Univ.-Prof. Dr. Radu Ioan Boţ

Vienna, October 31, 2019
To be discussed on November 8, 2019

Nonlinear Optimization

Exercise session 3

13. (a) Solve

$$\begin{array}{ll} \min & -x_1 - 2x_2. \\ \text{s.t.} & x_1^2 + x_2^2 \leq 4 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

(b) Verfiy if $x = (2,4)^T$ is an optimal solution of the optimization problem

$$\begin{array}{ll} \min & (x_1 - 4)^2 + (x_2 - 3)^2. \\ \text{s.t.} & x_1^2 \leq x_2 \\ & x_2 \leq 4 \end{array}$$

Determine a KKT point of this optimization problem.

(3 points)

14. Solve the following optimization problem

$$\begin{array}{ll} \min & \sum_{i=1}^n (x_i - a_i)^2. \\ \text{s.t.} & \sum_{i=1}^n x_i^2 \leq 1 \\ & \sum_{i=1}^n x_i = 0 \end{array}$$

(3 points)

15. Let be the function

$$f : \mathbb{R}^2 \to \mathbb{R}, f(x) = 3x_1^4 - 4x_1^2 x_2 + x_2^2.$$

Prove that the following statements are true:

(a) $x^* = (0,0)^T$ is a critical point of f;

(b) $x^* = (0,0)^T$ is a strict local minimum of f along any line going through the origin;

(c) $x^* = (0,0)^T$ is not a local minimum of f.

(3 points)

16. (a) Formulate a statement concerning the solutions of the optimization problem

$$\begin{array}{ll} \max & x_1 \\ \text{u.d.N.} & x_1^2 + x_2^2 \leq 1 \\ & (x_1 - 1)^2 + x_2^2 \geq 1 \\ & x_1 + x_2 \geq 1 \end{array}$$

by using geometric arguments and verify this statement by means of analytical arguments.

(b) Verfiy if $x^* = (1,1)^T$ fulfills the constraint qualifications (LICQ), (MFCQ) and (ABADIE-CQ) for the optimization problem

$$\begin{array}{rl} \min & x_1. \\ \text{s.t.} & x_1 + x_2 - 2 \leq 0 \\ & x_1 x_2 \geq 1 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

(3 points)

17. Find out, by usig second order optimality conditions, if $x^* = (0,0)^T$ is a local minimum of

$$\begin{array}{rl} \min & -x_1^2 + x_2, \\ \text{s.t.} & -x_1^3 + x_2 \geq 0 \\ & -m x_1 + x_2 \leq 0 \end{array}$$

where $m > 0$. (3 points)

18. Let $\{t_k\}_{k \geq 0} \subseteq \mathbb{R}$ be a monotonically decreasing sequence and t^* an accumulation point of it. Show that the whole sequence $\{t_k\}_{k \geq 0}$ converges to t^*. (2 points)

19. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a twice continuously differentiable function and $x^0 \in \mathbb{R}^n$, such that the lower level set $\mathcal{L}(x^0) := \{z \in \mathbb{R}^n : f(z) \leq f(x^0)\}$ is convex and bounded. Prove that ∇f is Lipschitz continuous on $\mathcal{L}(x^0)$. (3 points)