COMMENT ON ZINOVIEV'S PAPER

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To put first things first it should be noted — and the author does not — that more than 15 years ago, J. C. Shepherdson established that, in the natural formal systems for the *elementary* arithmetic of the ring of integers,

$$xyz = 0 \vee x^n + y^n \neq z^n, \ FGT_n \text{ for short}$$

can of course be stated locally, that is, for each fixed n, but cannot be proved for any n; cf. The Theory of Models, North Holland 1965 (Proc. Conference, Berkeley 1963). Evidently since for many $n > 2$, FGT_n *can* be proved by correct methods, Shepherdson's result shows that the systems he considers are not 'sufficient for examining ... FGT'. This should be compared with Z10, 1. 14 of Zinoviev's paper. Even if he has recognized the difference between global and local unprovability of FGT_n (that is for variable n, resp. for some fixed n), he shows no evidence of having recognized the significance of the difference; in particular, he has not pointed out a step in his purported proof which does not apply to some (suitable) fixed n.

Given Shepherdson's result, it is clear that the interest of Zinoviev's claim depends dramatically on the kind of *provability, that is, on the consequence relations* meant; more formally, on the relations which satisfy his axioms for the monadic and binary propositional relations T and $\vdash$ (validity and consequence resp.)

This interest is highly questionable, quite independently of the lack of precision in his presentation of the formal rules and possible errors in his formal manipulations. In the very first paragraph of the paper, its ideas are said to be understandable independently (of the author's papers and novels). Now, which commonly understood notion of *consequence* is supposed to satisfy the axioms for $\vdash$? (a) The model-theoretic notion is excluded, at least legalistically, by the business on p. Z2, l. 5 to 1.7 about 'linguistic definitions' since the 'source' of model theoretic consequence involves other things too, for example, realizations of a language. (b) Consequence in any particular formal system is excluded by A(3) or the related C(4) in para 1, 2. Incidentally, the axiom schemata at the beginning of his list on p. Z2 are familiar enough from modal logic if

'T x' replaced by '$\Box$x' and 'x $\vdash$ y' by '$\Box$(x $\rightarrow$ y)'.

Those familiar schemata have the property that they remain valid if '$\Box$' is dropped throughout. As a corollary: *if* one had a proof from such schemata of

$$\sim \Box[(\forall c, a, b) \sim (c^n = a^n + b^n)] \text{ (cf. p. Z12), for some } n > 2,$$

one would actually have *refuted* FGT_n.

In paragraph 1.3 on p. Z3 Zinoviev introduces novel rules, involving $\sim(x \vdash y)$ — speaking of his 'own point of view' which allows such 'negative' rules: though '$\sim(x \vdash y)$' occurs in the familiar schemata mentioned in (b) above too. The rules are quite odd, and seem to exclude any notion of consequence (which one would ordinarily wish to consider). For example, $\sim(x \vdash y)$ is asserted in para. 1.3(1) whenever y contains a variable which does not occur in x (hence also for provable sentences x). So why should T y ever hold when y contains a variable ?

To avoid misunderstanding, let me stress that I have not tried to locate a place in Z 11-15 where these odd rules have been used, if at all. Nor have I tried to settle the consistency of those rules: they are teratological, and hence neither their consequences nor their metamathematical status can be considered to be scientifically significant. On the other hand, if those rules have not been used, all that is left are inequalities like

$$(n + 4)^n > (n + 2)^n + (n + 1)^n \text{ on p. Z13,}$$

which are surely inadequate for settling anything about FGT.

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