The program for Generalized Bäcklund-Darboux transformation (GBDT)

The transformed via GBDT solution is expressed in terms of matrix functions Π_1, Π_2^* and S, or, more precisely, via matrix-function $\Pi_2^* S^{-1} \Pi_1$. We differentiate Π_1 and Π_2^* with respect to x using (7.16) and differentiate S using (7.17). Here and further we refer to formulas from the book "Inverse Problems and Nonlinear Evolution Equations. Solutions, Darboux Matrices and Weyl-Titchmarsh Functions" (Studies in Mathematics, Vol. 47, De Gruyter, Berlin, 2013) by A. L. Sakhnovich, L. A. Sakhnovich and I. Ya. Roitberg. We also use the direct analogs of (7.16) and (7.17) in order to differentiate Π_1, Π_2^* and S with respect to t. We substitute $S^{-1}A_1 + \dots$ instead of $A_2 S^{-1}$ using the matrix identity (7.26). We use the mentioned above formulas (or their reductions) for the important case when the auxiliary linear systems do not have poles with respect to the spectral parameter z. The definitions for performing the differentiation are all gathered inside the module **noPole** and can be invoked by evaluating the command **noPole**.

In the case where auxiliary systems have poles with respect to the spectral parameter, we use formulas (7.53), (7.54) and (7.55) for the differentiation with respect to x and formulas (7.68), (7.69) and (7.70) for the differentiation with respect to t. The corresponding definitions are contained in **multiPole**.

In our program, we use the following notations:
`tau` for the function τ, `tildetau` for $\tilde{\tau}$, `P1` for Π_1, `P2star` for Π_2^* and `q[n_]:=f[tau, n]`, `Q[n_]:=F[tau, n]` for the coefficients at the nth power of the spectral parameter. The module for converting the given expression into LaTeXconverts `P1` into P_1 and `P2star` into P_2^*. The notations for the transformed coefficients $\tilde{q}_n$ and $\tilde{Q}_n$ inside the program are `tildeq[n]` and `tildeQ[n]`, respectively. In order to preserve the initial structure, the transformed coefficients $\tilde{q}_n$ and $\tilde{Q}_n$ should be equal to `qtt[n_]:=f[tildetau, n]`, `Qtt[n_]:=F[tildetau, n]`, respectively. We check these equalities using the program.

The program was tested by checking the fulfillment of formula (7.61) (see module `test761`). The program was applied to various equations. For each equation all necessary definitions are gathered in the corresponding module and can be invoked by evaluating a single command.

Our program was successfully run in the cases of coupled Schrödinger equation (see module `Schrodinger`) and coupled modified Korteveg–de Vries equation (see `KdV`). The symbolic computations provided by the program show that GBDT is applicable in those cases.

On the other hand, GBDT in the present form is not applicable to the derivative nonlinear Schrödinger equation (see, e.g., the modules `DNLSGI` and `KaupNewell`).

Running the program

To run the program, NCAlgebra is required.

- Install NCAlgebra following the instructions at `http://math.ucsd.edu/~ncalg/`
- Open BDT.nb in Mathematica.
- Use Evaluation ▷ Evaluate Notebook menu item.
- Use File ▷ New ▷ Notebook or Ctrl+N to begin your computations using the program from the clear sheet.

- In case of conflicting definitions or other errors, use menu item Evaluation▷ Quit Kernel and evaluate the BDT.nb with Evaluation▷ Evaluate Notebook.
- Define the required constants and rules manually or using the predefined modules (e.g., evaluate the commands KaupNewell; noPole;).
- Use result=c7e[expression] to obtain the result of iterative application of the defined formulas to the expression.
- Use resultWindow[result] in order to change the obtained expression into LaTeX format.