Proseminar zu Lie-Gruppen

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25. Let $i : (0, 2\pi) \to \mathbb{R}^2$, $i(s) = (\sin(2s), \sin(s))$. Then $M' := i((0, 2\pi))$ is an immersive submanifold of $\mathbb{R}^2$ (the figure-eight manifold) with global chart $\varphi : M' \to (0, 2\pi)$, $(\sin(2s), \sin(s)) \mapsto s$.

    Let $f : S^1 \to \mathbb{R}^2$, $f(x_1, x_2) = (2x_1x_2, x_2)$. Show that although f is smooth and $f(S^1) = M'$, the induced map from S^1 to M' is not smooth (in fact, not even continuous).

26. Let $H := \mathbb{R}^2 \times \{0\} \subseteq \mathbb{R}^3$. Then H is a 2-dimensional submanifold of $\mathbb{R}^3$ that is a Lie subgroup of $(\mathbb{R}^3, +)$. Show that the charts $\varphi_y : (x, y, 0) \mapsto x$ ($y \in \mathbb{R}$) form an atlas on H for a new differentiable structure in which H is a 1-dimensional Lie subgroup of $(\mathbb{R}^3, +)$.

27. Let H be a subgroup of a Lie group G such that H can be endowed with two manifold structures H_1, H_2, both second countable, such that H_1 and H_2 are Lie subgroups of G. Show that $H_1 = H_2$.

28. Give an example that shows that the statement in the previous problem is false if the assumption of second countability is dropped.