Genocchi Numbers and f-Vectors of Simplicial Balls

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Outline

1. f-Vectors of Simplicial Balls
2. Genocchi Numbers
3. Genocchi Numbers and f-Vectors of Simplicial Balls

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1 f-Vectors of Simplicial Balls

2 Genocchi Numbers

3 Genocchi Numbers and f-Vectors of Simplicial Balls

Simplicial Balls

- B an $(n-1)$-dimensional simplicial ball
- i.e. simplicial complex homeomorphic to a ball
- $f_i(B)$ number of i-dimensional faces
- boundary ∂B of B is a simplicial sphere with face numbers $f_i(\partial B)$
- interior $\operatorname{int} B$ of B (not a simplicial complex)
- $f_i(\operatorname{int} B) := f_i(B) - f_i(\partial B)$
- relation beetween $f(\partial B)$ and $f(\operatorname{int} B)$

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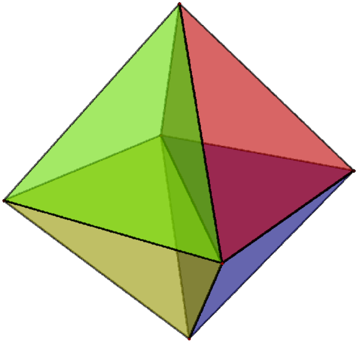
Example: Triangulated Polytopes

- polytope P: convex hull of finitely many points
- triangulation T: collection of simplices whose union is P
- relation between triangulation of the boundary and T



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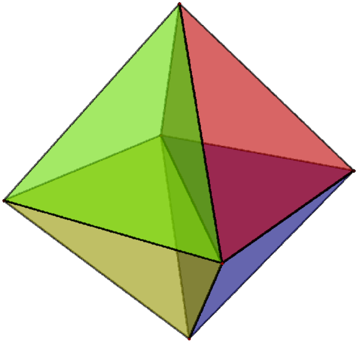

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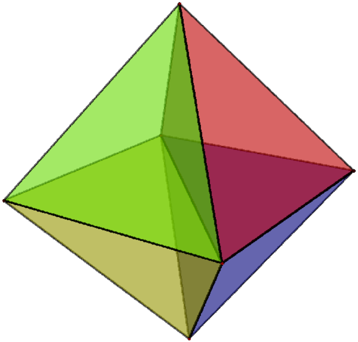

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- simplicial polytope (McMullen 2004)
  - all faces of P are simplices
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Dehn-Sommerville Relations

- $h_k = \sum_{i=0}^{k} (-1)^{k-i} \binom{n+1-i}{n+1-k} f_i$, $g_0 = 1$ and $g_k = h_k - h_{k-1}$ for $k \geq 1$

Theorem (Dehn-Sommerville)

S simplicial $(n-1)$-sphere

$$h_k(S) = h_{n-k}(S) \quad .$$

Theorem (McMullen & Walkup 1971)

B simplicial $(n-1)$-ball

$$g_k(\partial B) = h_k(B) - h_{n-k}(B) \quad .$$

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Relations for the f-Vector

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Genocchi Numbers

Definition

The Genocchi numbers are defined by means of the generating function

$$\frac{2t}{e^t+1} =: \sum_{n=0}^{\infty} G_n \frac{t^n}{n!} = t + \sum_{n=1}^{\infty} G_{2n} \frac{t^{2n}}{(2n)!} \quad .$$

- The first few numbers are
- -1, 1, -3, 17, -155, 2073, -38227, 929569, -28820619, 1109652905
- One has

$$G_n = 2(1-2^{2n})B_n,$$

where B_n are the Bernoulli numbers.

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- investigation goes back to Leonhard Euler
- intensively studies by Eric T. Bell (1926, 1929)
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Combinatorical Interpretation

- combinatorial interpretation due to Dominque Dumont (1974):
- absolute value of G_{2n+2} equals the number of permutations τ of $\{1, 2, \ldots, 2n\}$ such that $\tau(i) > i$ if and only if i is odd.
- $n = 2$:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

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Recursion Formulae

- There are a lot of formulae to compute G_n

Proposition

$$G_{2n} = -n - \frac{1}{2}\sum_{k=1}^{n-1}\binom{2n}{2k}G_{2k} \quad \text{and} \quad G_{2n} = -1 - \sum_{k=1}^{n-1}\binom{2n}{2k-1}\frac{G_{2k}}{2k}$$

Lemma

For $n \geq 2$ we have

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Main Theorem

Theorem (H. 2007)

For each simplicial $(n-1)$*-ball* B *and* $n-k$ *even we have*

$$f_k(\operatorname{int} B) = \sum_{i=1}^{\lfloor \frac{n-k}{2} \rfloor} \frac{G_{2i}}{2i} \left(\binom{k+2i-1}{k+1} f_{k+2i-2}(\partial B) - \binom{k+2i}{k+1} f_{k+2i-1}(\operatorname{int} B) \right) .$$

- *f_k for $n-k$ odd can be computed from the even ones*

- $f_k(\operatorname{int} B)$ depending an $\lfloor (n-k)/2 \rfloor$ $f_i(\operatorname{int} B)$

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Simplicial Balls without Small Interior Faces

- B a simplical $(n-1)$-ball without interior faces of dimension $\leq e$
- $n-k$ even and $k \leq e$

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- $\lfloor (e+2)/2 \rfloor$ $\left(\lfloor (e+3)/2 \rfloor\right)$ equations
- $\lfloor (n-1-e)/2 \rfloor$ $(\left(\lfloor (n-e)/2 \rfloor\right)$ unknowns $f_k(\operatorname{int} B)$
- similar equations for the h-vector computed and used: H. & Joswig 2007

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- $n-k$ even and $k \leq e$

$$\sum_{i=1}^{\lfloor\frac{n-k}{2}\rfloor} \frac{G_{2i}}{2i}\binom{k+2i-1}{k+1} f_{k+2i-2}(\partial B) = \sum_{i=1}^{\lfloor\frac{n-k}{2}\rfloor} \frac{G_{2i}}{2i}\binom{k+2i}{k+1} f_{k+2i-1}(\operatorname{int} B)$$

- $\lfloor(e+2)/2\rfloor \; \big(\lfloor(e+3)/2\rfloor\big)$ equations
- $\lfloor(n-1-e)/2\rfloor \; (\big(\lfloor(n-e)/2\rfloor\big)$ unknowns $f_k(\operatorname{int} B)$
- similar equations for the h-vector computed and used: H. & Joswig 2007