Proseminar zu Lie-Gruppen

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WS2017/18

5. Determine the dimension of the Lie group $O(n)$ and find the Lie algebra $\mathfrak{o}(n) = T_I O(n)$.

   Hint: Consider the map $f : GL(n,\mathbb{R}) \to M_{sym}(n,\mathbb{R})$, $f(A) := AA^t - I$, where $M_{sym}(n,\mathbb{R})$ is the space of symmetric $n \times n$ matrices. Show that $Df(A)$ is surjective for every A and calculate $\ker(Df(I))$.

6. Let G be the set of all matrices of the form

$$A = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$

   Find a global chart for G and show that G with the usual matrix multiplication is a Lie group.

7. Let G be a Lie group and for $g \in G$ let $\mathrm{conj}_g := h \mapsto ghg^{-1}$. Show that

$$\begin{aligned} \phi : G &\to GL(T_eG) \\ g &\mapsto T_e\mathrm{conj}_g \end{aligned}$$

   is a Lie group homomorphism.

8. Show that the right-trivialization

$$\begin{aligned} R : G \times \mathfrak{g} &\to TG \\ (g, v) &\mapsto R^v(g) = T_eR_g(v) \end{aligned}$$

   is a diffeomorphism with inverse $v_g \mapsto (g, T_gR_{g^{-1}}(v))$.