CANONICAL FORMS OF BOREL MEASURABLE MAPPINGS

$\Delta : [\omega]^{\omega} \to \mathbb{R}$

H.J. Prömel, B. Voigt

Bielefeld

Basically, this talk reports the main result from [5].

A well-known result of Kuratowski says that for every Baire mapping $\Delta : X \to Y$ between separable metric spaces there exists a meager set M such that the restriction $\Delta \upharpoonright X \setminus M$ is continuous.

Here we investigate the metric space $[\omega]^{\omega}$ of infinite subsets of ω, endowed with the usual Tychonoff product topology, cf., [2].
From Louveau and Simpson [3] it follows that for every Borel measurable mapping $\Delta : [\omega]^{\omega} \to X$, where X is a metric space, there exists an $A \in [\omega]^{\omega}$ such that the restriction $\Delta \upharpoonright [A]^{\omega}$ is continuous.

But this is not yet the end of the story. We show that for every continuous mapping $\Delta : [\omega]^{\omega} \to X$ there exists an $A \in [\omega]^{\omega}$ and there exists a continuous mapping $\Gamma : [A]^{\omega} \to [A]^{\leq\omega}$ with $\Gamma(B) \subseteq B$ such that for all $B, C \in [A]^{\omega}$ it follows that $\Delta(B) = \Delta(C)$ iff $\Gamma(B) = \Gamma(C)$. So, the image $\Delta(B)$ is determined by a subset of B, viz., $\Gamma(B)$.

In a sense, this generalizes the Erdös/Rado canonization theorem [1]. Also, this extends a result of Pudlak and Rödl [4], which is the particular case dealing with continuous mappings $\Delta : [\omega]^{\omega} \to \mathbb{N}$.

Additionally, we show that Γ is determined by a mapping $\gamma : [A]^{<\omega} \to \{0,1\}$ in such a way that $\Gamma(B) = \{k \in B \mid \gamma(B \cap k) = 1\}$ for all $B \in [A]^{\omega}$.

References

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[3] A. Louveau, S.G. Simpson, *A separable image theorem for Ramsey mappings*, Bull. Acad. Polonaise de Sci., 30 (1982), 105-108.

[4] P. Pudlak, V. Rödl, *Partition theorems for systems of finite subsets of integers*. Disc. Math 39 (1982), 67-73.

[5] H.J. Prömel, B. Voigt, *Canonical forms of Borel measurable mappings* $\Delta : [\omega]^{\omega} \to \mathbb{R}$, to appear in J. of Combin. Th. (A).