# A DUAL FORM OF ERDÖS-RADO'S CANONIZATION THEOREM

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This reports about a joint work with S.G. Simpson (Pennsylvania State University) and B. Voigt (Universität Bielefeld)

In [1] , Carlson and Simpson proved a theorem, which is, in a certain sense, a dual form of Ramsey's theorem. Moreover, their result can be viewed as an infinite generalization of the Graham-Rothschild partition theorem for n-parameter sets [3]. A canonizing version of the Graham-Rothschild theorem has been given in [5], extending the original partition theorem for n-parameter sets much in the same way as the Erdös-Rado canonization theorem [2] extends Ramsey's theorem.

The purpose of our work is to establish a canonizing version of the Carlson-Simpson result. This can be regarded as a dual form of the Erdös-Rado canonization theorem.

As corollaries, we obtain results which also are of interest in their own sake, e.g.,

Theorem A Let $P(\omega)$ be the powerset lattice of $\omega$ , topologized as $2^{\omega}$ (Cantor-space). Let $\pi$ be a Baire-partition on $P(\omega)$ . Then there exists a $P(\omega)$-sublattice $L \subseteq P(\omega)$ such that eigher $X \approx Y \bmod \pi$ for all $X,Y \in L$ or no two different elements from $L$ are equivalent modulo $\pi$ .

Corollary Let $P(\omega)$ we topologized as before and let $\Delta : P(\omega) \to \omega$ be a Baire-measurable mapping. Then there exists a $P(\omega)$ - sublattice $L \subseteq P(\omega)$ such that $\Delta \restriction L$ is a constant mapping.

Theorem B  Let $\pi$ be a Baire-partition on $\mathbb{R}$, the set of real numbers. Then there exists a sequence $(a_i)_{i\in\omega}$ of positive real numbers with $\Sigma_{i\in\omega} a_i \leq 1$ such that one of the following three possibilities holds for all nonempty subsets $I,J \subseteq \omega$ :

(1) $\Sigma_{i\in I}\, a_i \approx \Sigma_{j\in J}\, a_j \pmod{\pi}$

(2) $\Sigma_{i\in I}\, a_i \approx \Sigma_{j\in J}\, a_j \pmod{\pi}$ iff $\min I = \min J$

(3) $\Sigma_{i\in I}\, a_i \approx \Sigma_{j\in J}\, a_j \pmod{\pi}$ iff $I = J$ .

Recall that Hindman's theorem on finite sums [4] asserts that for every partition of $\omega$ into finitely many sets, $\omega = \bigcup_{j<r} C_j$ , there exist positive integers $(a_i)_{i\in\omega}$ such that all finite sums (without repetition) of the $a_i$'s belong to the same $C_j$ .

A canonizing version of Hindman's theorem has been established by Taylor [6] . He showed that for every mapping $\Delta : \omega\to\omega$ there exists positive integers $(a_i)_{i\in\omega}$ such that one of the following five cases holds for all finite and non-empty subsets $I,J \subseteq \omega$ :

(1) $\Delta(\Sigma_{i\in I}\, a_i) = \Delta(\Sigma_{j\in J}\, a_j)$

(2) $\Delta(\Sigma_{i\in I}\, a_i) = \Delta(\Sigma_{j\in J}\, a_j)$ iff $\min I = \min J$

(3) $\Delta(\Sigma_{i\in I}\, a_i) = \Delta(\Sigma_{j\in J}\, a_j)$ iff $I = J$

(4) $\Delta(\Sigma_{i\in I}\, a_i) = \Delta(\Sigma_{j\in J}\, a_j)$ iff $\max I = \max J$

(5) $\Delta(\Sigma_{i\in I}\, a_i) = \Delta(\Sigma_{j\in J}\, a_j)$ iff $\min I = \min J$ and $\max I = \max J$ .

As one easily observes, under the circumstances of Taylor's result, none of the five patterns can be omitted. Theorem B shows that, with respect to Baire-measurable mappings, a stronger result requires only three different patterns. And in fact, (2) cannot be omitted. Consider, e.g., the mapping $\Delta : ]0,1[\to$ with $\Delta(a) = i$ iff $i$ is minimal satisfying $2^i \cdot a \geq 1$ .

The requirement that $\Delta$ be Baire-measurable is necessary. Using the axiom of choive, theorem B fails if arbitrary mappings are allowed.

Details and proofs will appear elsewhere.

## References


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