

**MR0220590 (36 #3644) 02.60****Esenin-Vol'pin, A. S.****The present state of the foundations of set theory. (Russian) 1964***Proc. Fourth All-Union Math. Congr. (Leningrad, 1961) (Russian), Vol. II pp. 64–72 Izdat. “Nauka”, Leningrad*

This is a short and non-formal review of main results in the foundations of set theory. Three systems of axioms are considered as basic: the simple theory of types, Zermelo-Fraenkel's and the new foundations of Quine. There is a short discussion of different kinds of additional axioms (founding, constructivity, existence of inaccessible cardinals, etc.). Systems  $\Sigma$  of Gödel and  $\Theta$  of Mostowski are also considered. The author mentions various results on relative consistency of all these systems.

The axioms of the set theory use the idea of actual infinity which involves impredicative definitions. Gödel's incompleteness theorem shows that solving the problem of absolute consistency of set theory is impossible within the framework of modern mathematics formalized in the set theory. The author proposes to consider a new theory (which he calls “genetic”) one of whose main features is the existence of two (or more) non-isomorphic series of natural numbers. This theory is based on the ultraintuitionist criticism of mathematics [the author, *Infinitistic methods* (Proc. Sympos. Foundations of Math., Warsaw, 1959), pp. 201–223, Pergamon, Oxford, 1961; [MR0147389 \(26 #4905\)](#)]. In this theory one can prove the consistency of ZF and the independence of the axiom of choice and the continuum hypothesis. It is possible also to prove the consistency of some strengthened forms of ZF (e.g., with the axiom of existence of one or many inaccessible cardinals). The author proved this in his papers [*Application of logic to science and technology* (Russian), pp. 22–118, Izdat. Akad. Nauk SSSR, Moscow, 1960; *Logical studies* (Russian), pp. 218–262, Izdat. Akad. Nauk SSSR, Moscow, 1959; [MR0125774 \(23 3071\)](#)]. He mentions that the formulation of his theory as given in the just cited two papers suffers a certain lack of clarity. At the time when the paper under review was written the author had succeeded in finding a “sufficiently exact” formulation of his views. It is published in brief in the first of the above-mentioned papers. “At any rate, remaining difficulties are not connected with the actual infinity in its traditional meaning”, states the author.

{Reviewer's remark: The author reported on the further development of his ideas at the International Congress of Mathematicians, Moscow, 1966.}

Reviewed by *B. M. Schein*