Proseminar zu Lie-Gruppen

Michael Kunzinger

WS2017/18

9. Prepare a short presentation about power series in Banach algebras. In particular, discuss radius of convergence, identity theorem for power series, Neumann series, exponential map and logarithm.

10. Let $G := S^1 \times (\mathbb{R}^+, \cdot)$. Pick local coordinates (φ, x) (with φ the angle on S^1, $x \in \mathbb{R}^+$). Prove that the vector field $X := \frac{\partial}{\partial \varphi} + x\frac{\partial}{\partial x}$ is left-invariant.

11. Let G be the Lie group consisting of the matrices

$$A = \begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}$$

(cf. ex. 6). Show that $\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, x\frac{\partial}{\partial y} + \frac{\partial}{\partial z}\}$ forms a basis of $\mathfrak{X}_L(G)$.

12. Let G be the set of all matrices of the form

$$A = \begin{pmatrix} x & 0 & y \\ 0 & x & z \\ 0 & 0 & 1 \end{pmatrix}$$

($x, y, z \in \mathbb{R}$, $x > 0$). Verify that:

(a) With the differentiable structure induced by the global chart $A \mapsto (x, y, z)$, G is a Lie group.

(b) $\{x\frac{\partial}{\partial x}, x\frac{\partial}{\partial y}, x\frac{\partial}{\partial z}\}$ is a basis of $\mathfrak{X}_L(G)$.