Quasisymmetric functions and Young diagrams

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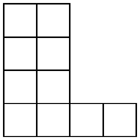
What is this talk about?

- Representations of symmetric groups

  irreducible representations
  $\simeq$ *Young diagrams*



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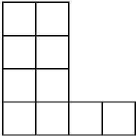

Partition $(4,2,2,2)$

- Kerov's and Olshanski's approach :

  Consider normalized character values $\chi^\lambda(\sigma) = \dfrac{\mathrm{tr}\left(\rho^\lambda(\sigma)\right)}{\dim(V_\lambda)}$
  as functions on Young diagrams $\lambda \mapsto \chi^\lambda(\sigma)$ *(*σ *fixed).*

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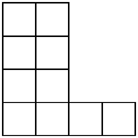

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  as functions on Young diagrams $\lambda \mapsto \chi^\lambda(\sigma)$ *(*σ *fixed).*

- *In this talk:* we explain that these functions live in the ring of quasisymmetric functions*.

  * *a natural extension of symmetric functions.*

Outline of the talk

1. Existing theory: symmetric functions on Young diagrams

2. An extension: quasisymmetric functions on Young diagrams

Kerov's and Olshanski's approach

Fix $\mu \vdash k$. Let us define

$$\mathrm{Ch}_\mu : \begin{array}{ccc} \mathcal{Y} & \to & \mathbb{Q}; \\ \lambda & \mapsto & n(n-1)\dots(n-k+1)\chi^\lambda(\sigma), \end{array}$$

where $n = |\lambda|$, $k = |\mu|$
and σ is a permutation in S_n of cycle type $\mu 1^{n-k}$.

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Examples:

$$\begin{aligned} \mathrm{Ch}_\mu(\lambda) &= 0 \quad \text{as soon as } |\lambda| < |\mu| \\ \mathrm{Ch}_{1^k}(\lambda) &= n(n-1)\dots(n-k+1) \quad \text{for any } \lambda \vdash n \\ \mathrm{Ch}_{(2)}(\lambda) &= n(n-1)\chi^\lambda\big((1\ 2)\big) = \sum_i (\lambda_i)^2 - (\lambda'_i)^2 \\ \mathrm{Ch}_{\mu\cup 1}(\lambda) &= (n-|\mu|)\,\mathrm{Ch}_\mu(\lambda) \quad \text{for any } \lambda \vdash n \end{aligned}$$

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Proposition (Kerov and Olshanski, 1994)

The functions Ch_μ, when μ runs over all partitions, are linearly independent. Moreover, they span a subalgebra $\Lambda^\star$ of functions on Young diagrams.

Example: $\mathrm{Ch}_{(2)} \cdot \mathrm{Ch}_{(2)} = 4 \cdot \mathrm{Ch}_{(3)} + \mathrm{Ch}_{(2,2)} + 2\,\mathrm{Ch}_{(1,1)}$.

Description of elements of $\Lambda^\star$

Theorem (Kerov and Olshanski, 1994)

Functions in $\Lambda^\star$ are exactly

- *polynomials in λ_1, λ_2,...*
- *which are symmetric in $\lambda_1 - 1$, $\lambda_2 - 2$, ...*

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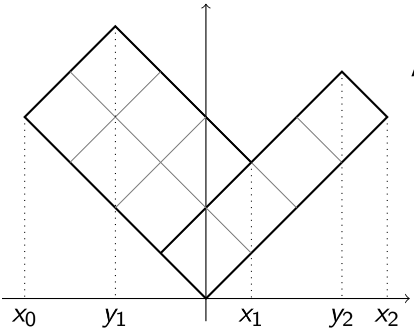
☹No geometric interpretation of the shifted symmetry.

Kerov's interlacing coordinates



Alternative description of Young diagrams

x-coordinates of lower corners

$x_0 = -4, \; x_1 = 1, \; x_2 = 4$

x-coordinates of higher corners

$y_1 = -2, \; y_2 = 3$

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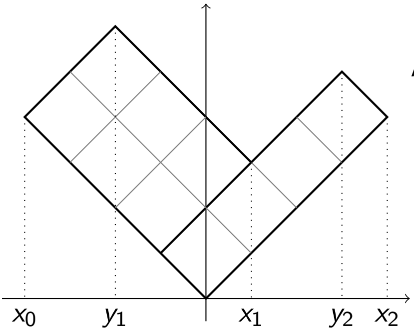

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Theorem (Kerov, 1999)

$\Lambda^\star$ *admits the following algebraic basis:*

$$\left(\lambda \mapsto \sum x_i^k - y_i^k\right)_{k \geqslant 2}.$$

λ-ring notation: $p_k(\mathbb{X} \ominus \mathbb{Y}) := \sum x_i^k - y_i^k.$

λ-rings in one slide

Reminder ($k \geqslant 1$): $p_k(\mathbb{X}) = \sum x_i^k$, $e_k(\mathbb{X}) = \sum_{i_1 < \cdots < i_k} x_{i_1} \cdots x_{i_k}$.

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Consider two (finite) alphabets $\mathbb{X}$ and $\mathbb{Y}$. We denote $\mathbb{X} \oplus \mathbb{Y}$ their union.

$$p_k(\mathbb{X} \oplus \mathbb{Y}) = p_k(\mathbb{X}) + p_k(\mathbb{Y}), \; e_k(\mathbb{X} \oplus \mathbb{Y}) = \sum_{i+j=k} e_i(\mathbb{X}) e_j(\mathbb{Y}).$$

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Empty alphabet is neutral. Imagine that $\mathbb{Y}$ has an inverse $\ominus\mathbb{Y}$.

$$p_k(\mathbb{Y} \ominus \mathbb{Y}) = p_k(\emptyset) = 0 \Rightarrow p_k(\ominus\mathbb{Y}) = -p_k(\mathbb{Y})$$

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$\ominus\mathbb{Y}$ does not exist but we can define $f(\ominus\mathbb{Y})$ for any symmetric function f!
(and it is compatible with multiplication $fg(\ominus\mathbb{Y}) = f(\ominus\mathbb{Y})g(\ominus\mathbb{Y})$).

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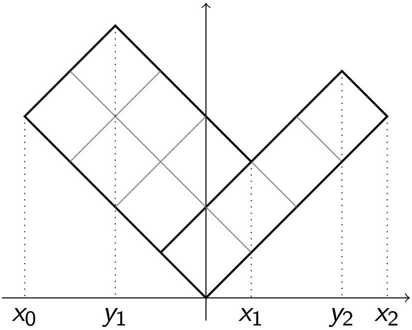
Finally $p_k(\mathbb{X} \ominus \mathbb{Y}) = p_k(\mathbb{X}) + p_k(\ominus\mathbb{Y}) = p_k(\mathbb{X}) - p_k(\mathbb{Y})$ as claimed.

Back to Kerov's interlacing coordinates



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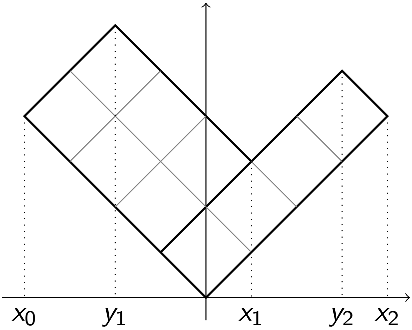

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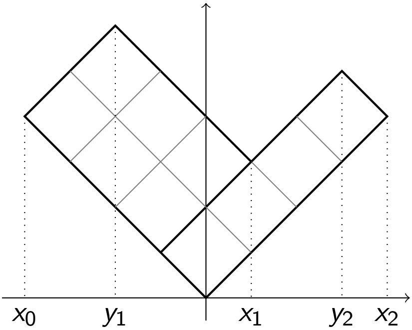

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🙂nice compact description.

🙁no geometric interpretation of the symmetry.

And what?

Theorem (F. 2006, conjectured by Stanley)

Let $\mu \vdash k$, $\pi \in S_k$ a permutation of type μ. For any Young diagram λ,

$$\mathrm{Ch}_\mu(\lambda) = \sum_{\substack{\sigma,\tau \in S_k \\ \sigma\tau=\pi}} \pm N_{\sigma,\tau}(\lambda),$$

for some nice functions $N_{\sigma,\tau}$ on all Young diagrams.

Here, nice means:

- has a combinatorial description;
- polynomial with respect to interlacing coordinates.

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☹ In general, $N_{\sigma,\tau} \notin \Lambda^\star$.

How to construct a bigger algebra?

2 approaches:

- Consider the algebra generated by the $N_{\sigma,\tau}$;
- Consider the algebra of nice functions (for some definition of nice).

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- Consider the algebra of nice functions (for some definition of nice).

In fact, both lead to the same algebra!

We will explain the second one.

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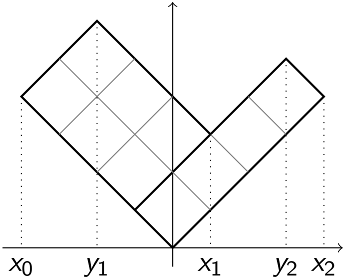

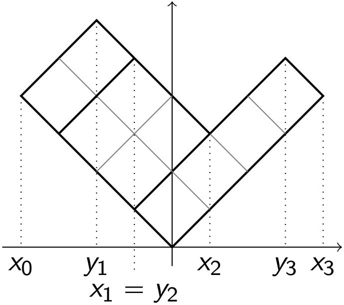

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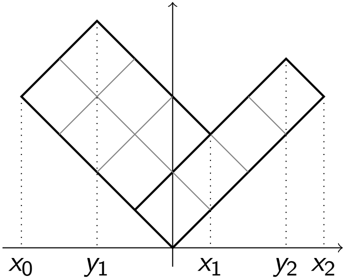

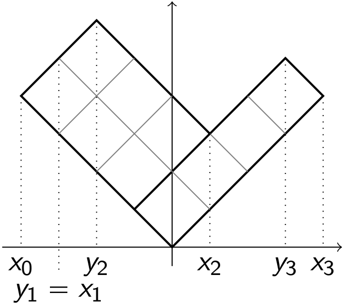

What is a nice function?

Question

Which polynomials (in infinitely many variables) fulfill

$$f(x_0, y_1, \dots, x_{i-1}, y_i, x_i, y_{i+1}, \dots)\Big|_{x_i = y_i} = f(x_0, y_1, \dots, x_{i-1}, y_{i+1}, \dots);$$

$$f(x_0, y_1, \dots, y_i, x_i, y_{i+1}, x_{i+1}, \dots)\Big|_{x_i = y_{i+1}} = f(x_0, y_1, \dots, y_i, x_{i+1}, \dots)?$$

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Answer

The algebra of solutions of the functional equation above is

$$\mathrm{QSym}((x_0) \ominus (y_1) \oplus (x_1) \ominus (y_2) \oplus (x_2) \dots)$$

QSym := quasisymmetric function ring (extension of symmetric function ring)

Explanation

Example of quasisymmetric function : $M_{1,2}(a_1, a_2, \dots) = \sum_{i<j} a_i a_j^2$.

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If $\mathbb{X}$, $\mathbb{Y}$ and $\mathbb{Z}$ are three lists of variables, denote $\mathbb{X} \oplus \mathbb{Y} \oplus \mathbb{Z}$ their concatenation.

$$\begin{aligned} M_{1,2}(\mathbb{X} \oplus \mathbb{Y} \oplus \mathbb{Z}) = {} & M_{1,2}(\mathbb{X}) + M_{1,2}(\mathbb{Y}) + M_{1,2}(\mathbb{Z}) \\ & + M_1(\mathbb{X}) M_2(\mathbb{Y}) + M_1(\mathbb{X}) M_2(\mathbb{Z}) + M_1(\mathbb{Y}) M_2(\mathbb{Z}). \end{aligned}$$

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Hence we define:

$$\begin{aligned} M_{1,2}((x_0) \ominus (y_1) \oplus (x_1)) = 0 + M_{1,2}(\ominus(y_1)) + 0 \\ + x_0 M_2(\ominus(y_1)) + x_0 x_1^2 + M_1((y_1))x_1^2 \end{aligned}$$

It remains to define $M_I(\ominus(y_1))$.

Explanation (2/2)

$M_I(\ominus(y_1))$ is computed as for symmetric functions.

$$M_1((y_1) \ominus (y_1)) = 0 = M_1(y_1) + M_1(\ominus(y_1)).$$

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Let now consider $M_{1,2}(\ominus(y_1)^{-1})$:

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Finally

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Rk: if we start from $M_{1,2}(\ominus(y_1) \oplus (y_1))$, we get the same result!

Back to the result

Theorem

The algebra of nice functions on Young diagrams is

$$\mathrm{QSym}((x_0) \ominus (y_1) \oplus (x_1) \ominus (y_2) \oplus (x_2) \dots)$$

Painful to compute, but

- easy to implement (there are explicit expression for $M_I(\ominus\mathbb{Y})$);
- it gives the algebraic structure of the space of nice functions ($\simeq$ QSym).

Conclusion

- There is a natural algebra of functions on Young diagrams which is isomorphic to QSym and contains Kerov's and Olshanski's algebra;
- helps to reprove some results.

Conclusion

- There is a natural algebra of functions on Young diagrams which is isomorphic to QSym and contains Kerov's and Olshanski's algebra;
- helps to reprove some results.
- Question: as

  symmetric functions on YD $\leftrightarrow$ *shifted symmetric functions*

  can we consider shifted quasisymmetric functions?
  Work in progress…