DISSECTIONS OF CONVEX SETS AND HOMOTOPY TYPES OF POSETS

by

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Let K be a convex set in $\mathbb{R}^d$, $\mathrm{int}(K) \neq \emptyset$. If we remove a finite family of affine hyperplanes from $\mathbb{R}^d$, we disconnect K into a family of convex parts, which is called a <u>topological dissection of K by an arrangement of hyperplanes</u>.

We study dissections from the point of view of the homological theory of posets and Möbius functions: we associate to a dissection a partially ordered set - the poset of regions and faces - and determine the homotopy types of the classifying spaces of this posets and of its intervals. By this technique, we succeed in computing the Möbius funtion of the poset of regions and faces, which turns out to take only values 0,1,-1; this gives, as an immediate consequence, generalizations of the Euler relation. In the case when the convex set is the whole space, the Euler relation for faces and for bounded faces are nothing but the upper and the lower recursion for the Möbius function, respectively. Finally, we describe connections between our approach and that based on the notion of cut-intersection poset due to T.Zaslavsky.