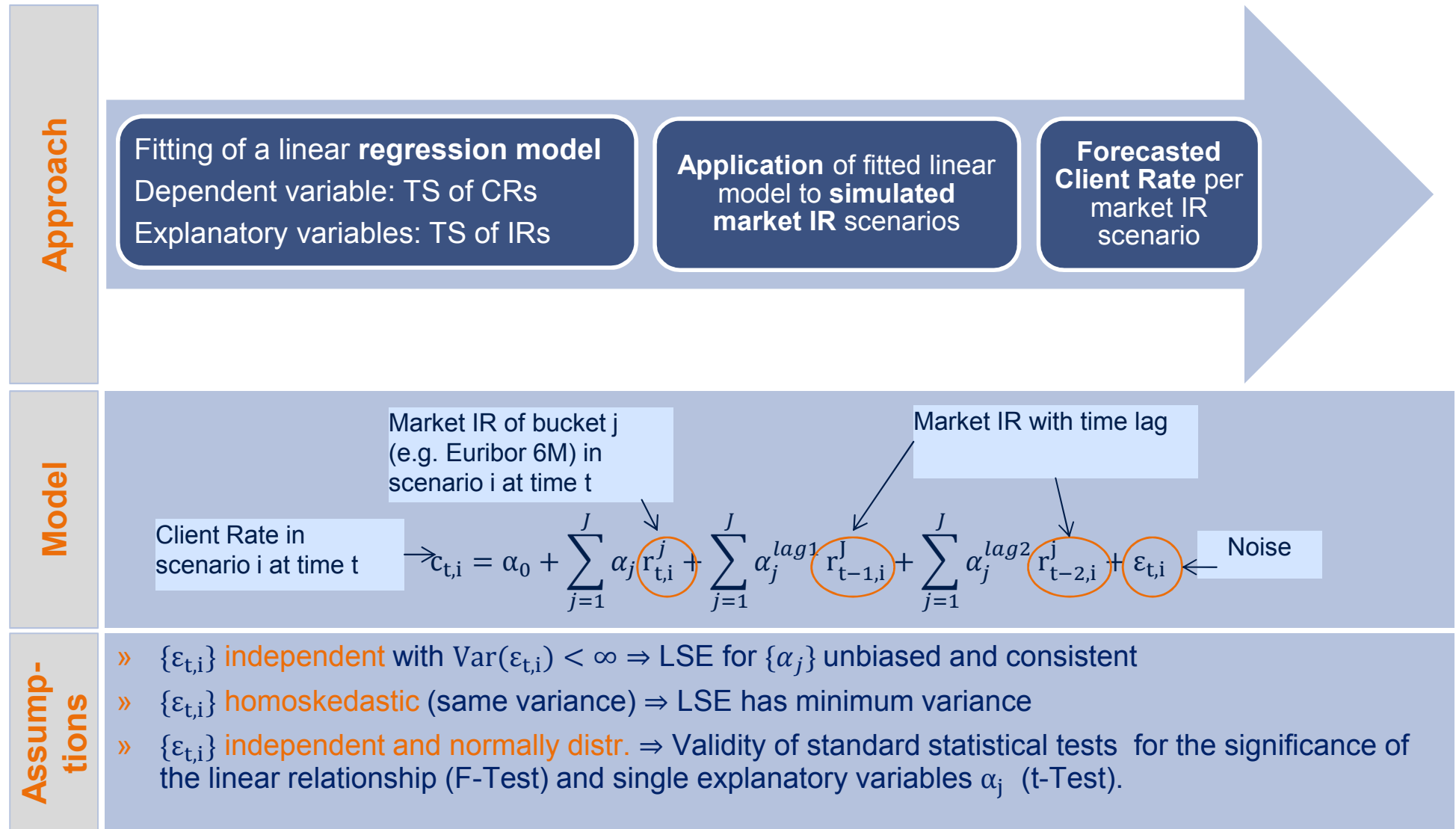
- » Deliver all data and functions (risk types, accounting, reg. reporting etc.) out of one harmonized source
- » Provide different perspectives on the same data
- » Establish common taxonomies in order to achieve consistent reporting across risk types
- » Enable integrated (stress) scenario evaluation across risk types
- » Avoid individual data sourcing and source system dependent business logic across the departments
- » Create transparency through central function architecture



# First Step: Linear Regression

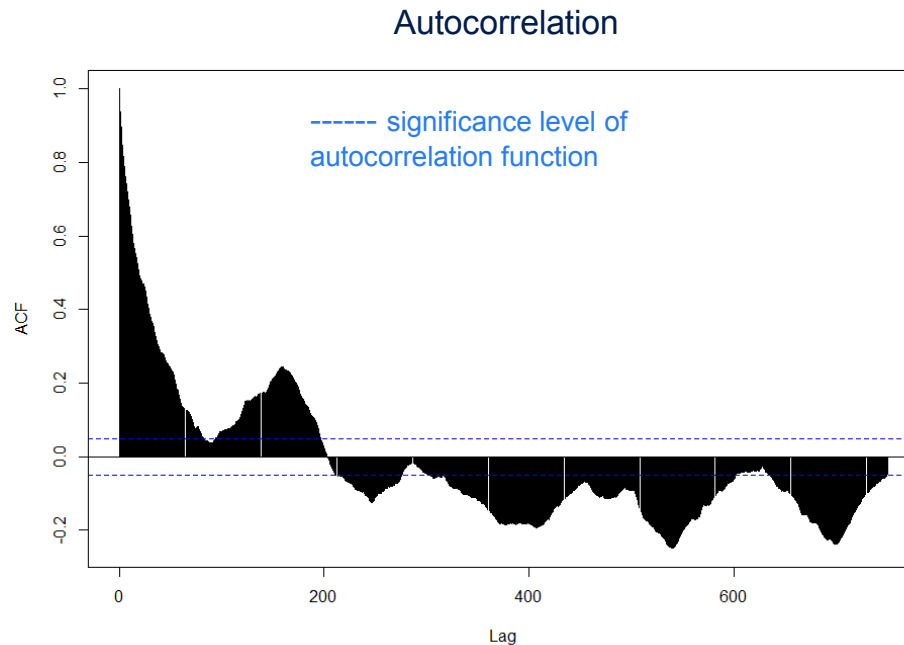


# Linear Regression to forecast Client Rates



# Analysis of Linear Regression Approach

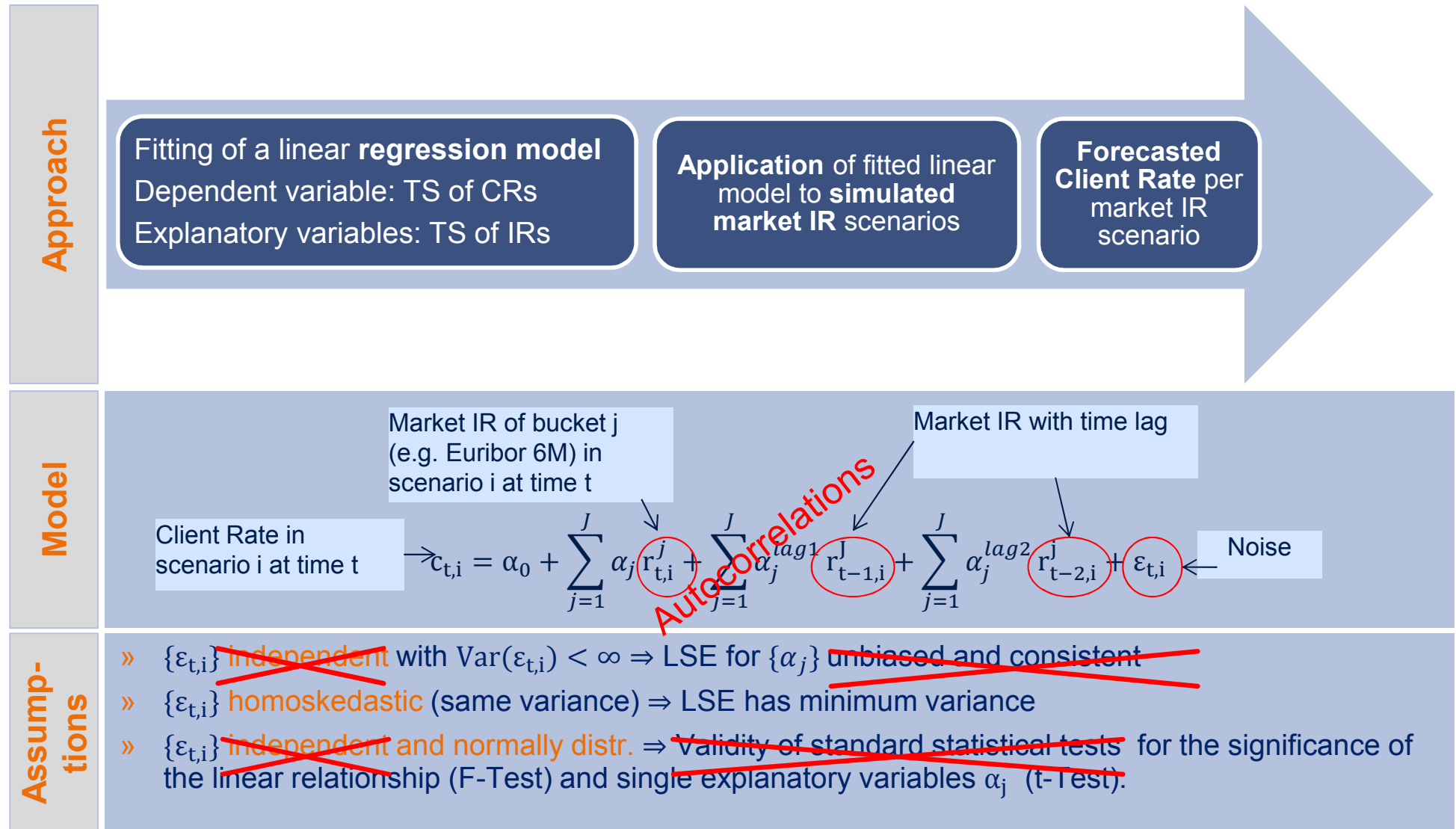
- » Very high **autocorrelation of the regression residuals** indicates **spurious regression**, i.e. the model may assume a **functional relationship without economic validity**



- » Consequence: standard **significance tests** for a good model-fit are **not valid**
- » This is also true for the regression constants
- » A high significance of the t-tests has therefore no value

The model for Client Rates should hence be based on uncorrelated data

# Linear Regression to forecast Client Rates



# Spurious Regression and Consequences for Prediction Quality

- » **Does the model work for prediction anyway?**
  - › **No!**
  - › Too many explanatory autocorrelated market IR time series cause **overfitting** since they act like a **set of basis functions**.
- » **Consequences of overfitting:**
  - › Almost **perfect fit for training data**
  - › Risk of a drastic **loss in prediction quality after structural disruptions** of the market
  - › Example: **Shift from normal to low IR regime**



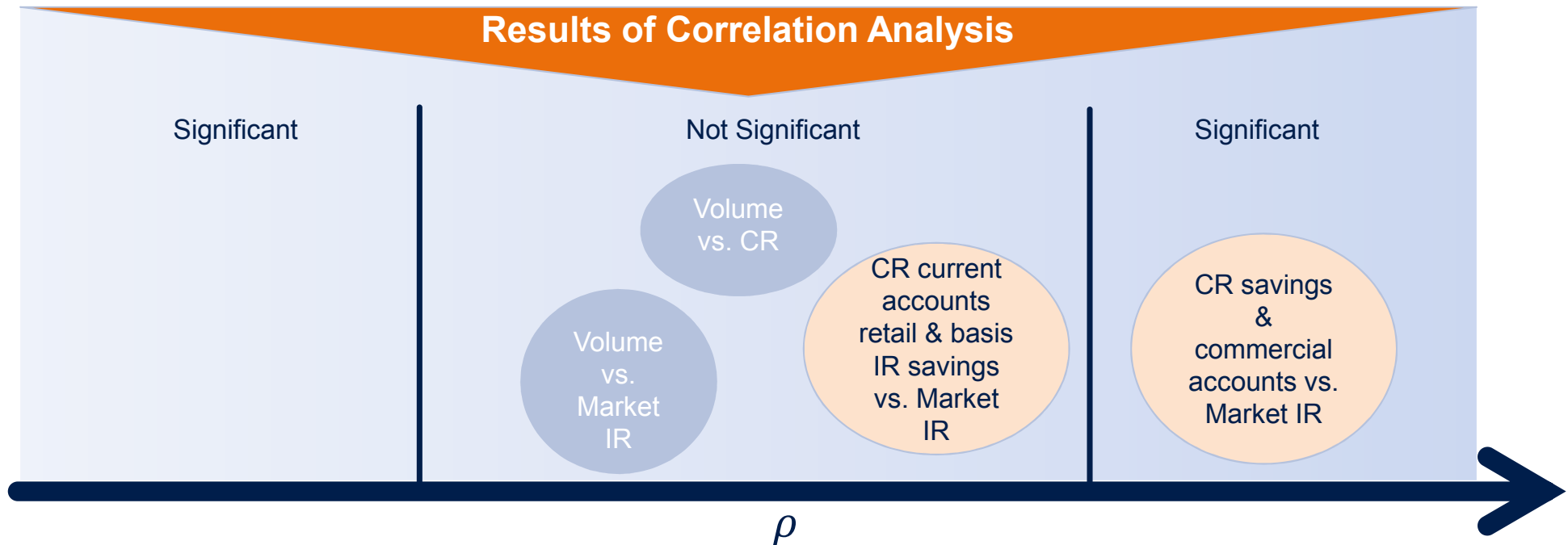
modeling goals: reduce overfitting and increase robustness of predictions under structural market changes

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# Fixing the Regression: Analyzing Returns

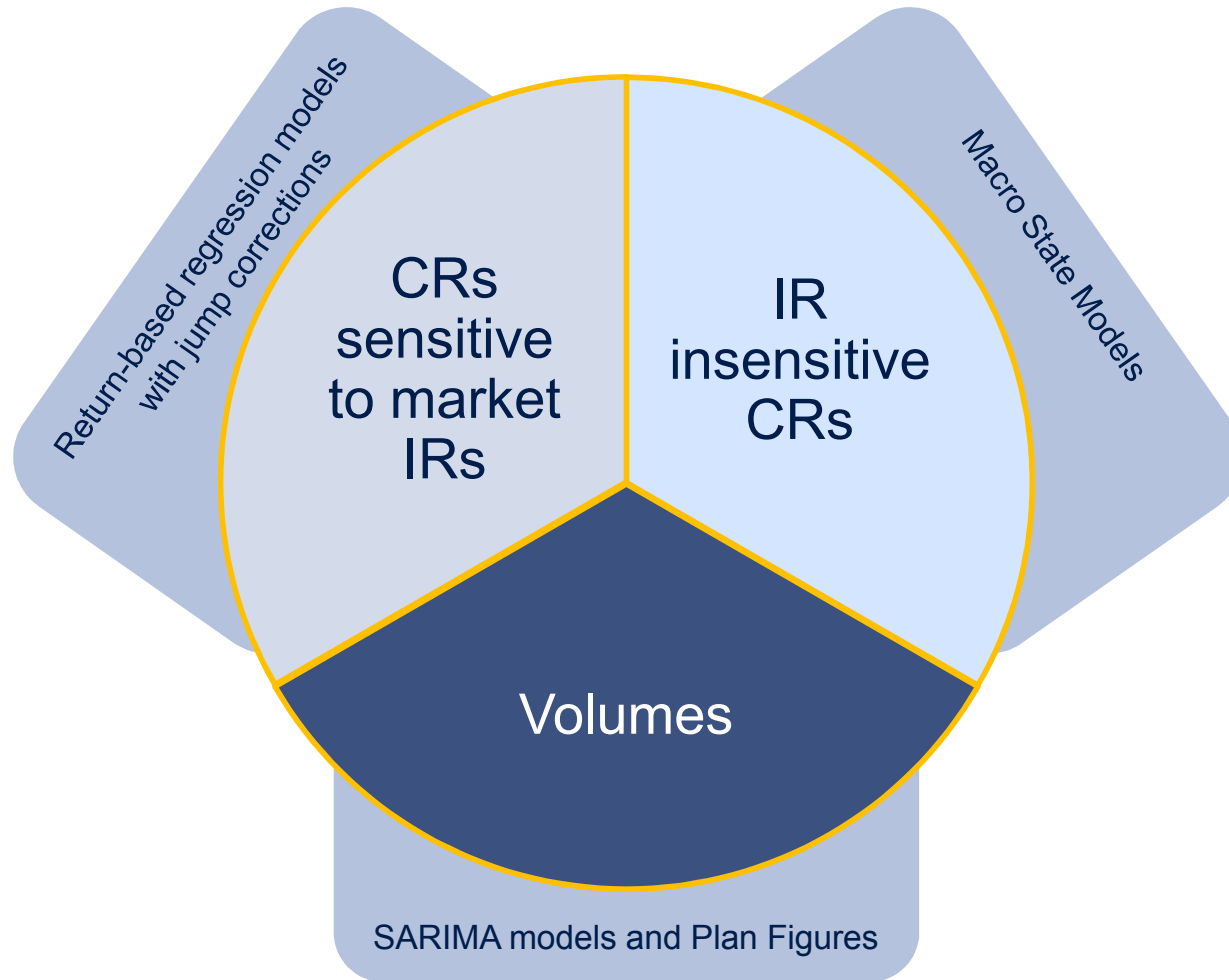
# Correlation Analysis of Returns

- » **Autocorrelation of returns** found to be **very small** for **Client Rates, Volumes** as well as **Market IRs**
- » **Return-based regression models** are therefore **not prone to spurious regression**
- » **Statistical tests** can therefore be used to **assess model validity**
- » **Variable Selection**
  1. **Correlation analysis** of returns **identifies products** where return-based **regression models may succeed** in forecasting Client Rates and/or Volumes based on market IRs
  2. **Backward elimination** identifies a „parsimonious Model“ to **avoid overfitting**





# Grouping of Modeling Approaches for Forecasting



Forecasts are based on three basic model types

# Return-based regression model

- » Linear model for **returns**

$$\Delta c_{t,i} = c_{t,i} - c_{t-1,i} = \alpha_0 + \sum_{j=1}^J \alpha_j (r_{t-\text{lag}(j),i}^j - r_{t-\text{lag}(j),1,i}^j) + \varepsilon_{t,i}$$

- » This yields the **estimators**

$$\Delta \hat{c}_{t,i} = \hat{\alpha}_0 + \sum_{j=1}^J \alpha_j (\hat{r}_{t-\text{lag}(j),i}^j - \hat{r}_{t-\text{lag}(j),1,i}^j),$$

where  $\hat{r}_{t,i}^j$  denotes the simulated market IR of maturity bucket  $j$  at time  $t$  in the  $i$ -th scenario.

- » **Forecast:**

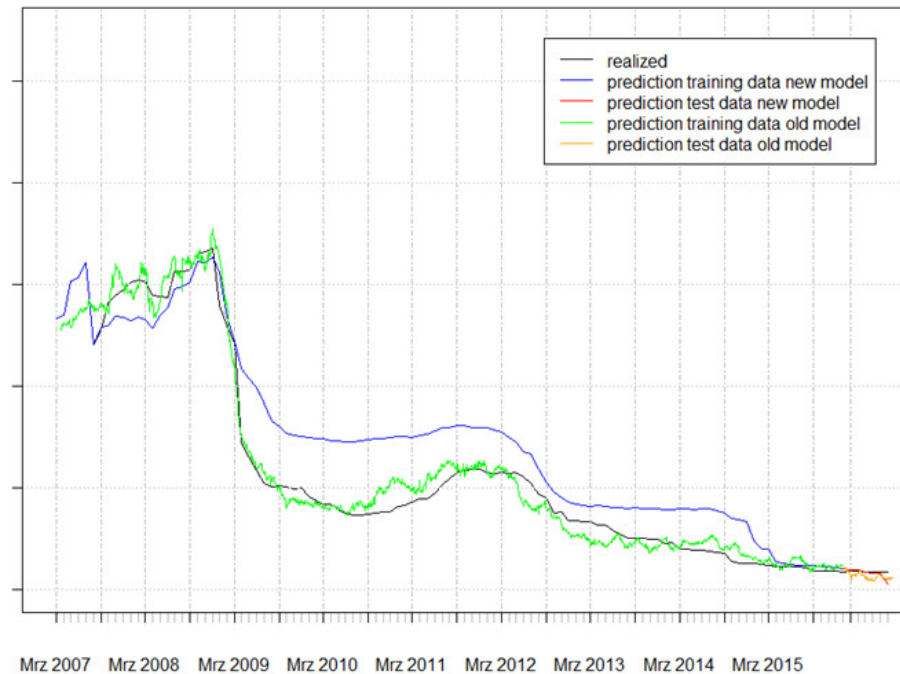
$$\hat{c}_{t,i} = \max\{c_{\text{floor}}, \hat{c}_{t,i-1} + \Delta \hat{c}_{t,i}\}, \quad t > 0$$
$$\hat{c}_{0,i} := c_0 \text{ (real observed starting Client Rate)}$$

- » **Floor-Parameter**  $c_{\text{floor}}$  applies to all retail products, set by **management decision** within legal bounds

## Drawbacks

- » **Misprediction of single returns** may lead to sustainable **bias in CR** levels with very slow decay
- » Model does not account for the **market** tending to keep the **spread between CR and market IRs at historically observed levels**
- » Introduction of a **jump component** was able to **reduce these shortcomings**

# Fitting Results for Pure Return-Based Regression Models

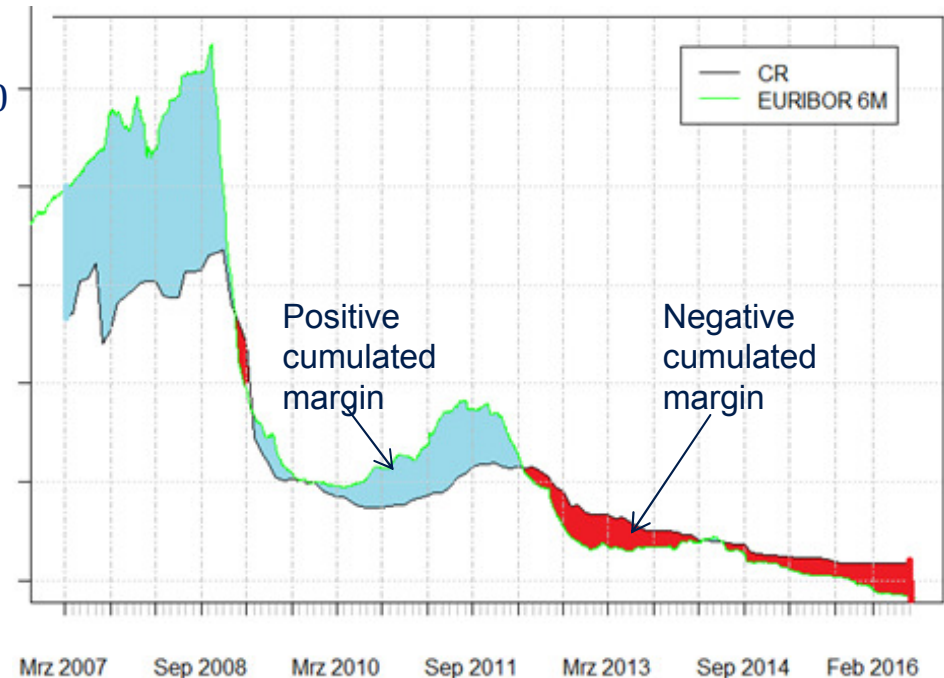


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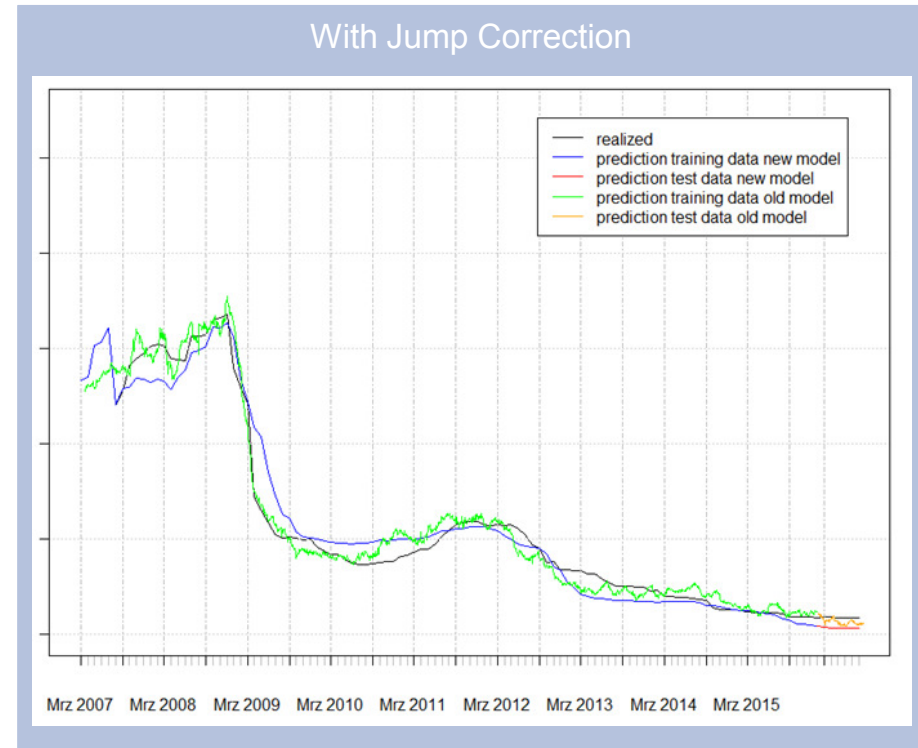
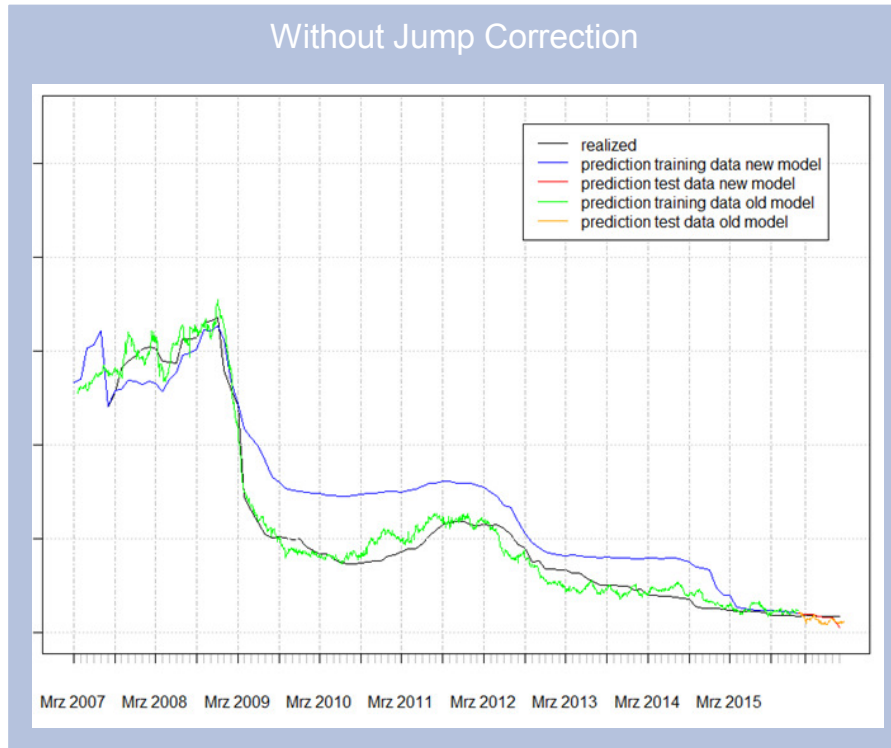
# Return-Based Regression Models: Design of the Jump Component

- » Compute cumulated margin between intersections of CR and a historical reference CR after shift by mean margin
- » Determine max and min cumulated margins  $S_{max} > 0$  (for  $IR > CR$ ) und  $S_{min} < 0$  (for  $IR < CR$ )
- » Suggest CR increment by regression model
- » Compute cumulated margin  $S_i(t)$  since last curve crossing
- » If  $CR > IR$ , a downshift occurs with probability  $p^d(t, i) = d \cdot S_i(t) / S_{min}$  and size  $p^d(\hat{r}_{t,i}(t) - \hat{c}_{t,i}) < 0$
- » If  $CR < IR$ , an upshift occurs with probability  $p^u(t, i) = u \cdot S_i(t) / S_{max}$  and size  $p^u(\hat{r}_{t,i}(t) - \hat{c}_{t,i}) > 0$



Model parameters  $u$  and  $d \in 0,1$  controlling jump-behavior are calibrated via Least-Squares-Minimization

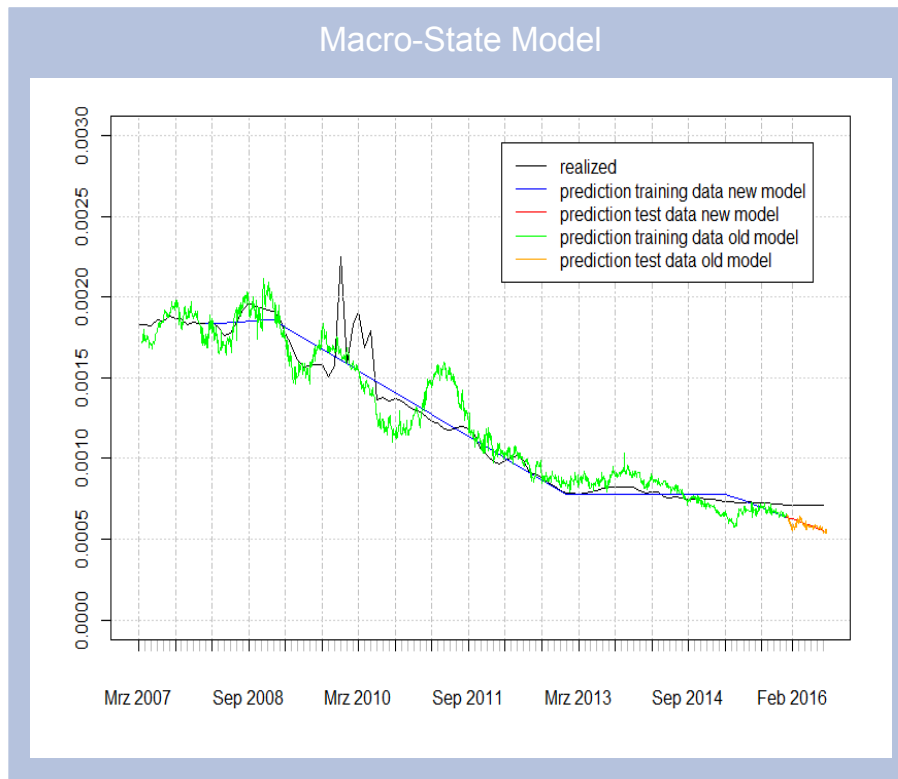
# Fitting Results: CR Forecast by Return-Based Regression with Jumps



Jump Component keeps margin at historically observed levels and corrects forecasts for level errors

# Macro State Model

- » Modeling Client Rates with infrequent adjustments:
  - › CRs for **new accounts** are set by the bank to few **fixed levels** (called macro states)
  - › CR **changes** only affect new accounts and and **propagate slowly**



Two modeling approaches capture diverse product-specific behavior of CRs



# The Model for Client Rates

## Direct Regression Model

### Basic assumption

- » CR is a linear combination of market IR curves
- » Statistical validity of regression approach relies on uncorrelated input data (This is severely violated by the data)

### Model parameters

- » (time lagged) market IR time series

### Calibration

- » Least Squares

### Scenario generation

- » Apply regression model to simulated market IRs

Model Evolution

## Return-Based Regression Models and Macro State Model

### Basic assumptions

- » IR-sensitive CRs:
  - » CR *returns* are a linear combination of market IR *returns*
  - » the bank tries to adjust spreads between CR and market IR to the historically observed levels
- » CRs with infrequent adjustments:
  - » CRs for new accounts are set by the bank to few fixed levels (called macro states)
  - » CR changes only affect new accounts and propagate slowly

### Calibration

- » Least Squares Calibration

### Scenario generation

- » Apply model to simulated returns of market IRs

For CRs with significant correlations to IR returns, return-based regression models avoid the pitfalls of spurious regression. Alternative modeling approaches cover products without significant correlations.



# Modeling Deposit Volumes

# Deposit Volume Modeling

## Basic observations

- » Deposit volumes show a significant trend component (can usually be steered by the bank)
- » Deposit volumes show an inter-month seasonal pattern, e.g. pre-holiday season gain, new year drop, ...
- » Current accounts also have a distinct intra-month pattern, i.e. pay-day and consecutive draw-downs

## Observations from calculating correlations

- » No correlation to Client Rates discernible, i.e. clients seem to be insensitive to rate changes (this clearly just holds within bounds and in the current interest rate environment)
- » No correlation to market rates discernible, i.e. clients do not change their saving behavior based on the interest rate environment (this should also be taken with a large grain of salt)

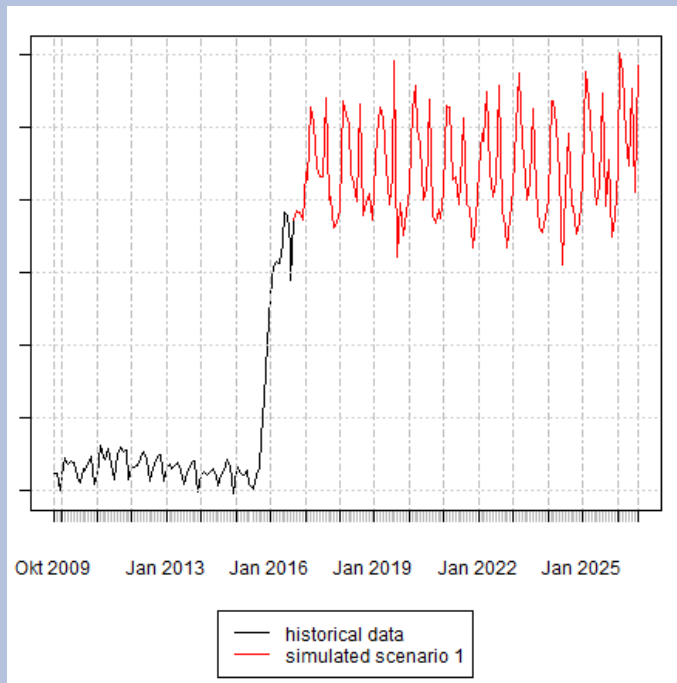
## Calibration and scenario generation

- » Detrend the historical time series
- » Fit a SARIMA time series model to the detrended data
- » Take trend from plan figures (constant extrapolation beyond planning horizon)
- » Simulate seasonal component via time series model
- » Add residuals by bootstrapping from the historical data

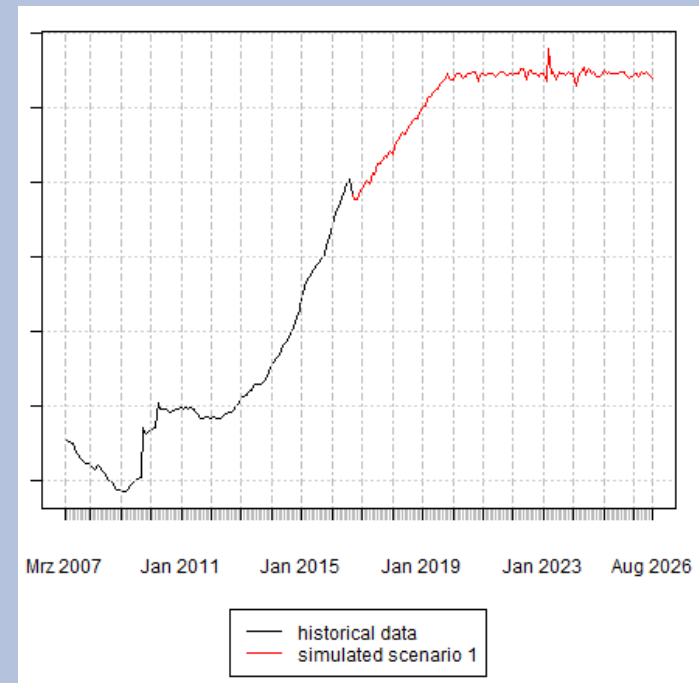
Volume forecasts can be based on SARIMA model, but assumptions should be re-evaluated carefully from time to time!

# Deposit Volume Forecast

Model for constant plan figures



Model for increasing plan figures with constant extrapolation beyond the planning horizon



Single model type captures diverse landscape of volume trajectories.

# Volume Prediction: Combining Plan Figures with Time Series Modeling

## Direct Regression Model

### Basic assumption

- » Volume is a linear combination of market IR curves and CR  
(This assumption is not supported by the data)
- » Statistical validity of regression approach relies on uncorrelated input data.  
(This assumption is severely violated by the data)

### Model parameters

- » (time lagged) market IR time series
- » CR

### Calibration

- » Least squares calibration on historical data

### Scenario generation

- » Apply regression model to simulated market IRs and CR prediction

## Model Evolution

## Seasonal Time Series Model with Plan Figures for Trend

### Basic assumptions

- » Volume trends develop according to plan figures
- » Inter-Month fluctuations can be captured by a seasonal time series model
- » Future fluctuations are similar to historically observed client behavior

### Calibration

- » Detrend the historical time series
- » Fit a SARIMA time series model to the detrended data

### Scenario generation

- » Take trend from plan figures (constant extrapolation beyond planning horizon)
- » Simulate seasonal component via time series model
- » Add residuals by bootstrapping from the historical data

Liquidity risk due to volume drops can be captured by classical time series models. Replacing the trend component by plan figures yields replication portfolios that are consistent with the plan.



# Interest Rate Scenario Generation



# The Hull-White 2 Factor Model

- » The Hull-White 2 factor model is a so called short-rate model. This model class models only the very front end of the yield curve
- » It uses two stochastic processes  $x(t)$  and  $y(t)$  to model the short rate

$$r(t) = x(t) + y(t) + \phi(t), \quad r(0) = r_0$$

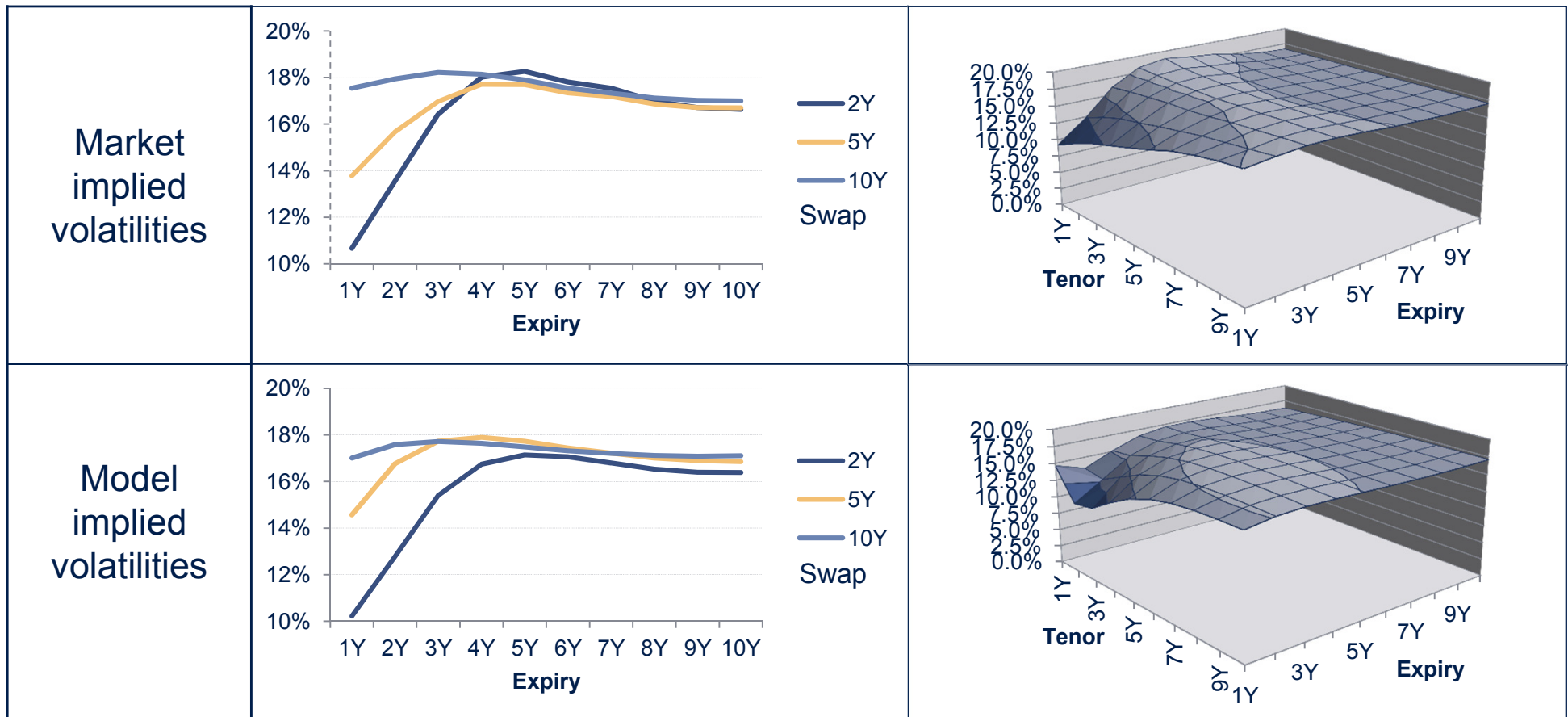
- »  $\phi(t)$  assures that the initial term structure of the yield curve is matched
- » The dynamics of the two stochastic processes  $x(t)$  and  $y(t)$  are described by the following stochastic differential equations (SDEs)

$$\begin{aligned} dx(t) &= -a \cdot x(t)dt + \sigma dW_1(t), & x(0) &= 0 \\ dy(t) &= -b \cdot y(t)dt + \eta dW_2(t), & y(0) &= 0 \\ dW_1(t) dW_2(t) &= \rho dt \end{aligned}$$

- » Hence, the level and shape of the yield curve at any future date is completely determined by two stochastic quantities

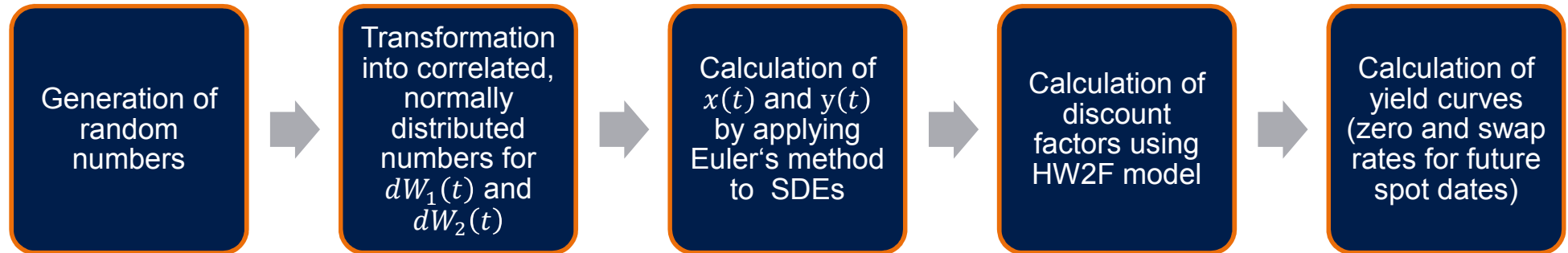
# Calibration of the Hull-White 2 Factor Model

- » The five free parameters  $a, b, \sigma, \eta, \rho$  are set by fitting the model to ATM swaption volatilities
- » Set of benchmark instruments: [1Y,2Y, 3Y...10Y] x [1Y,2Y, 3Y...10Y] swaptions
- » Use of vega-weighting improves stability and puts focus on long end of yield curve

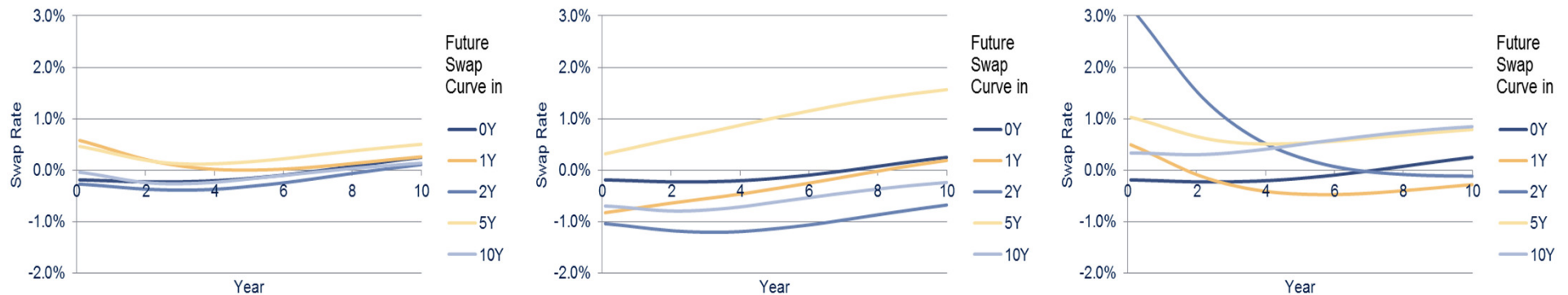


# Yield curve scenario generation using the Hull-White 2 factor model

## Steps required



## Exemplary scenarios



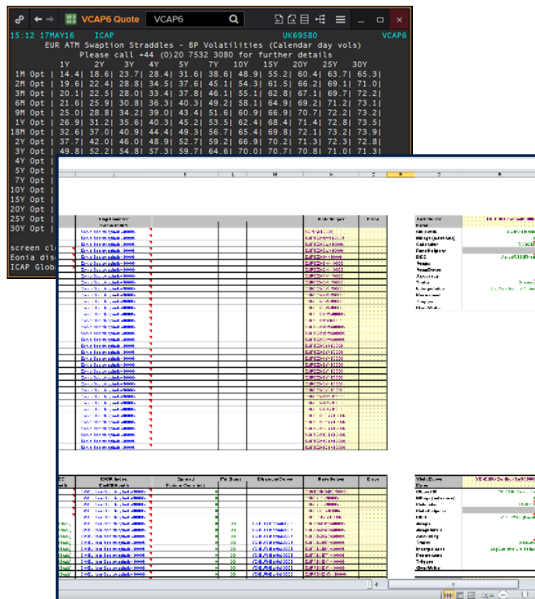
Simulation covers a variety of normally distributed scenarios

# New process and model yields robust calibration and scenario generation

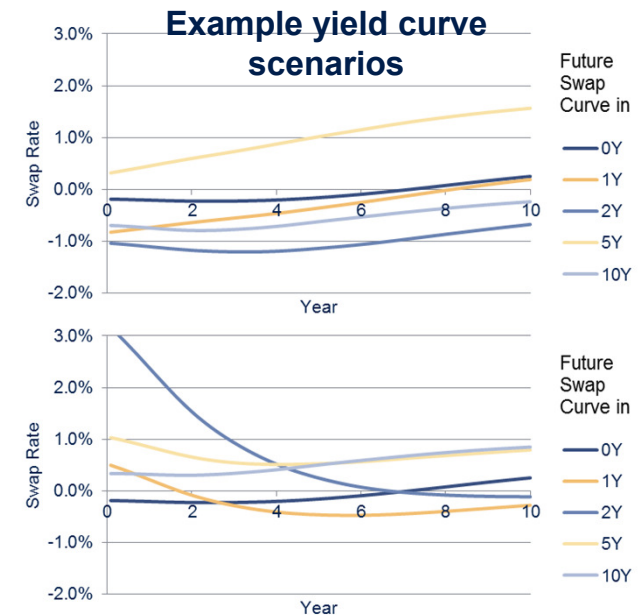
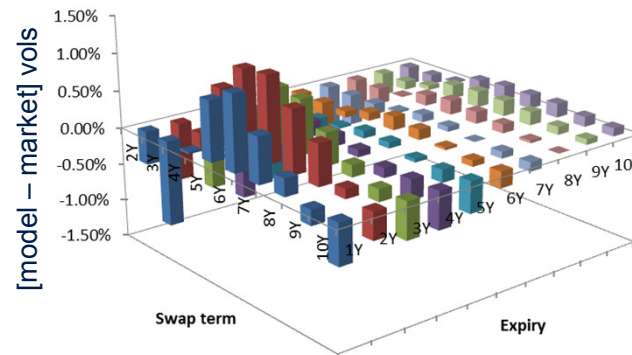
Market data input from Reuters to Excel/QuantLib

2-Factor Hull White model calibration in Excel/QuantLib/VBA

Scenario generation in Excel/VBA and output to CSV files



Example calibration results show variances between market and model-implied volatilities<sup>(1)</sup>



Though there are variances between market and model-implied volatilities (e.g. for short expiries/swap terms) the overall model fit is fairly good and fit for purpose

(1) Variances expressed in terms of shifted log-normal volatilities with 3% shift size



# Optimization Problem

# Optimization Problem

- » Consider **time-averaged margin** over the next 10 years and **minimize its variance**
- » **Liquidity Constraints**: maturing volume fraction > historical quantile of volume drops
- » Theoretical **Target Function**

$$f(w_1, \dots, w_n) = \text{Var} \left( \frac{1}{T} \sum_{j=1}^J w_j \sum_{t=1}^T m_j(t) \right) = w' \Sigma w \longrightarrow \min$$
- » **Covariance matrix**  $\Sigma = \langle \text{Cov}(\bar{m}_i, \bar{m}_j) \rangle$  of the time-averaged margins  $\bar{m}_j = \frac{1}{T} \sum_{t=1}^T m_j(t)$  **to be estimated**



## Monte-Carlo-Simulation

### Market IRs

- » **10,000 trajectories of market swap par rates** over the next 10 years (2-Factor Hull White Model)
  - › Normal distribution allows for modeling of negative IRs
  - › Five Parameters (for volatilities, mean reversion and correlation) fitted to ATM swaption

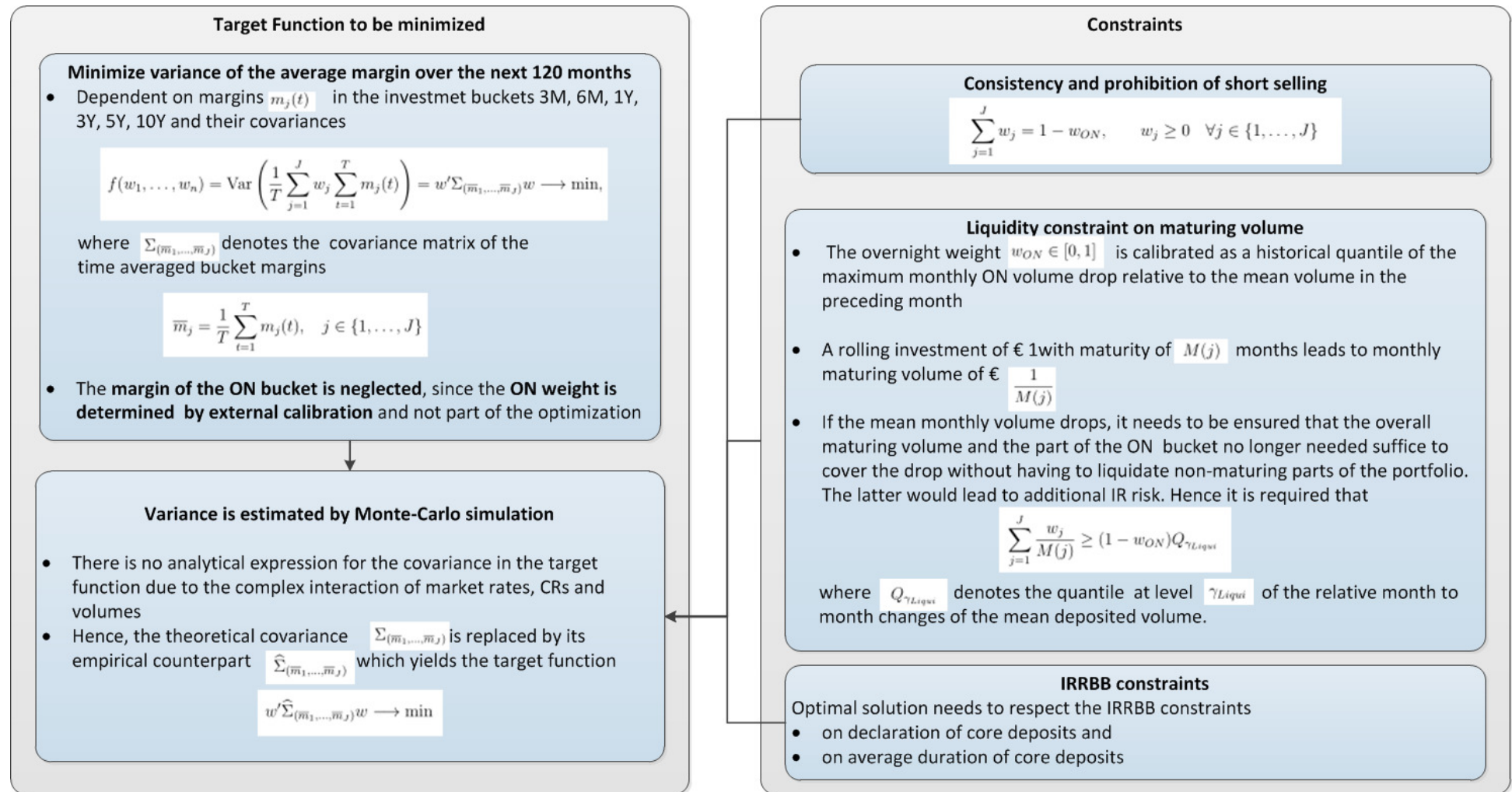
### Client Rates

- » **Prediction of Client Rates** that are consistent with market rates (correct correlation)

### Volumes

- » **Prediction of Deposited Volumes** reflecting historically observed client behavior

# Details on the Optimization Problem





# Time Series Forecasts with Automated Modell Selection



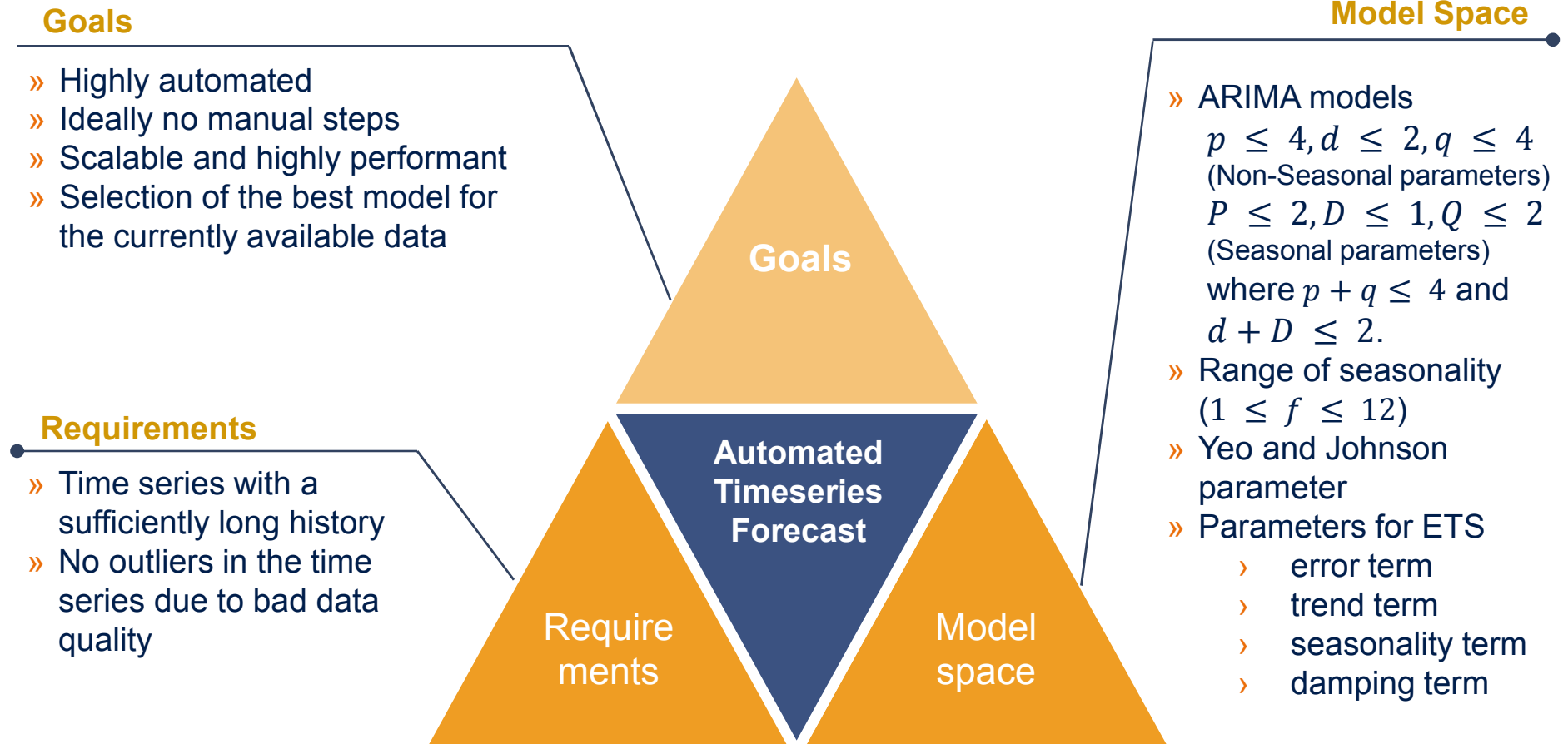
# Application of an Automated Time Series Forecast Procedure

- » Forecasts can be used to model a realistic expected behaviour of a time series firmly based in well understood mathematics, for example:
  - › **Customer deposits**
  - › **Volume of new business**
  - › Operating expenditures (non-project related)
- » Often the setup is highly manual and done very infrequently, but in principle the process can be automated

Pros and Cons of an automated time series forecast compared to a manual setup

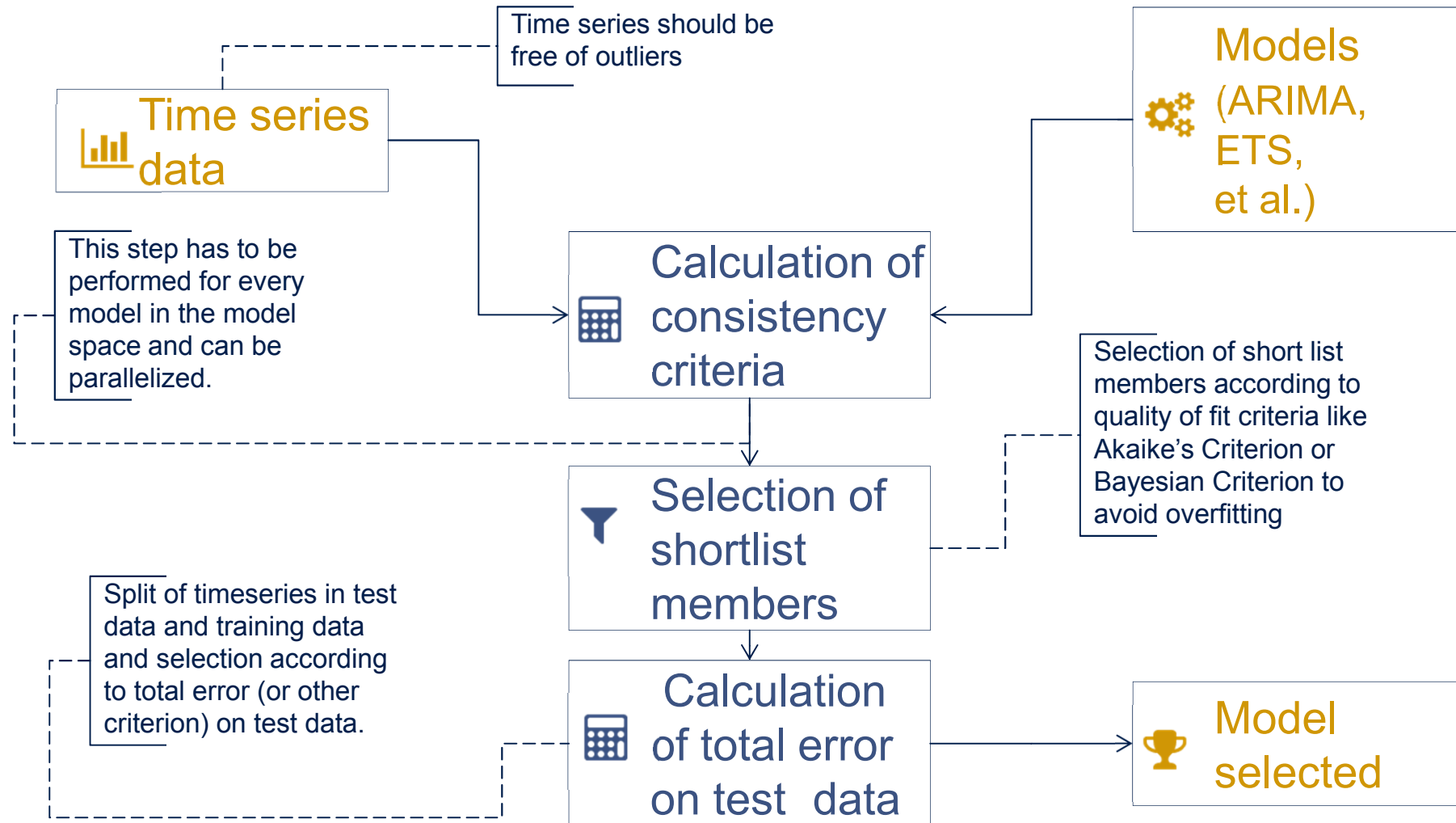
Pro	Contra
<ul style="list-style-type: none"><li>» Can expand a consistent forecast procedure to many products and different departments</li><li>» Can perform the forecast frequently and can steer accordingly</li><li>» Higher granularity possible</li><li>» Higher accuracy of the forecast</li></ul>	<ul style="list-style-type: none"><li>» Initial setup has to be performed</li><li>» Maintenance of the running system required</li><li>» Higher requirements for data quality</li><li>» Results may need more interpretation</li></ul>

# Basic Goals and Requirements of an Automated Time Series Forecast



Want to achieve a highly automated model selection over a large model space with minimal manual input

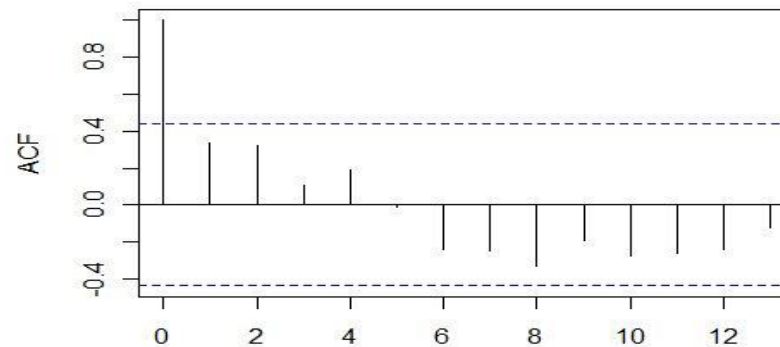
# Schematic Depiction of the Forecast Procedure



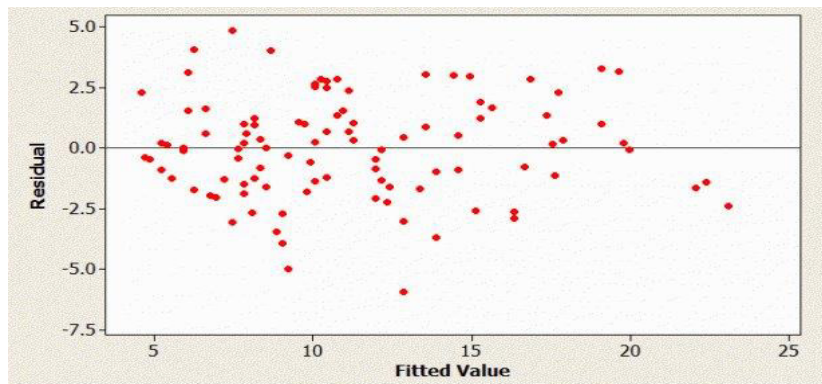
# From Models to Shortlist – Calculation of Consistency Criteria

» Only models that pass the following criteria are accepted for the shortlist:

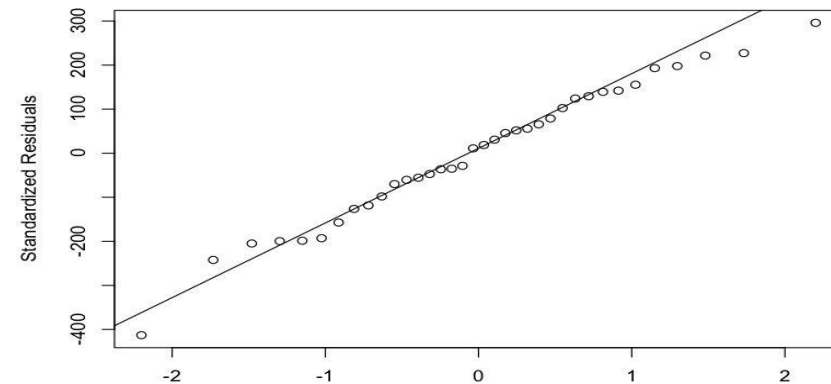
Small autocorrelation of residuals



Variance of residuals constant



Residuals follow a normal distribution



» Only models that pass basic consistency criteria are considered for the shortlist

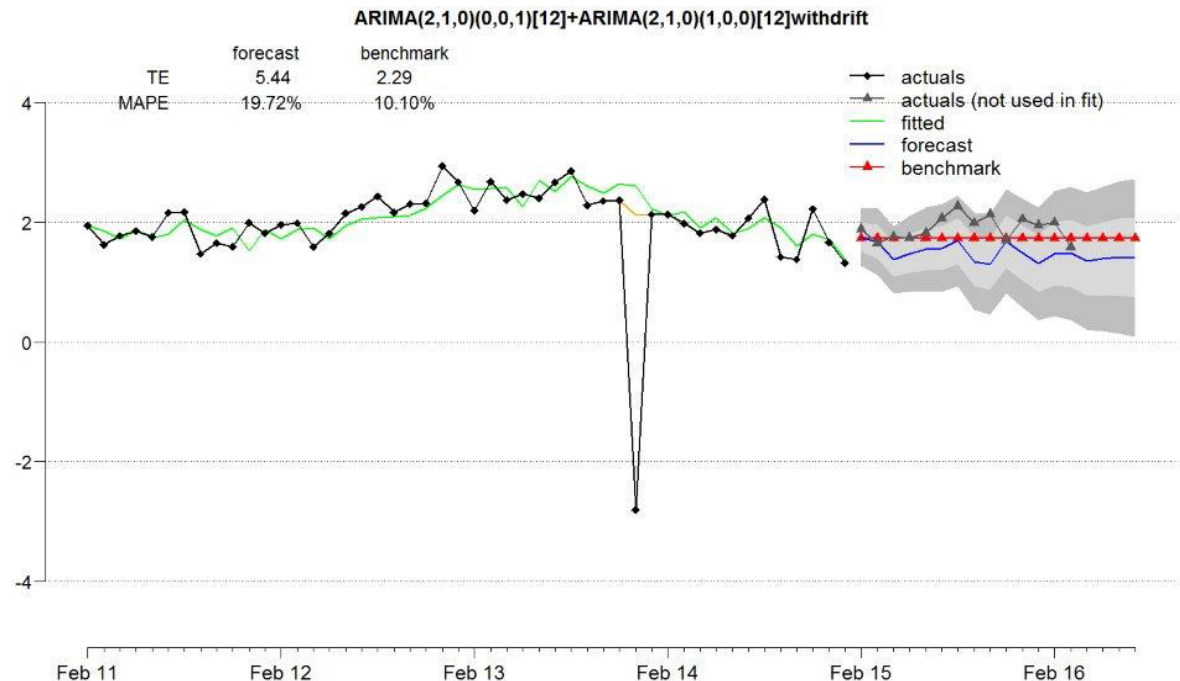
# From Shortlist to Model Selection

## » Select from the shortlist the top 10 models according to:

- › Sample corrected Akaike's Information Criterion  $AIC_c$  (preferred)
- › Akaike's Information Criterion AIC
- › Bayesian Information Criterion BIC

## » Selection of the model:

- › Divide data in training set and test set
- › Calculate expectation for models on the shortlist and combination of these models (i.e. sums of two models)
- › Compute forecast errors on the test data and select model according to:
  - › Mean absolute error (MAE) (preferred)
  - › Mean error (ME)
  - › Mean absolute scaled error (MASE)



The model that describes the test data set in the best way based on the training data is automatically selected

# Contact

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