Proseminar zu Lie-Gruppen

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17. Determine the Lie algebra $\mathfrak{g}$ of the Lie group G from problem 11 and show that $\exp^G : \mathfrak{g} \to G$ is a diffeomorphism.

18. Let G be a linear Lie group (i.e., a Lie group that is a subgroup of $GL(n, \mathbb{R})$).

    (a) Show that, for any $A \in G$, $\mathrm{Ad}(A) = B \mapsto ABA^{-1}$.

    (b) Verify that $\frac{d}{dt}\big|_0 \mathrm{Ad}(e^{tX})(Y) = \mathrm{ad}(X)(Y)$ and conclude from this that $\mathrm{ad}(X)(Y) = [X, Y]$.

19. Let M be the set of symmetric 2×2 matrices with two different eigenvalues.

    (a) Show that M is an open submanifold of the set of all symmetric matrices.

    (b) Call two elements A, B of M equivalent if there exists some invertible matrix T such that $B = TAT^{-1}$. Denoting by ρ this equivalence relation, show that M/ρ can be given the structure of a two-dimensional quotient manifold of M.

20. Let ρ be an equivalence relation on a manifold M such that $M' = M/\rho$ is a quotient manifold of M. Let $f : M \to M_1$ be an invariant of ρ and denote by $\tilde{f} : M' \to M_1$ the corresponding projection.

    (a) Show that if f is an immersion or submersion, then so is $\tilde{f}$.

    (b) In the situation of problem 19, determine the projections of the determinant and the trace map.