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Convex Optimization

Exercise session 7

37. (Proximal-gradient algorithm) Let $f : \mathbb{R}^n \to \overline{\mathbb{R}}$ be a proper, convex and lower semicontinuous function and $g : \mathbb{R}^n \to \mathbb{R}$ a convex and differentiable function with $\frac{1}{\beta}$-Lipschitz continuous gradient. To solve the convex optimization problem

$$\inf_{x \in \mathbb{R}^n} \{f(x) + g(x)\},$$

   (i) implement the proximal-gradient algorithm with relaxation variable $\lambda_k = 1, k \geq 0$, and constant stepsize β;

   (ii) implement the accelerated proximal-gradient algorithm (FISTA) with $\alpha_k = \frac{t_k - 1}{t_{k+1}}, k \geq 1$, where $t_1 = 1$ and $t_{k+1} = \frac{1+\sqrt{1+4t_k^2}}{2}, k \geq 1$;

   (iii) implement the accelerated proximal-gradient algorithm (FISTA) with $\alpha_k = \frac{t_k - 1}{t_{k+1}}, k \geq 1$, where $t_1 = 1$ and $t_k = \frac{k+a-1}{a}, k \geq 1$, by considering different values for $a > 3$.

   Test the algorithms for

$$n = 50, \quad f(x) = \|x\|_1, \quad g(x) = ^1\left(\frac{1}{5}\|\cdot\|\right)(x) = \begin{cases} \frac{1}{2}\|x\|^2, & \text{if } \|x\| \leq \frac{1}{5} \\ \frac{1}{5}\|x\| - \frac{1}{50}, & \text{otherwise,} \end{cases}$$

   by using as stopping criterion $(f + g)(x_k) - (f + g)(x^*) \leq 10^{-3}$, where x^* denotes the unique minimizer of $f + g$.

   For all three algorithms display the objective function values $((f + g)(x^k) - (f + g)(x^*), k = 0, 1, 2, ...)$ as well as the theoretical upper bounds for $(f + g)(x^k) - (f + g)(x^*)$ as functions of the number of iterations k (in a single plot).

   For all three algorithms display the RMSE ($\frac{1}{\sqrt{50}}\|x_k - x^*\|, k = 0, 1, 2, ...$) as functions of the number of iterations k (in a single plot).

   Send the generated files by July 2, 2019 at `radu.bot@univie.ac.at`.

38. (MAP versus DR) Implement the method of alternating projections and the Douglas-Rachford algorithm for determining an element in the intersection of two sets. Apply the algorithms to find an element in the intersection of the sets

$$S = \mathbb{R}^2_+ \text{ and } T = \{(u, v) \in \mathbb{R}^2 : u + 5v = 6\}.$$

   (i) Use $d_T(x_k) \leq 10^{-4}$ as stopping criterion for the method of alternating projections.

(ii) Use $d_S(P_T(x_k)) \leq 10^{-4}$ as stopping criterion for the Douglas-Rachford algorithm.

(iii) Use $x_0 \in \{(u_0, v_0) \in \mathbb{Z} \times \mathbb{Z} : u_0 \in [0, 100], v_0 \in [-100, 0]\}$ as starting points for both algorithms.

Plot for both algorithms the number of iterates needed to satisfy the stopping criteria.

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