Parking functions in types C and B

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joint work with Marko Thiel

Universität Wien

Ellwangen, 23rd March 2015

Outline

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- Regions of the Shi arrangement

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- Finite torus (parking functions)

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$$H_{\alpha,k} = \{x \in V : \langle x, \alpha \rangle = k\}.$$

Let $s_{\alpha,k}$ be the reflection through $H_{\alpha,k}$.

$$s_{\alpha,k}(x) = x - \frac{2\langle x, \alpha \rangle - 2k}{\langle \alpha, \alpha \rangle}\alpha.$$

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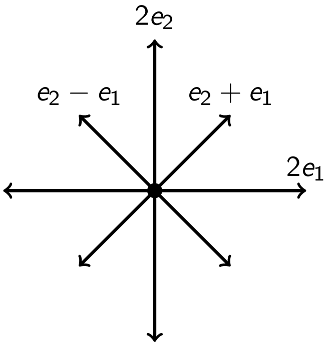

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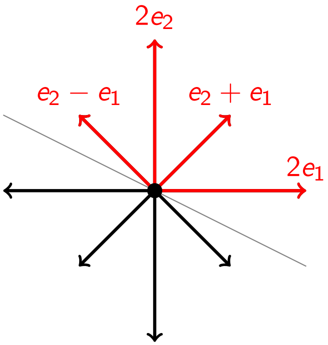

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We only consider irreducible root systems.

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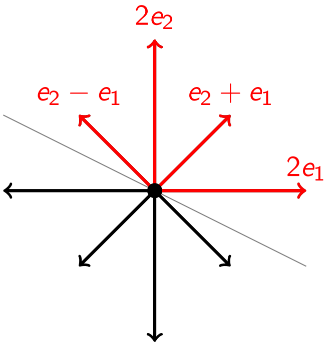

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The positive roots are given by

$$\Phi^+ = \{e_j \pm e_i : 1 \leq i < j \leq n\} \\ \cup \{2e_i : 1 \leq i \leq n\}.$$

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$$f = (f_1, f_2, \ldots, f_n)$$

with nonnegative entries such that there exists a permutation $\sigma \in \mathfrak{S}_n$ with

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$$\begin{aligned} \sigma \cdot f &= (0, 0, 1, 1, 4, 4, 4) \\ &\leq (0, 1, 2, 3, 4, 5, 6) \end{aligned}$$

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The set of classical parking functions of length n is a natural system of representatives for the finite torus of type A_{n-1}.

Recall that $\mathbb{Z}^n/(2n+1)\mathbb{Z}^n$ is the finite torus of type C_n.

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We define parking functions of type C as integer vectors $f = (f_1, f_2, \ldots, f_n)$ where $-n \leq f_i \leq n$.

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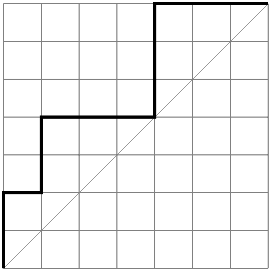

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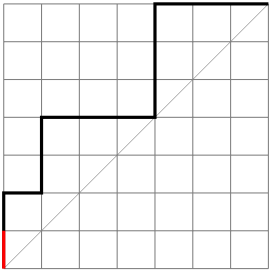

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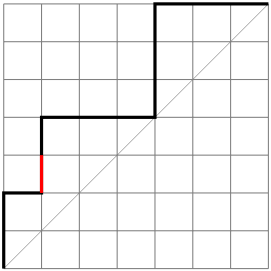

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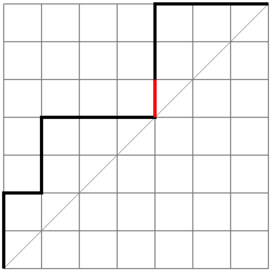

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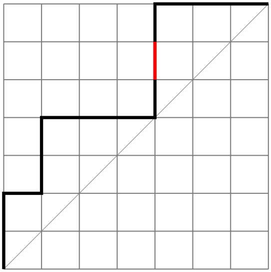

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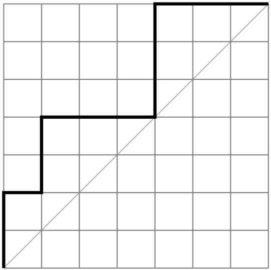

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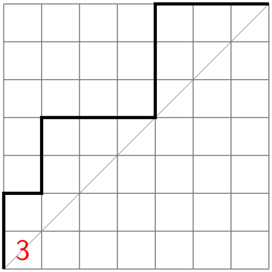

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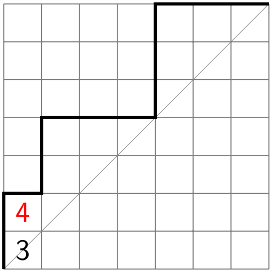

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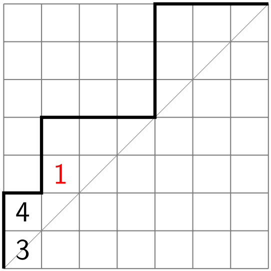

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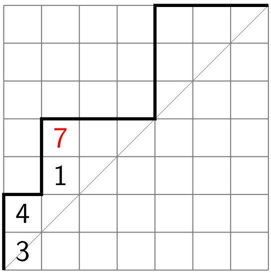

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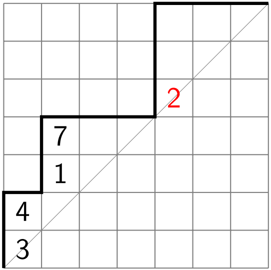

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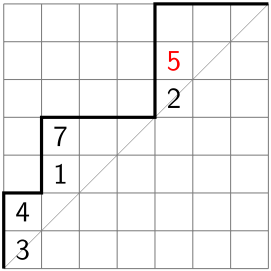

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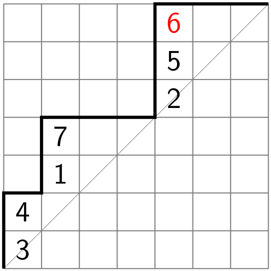

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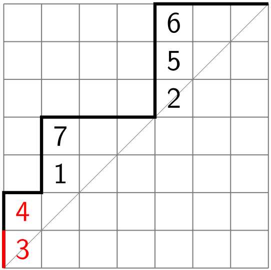

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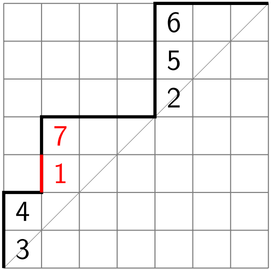

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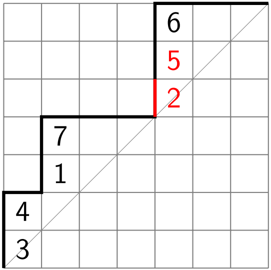

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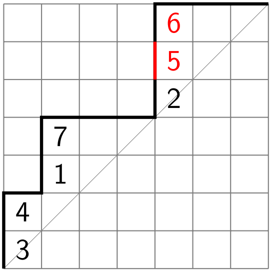

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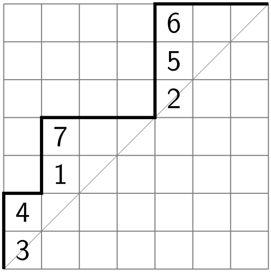

The rises of π are $i = 1, 3, 5, 6$. Let $\sigma = 3417256$. The pair (π, σ) is a vertical labelling.

From vertically labelled Dyck paths to parking functions

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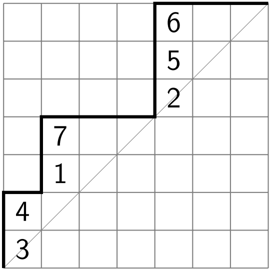

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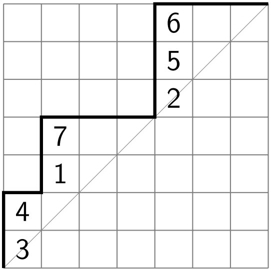

Classical parking functions

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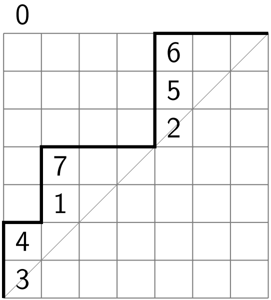

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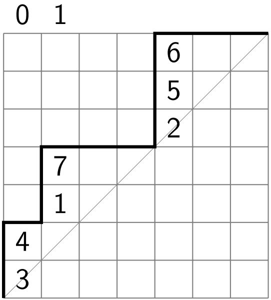

Classical parking functions

There is a natural way to construct the parking function corresponding to a vertically labelled Dyck path.

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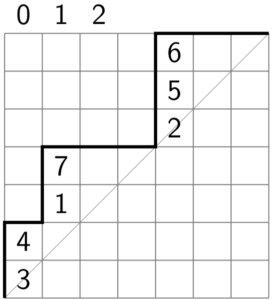

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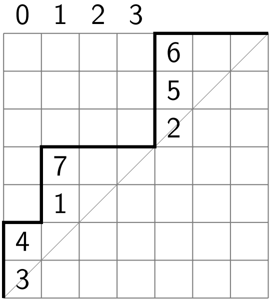

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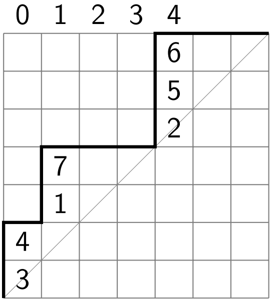

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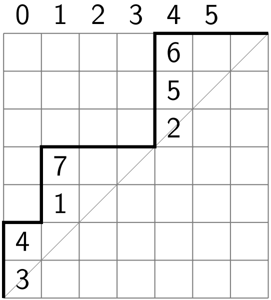

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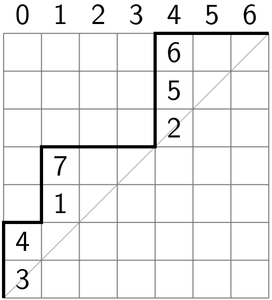

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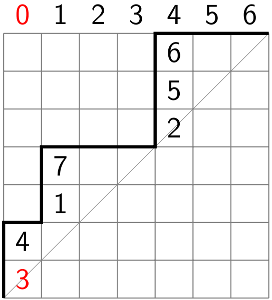

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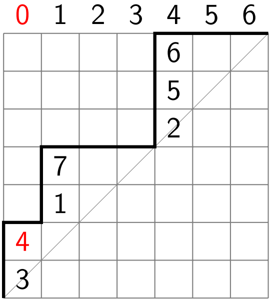

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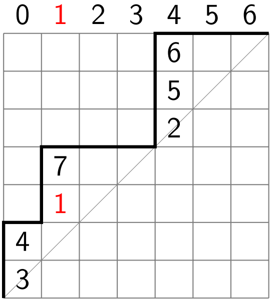

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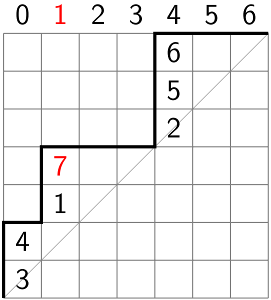

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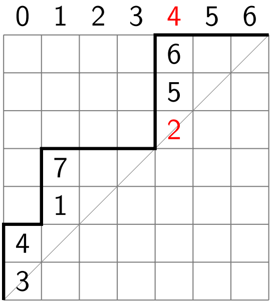

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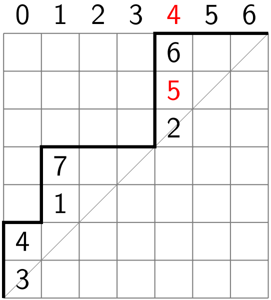

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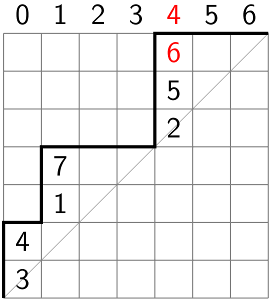

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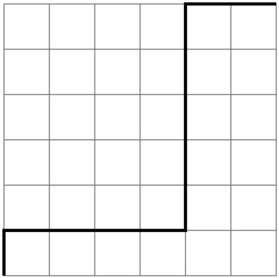

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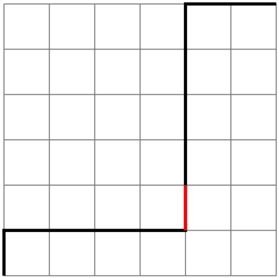

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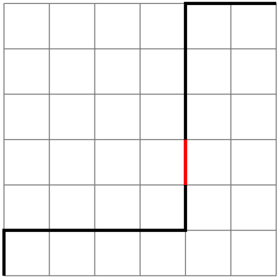

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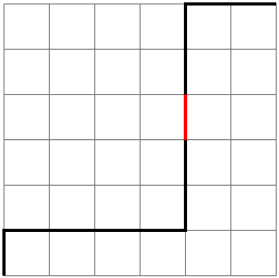

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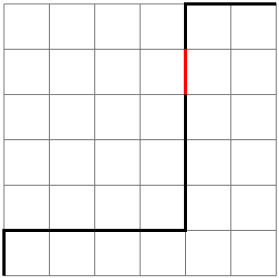

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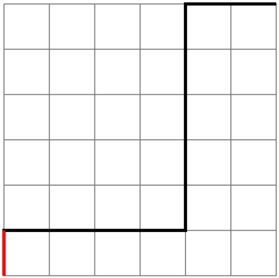

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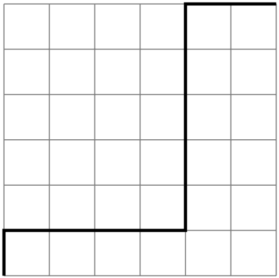

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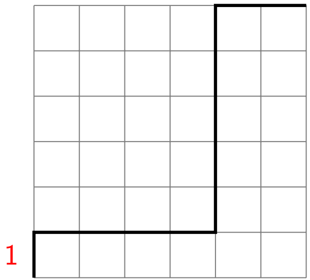

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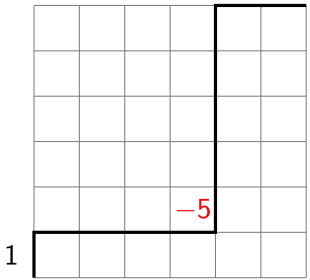

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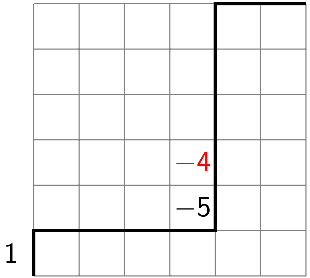

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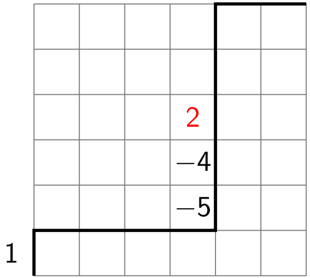

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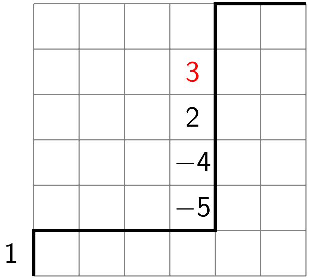

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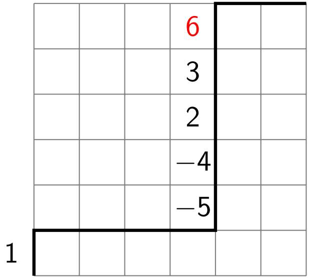

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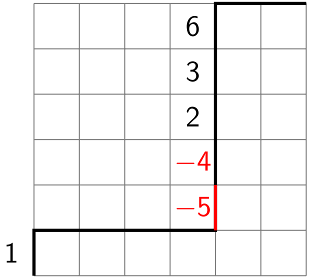

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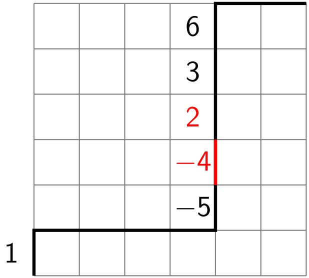

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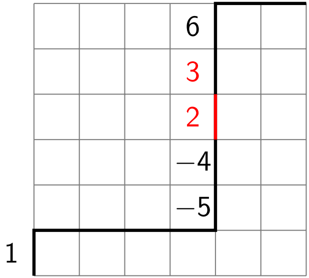

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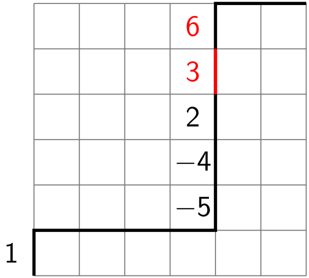

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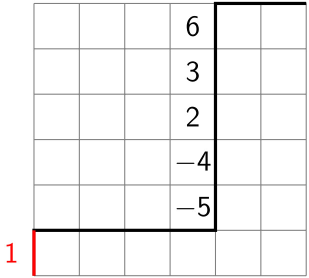

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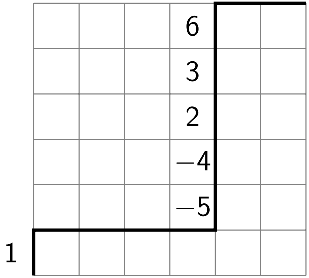

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Let $\sigma = 1(-5)(-4)236$. The pair (π, σ) is a vertical labelling.

From vertically labelled lattice paths to parking functions

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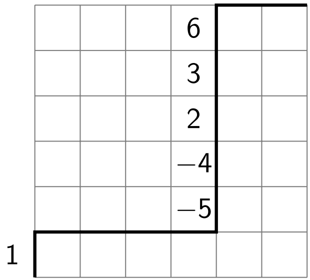
Vertically labelled lattice paths



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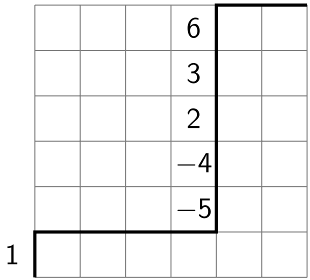

Type C parking functions

There is a natural bijection between type C parking functions and vertically labelled lattice paths.

$$f = (\; , \; , \; , \quad , \quad , \;)$$

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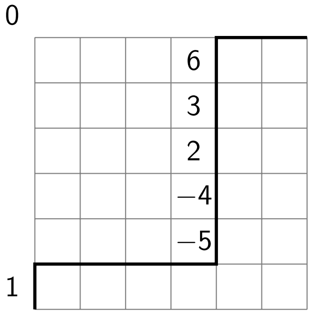
Vertically labelled lattice paths



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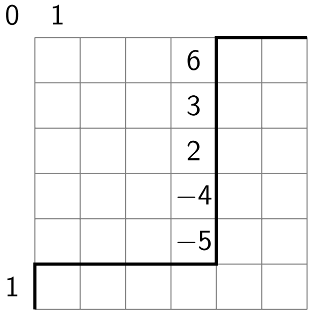

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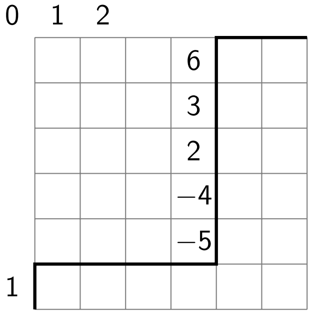

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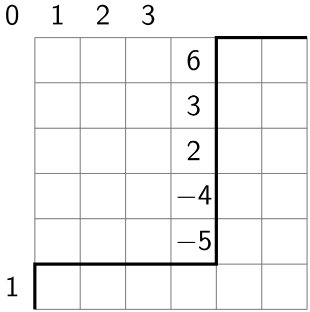

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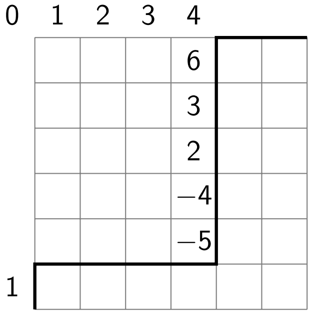
From vertically labelled lattice paths to parking functions

Vertically labelled lattice paths



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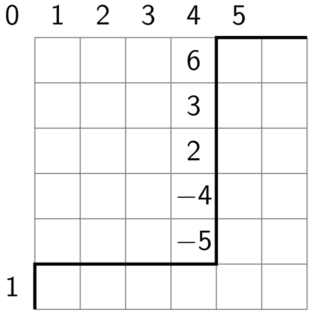

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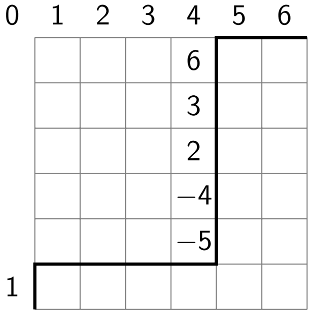

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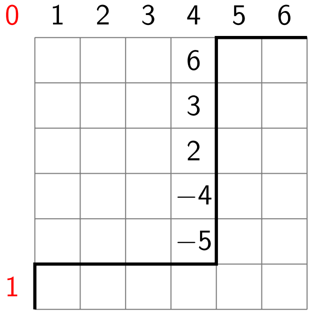

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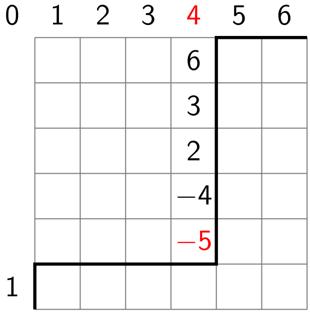

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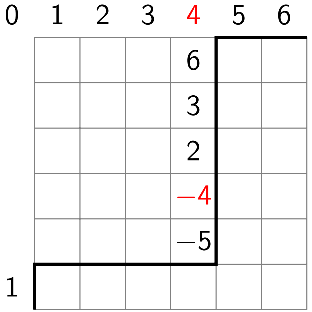

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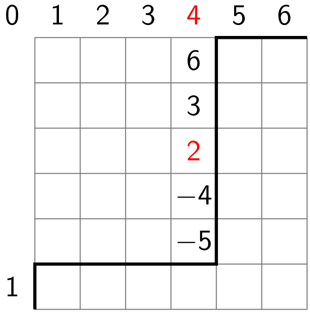

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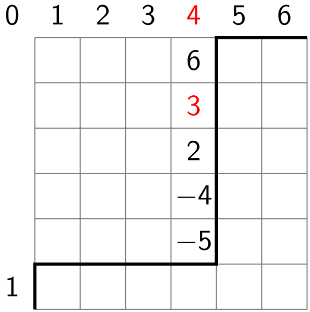

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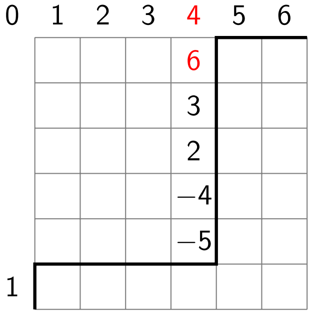

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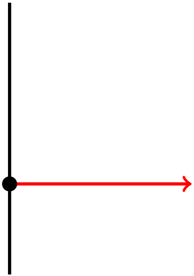

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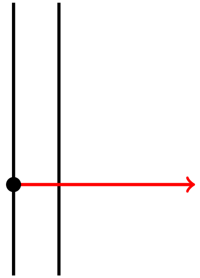

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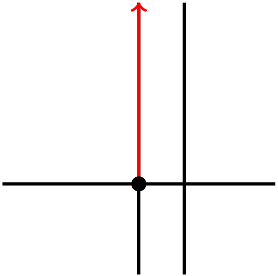

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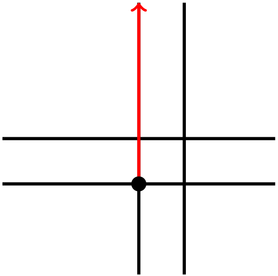

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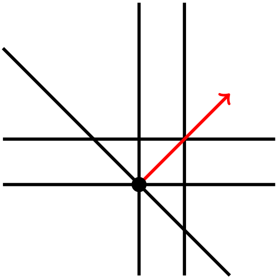

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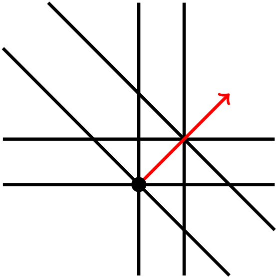

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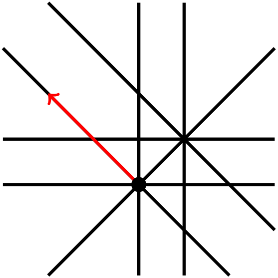

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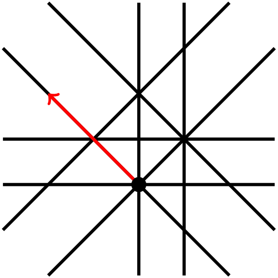

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The connected components of

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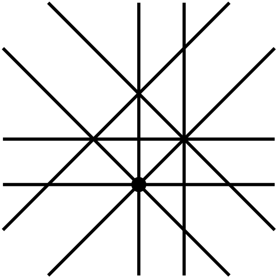

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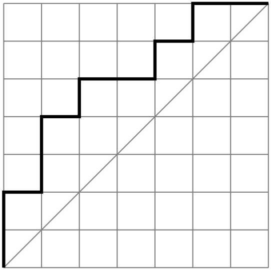

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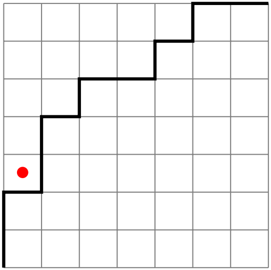

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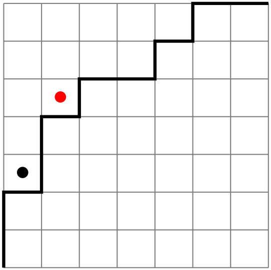

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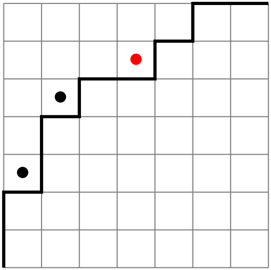

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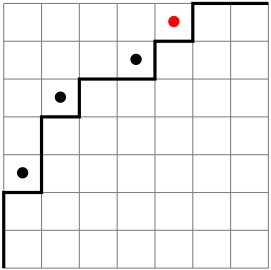

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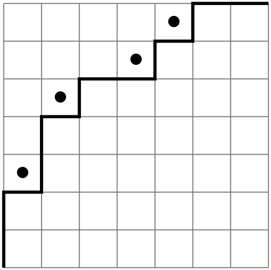

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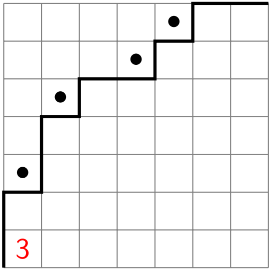

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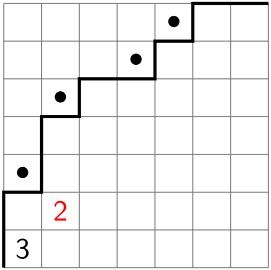

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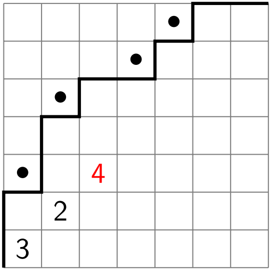

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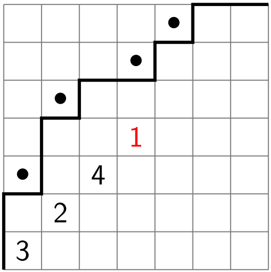

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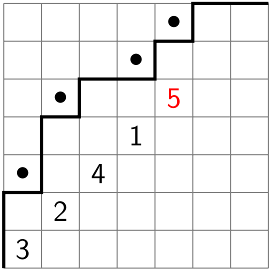

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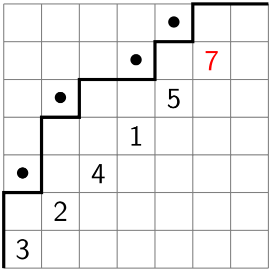

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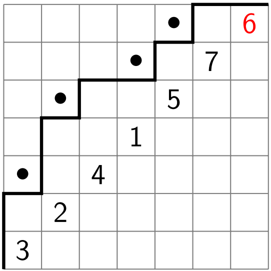

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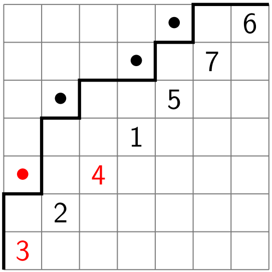

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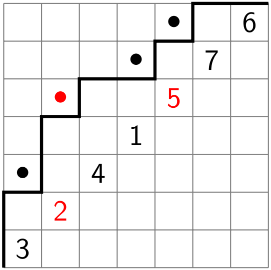

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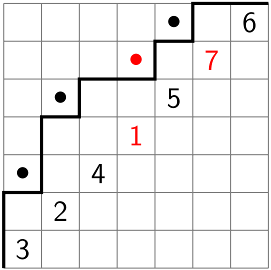

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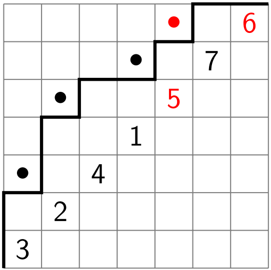

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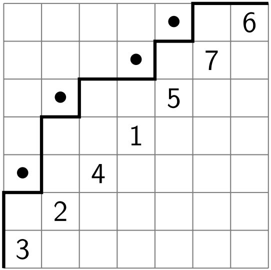
A diagonally labelled Dyck path is a pair (π, σ) of a Dyck path $\pi \in \mathcal{D}_n$ and a permutation $\sigma \in \mathfrak{S}_n$ such that

$$\sigma_i < \sigma_j$$

for each valley (i, j) of π.

We call (i, j) a valley if the i-th East step is followed by the j-th North step.

Example



The valleys of π are $(1, 3)$, $(2, 5)$, $(4, 6)$, $(5, 7)$.
Let $\sigma = 3241576$. The pair (π, σ) is a diagonal labelling.

Ballot paths

Ballot paths

Definition

A ballot path is a lattice path starting at $(0,0)$ consisting of $2n$ North and/or East steps that never goes below the main diagonal $x = y$.
Let $\mathcal{B}_n$ denote the set of such paths.

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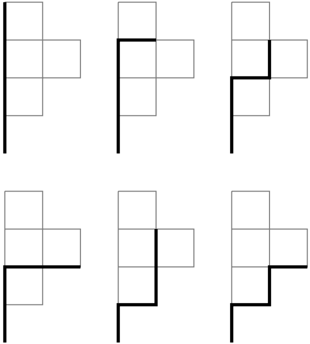

The paths in $\mathcal{B}_2$.

Diagonally labelled ballot paths

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Definition

A diagonally labelled ballot path is a pair (β, w^σ) of a ballot path $\beta \in \mathcal{B}_n$ and a word w^σ depending on a signed permutation $\sigma \in \mathfrak{H}_n$ such that

$$w_i^\sigma > w_j^\sigma$$

for each valley (i, j) of β, and such that

$$0 < w_i^\sigma$$

if β ends with its i-th East step.

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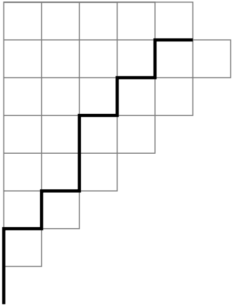

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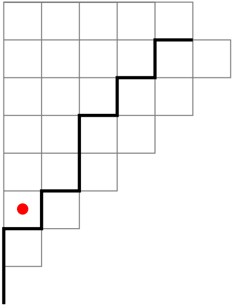

The valleys of β are $(1, 3)$, $(2, 4)$, $(3, 6)$, $(4, 7)$.

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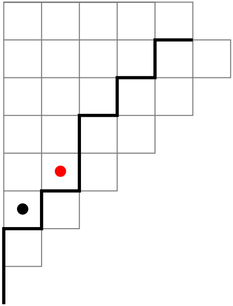

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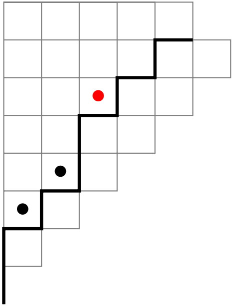

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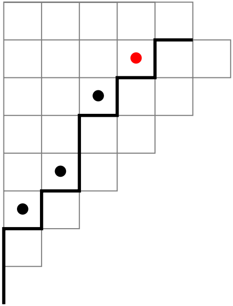

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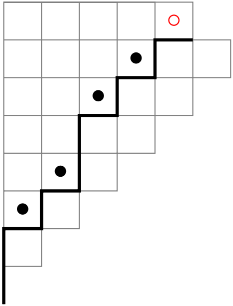

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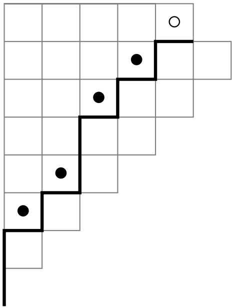

The valleys of β are $(1,3)$, $(2,4)$, $(3,6)$, $(4,7)$. Moreover, β ends with the fifth East step.
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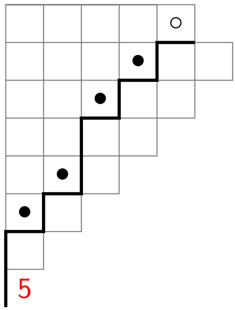

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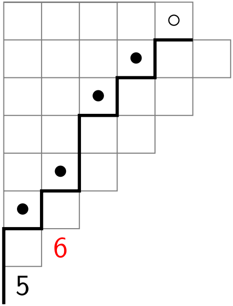

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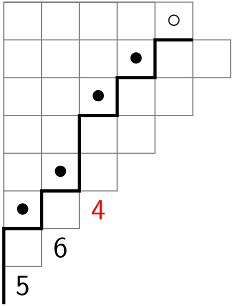

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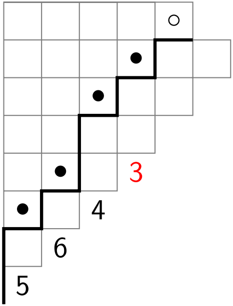

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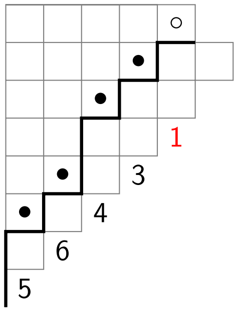

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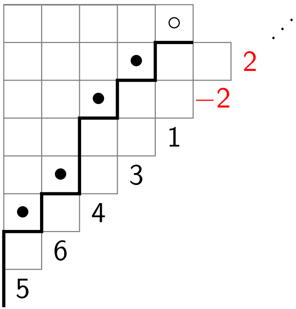

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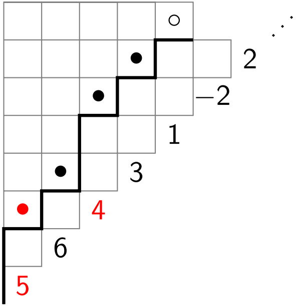

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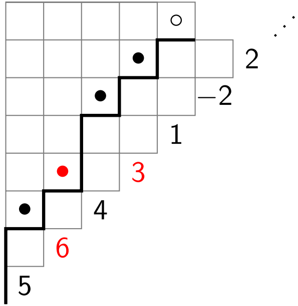

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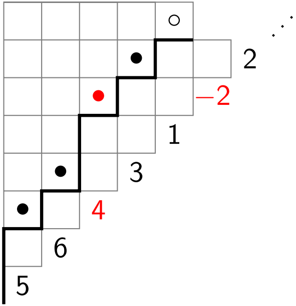

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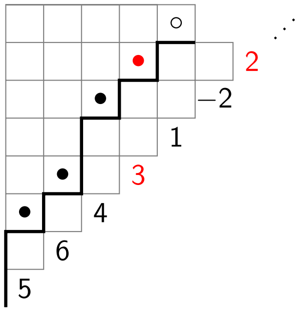

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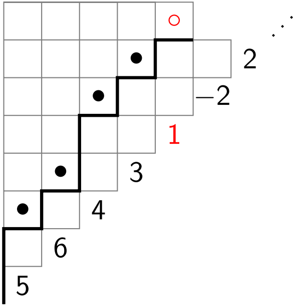

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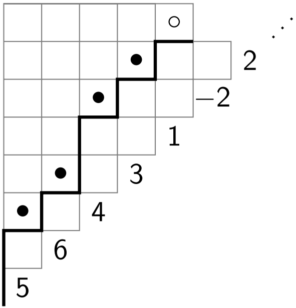

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The Haglund–Loehr zeta map

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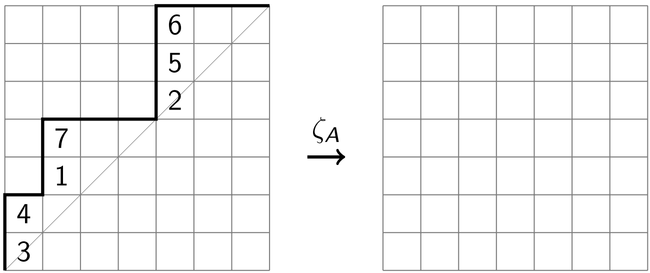

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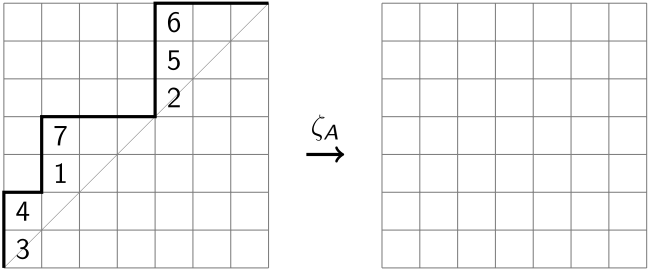
For $i = 0, 1, \ldots, n$ read the labels of rows with area equal to i from bottom to top and insert them in the diagonal.



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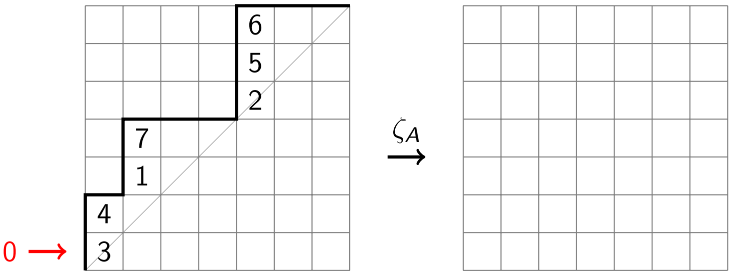

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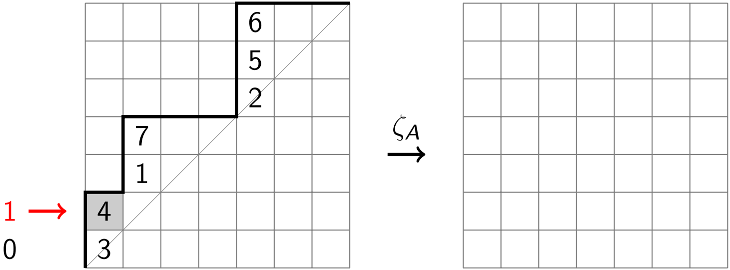

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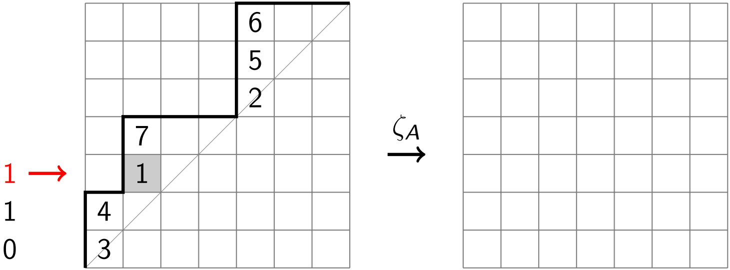

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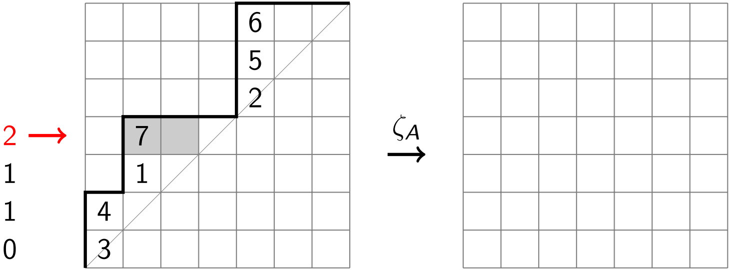

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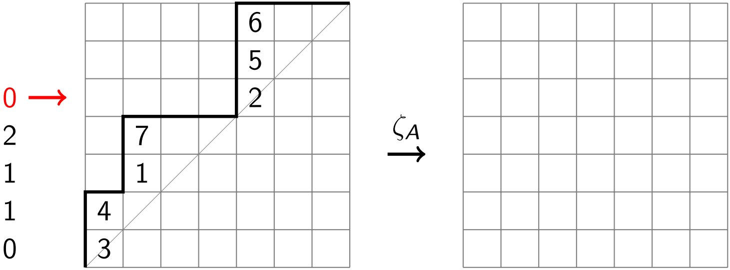

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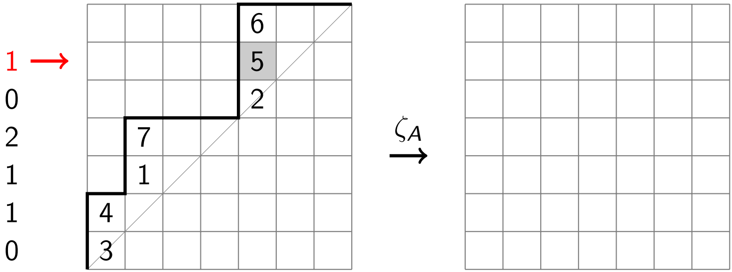

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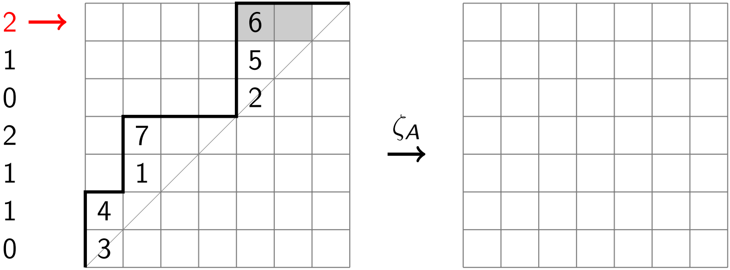

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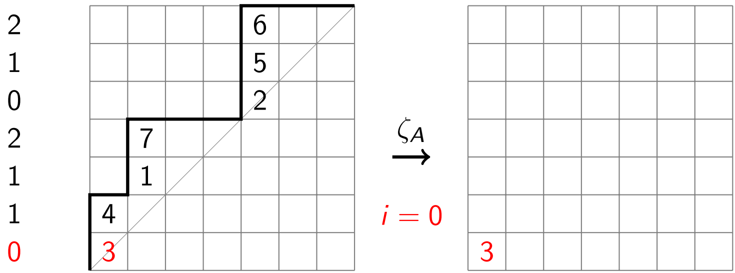

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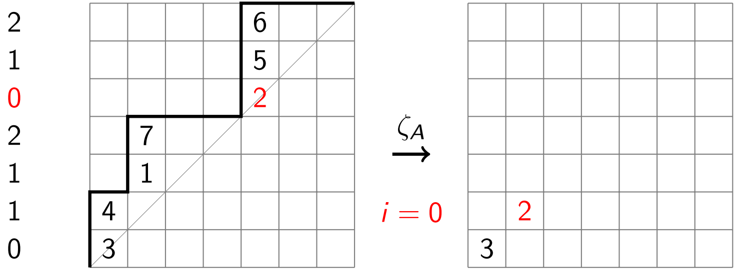

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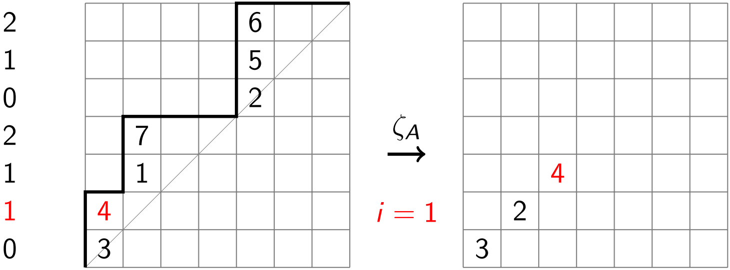

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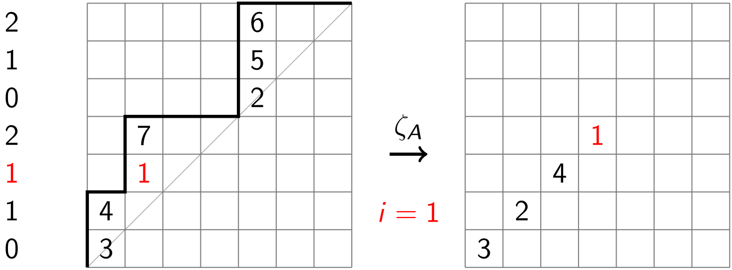

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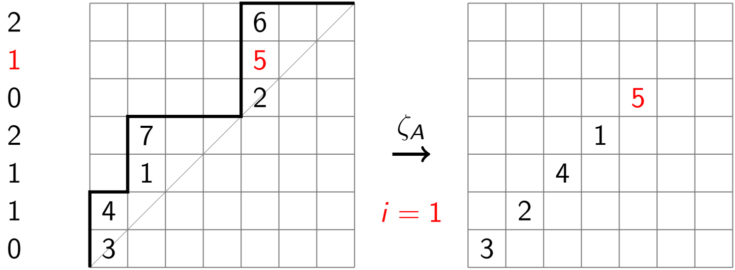

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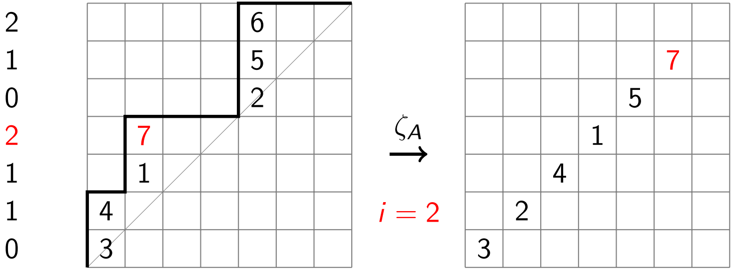

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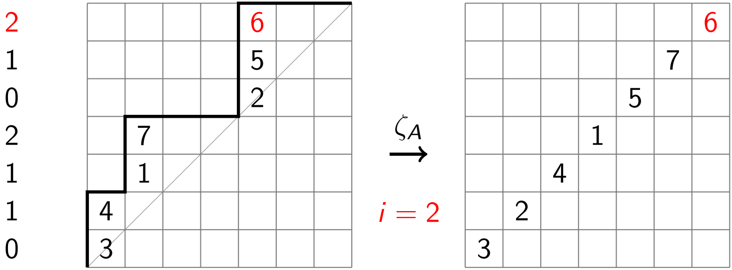

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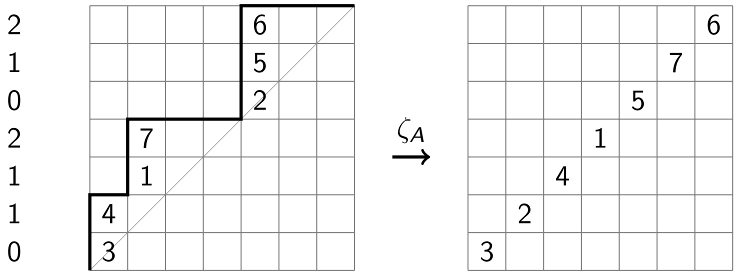

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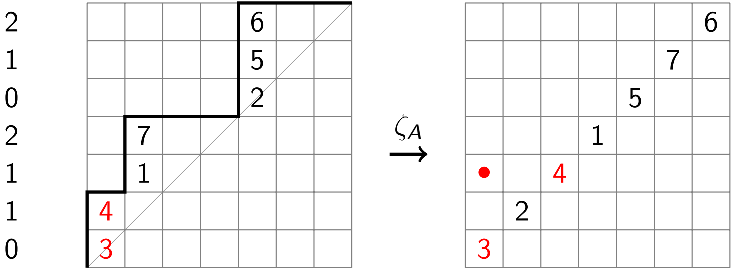

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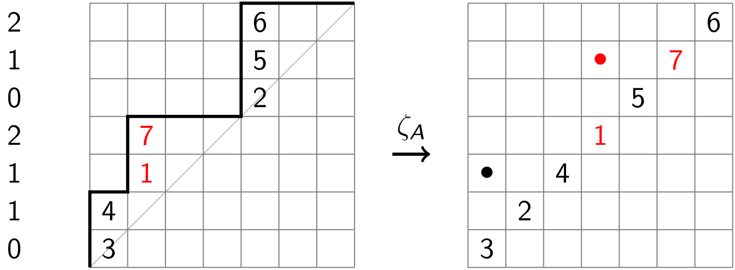

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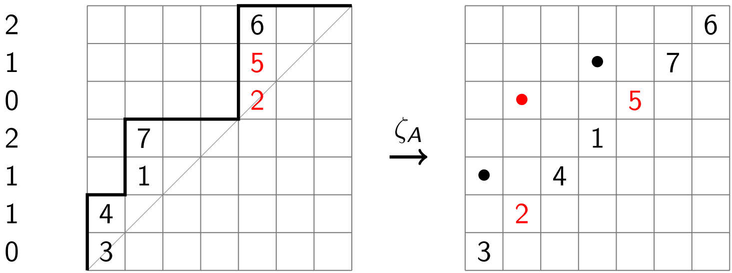

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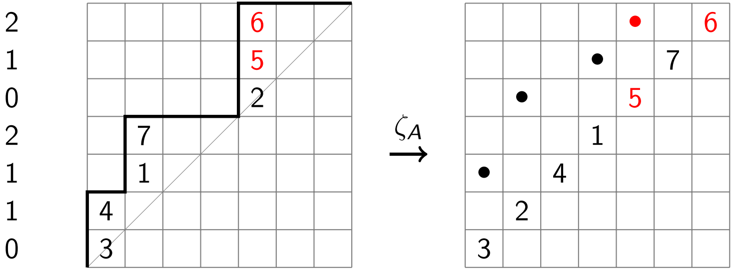

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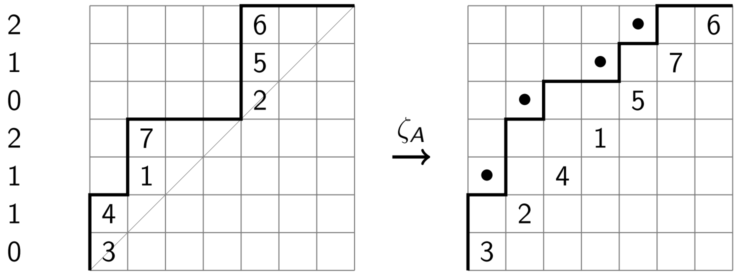

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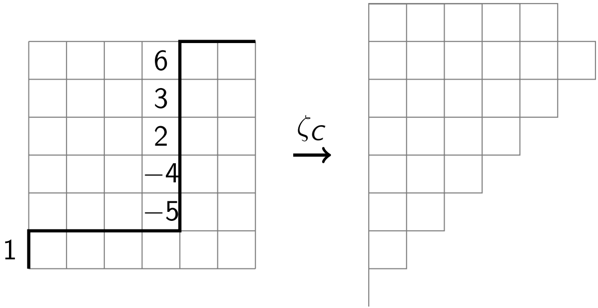

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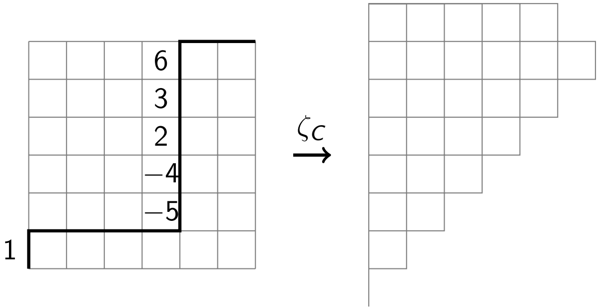

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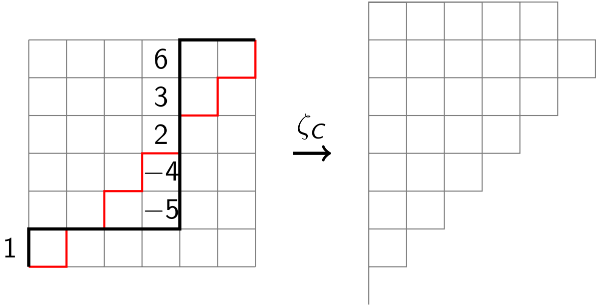

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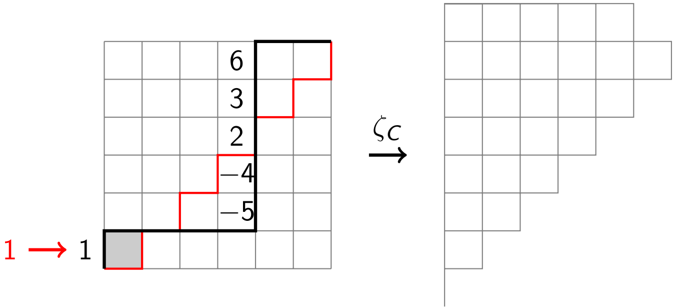

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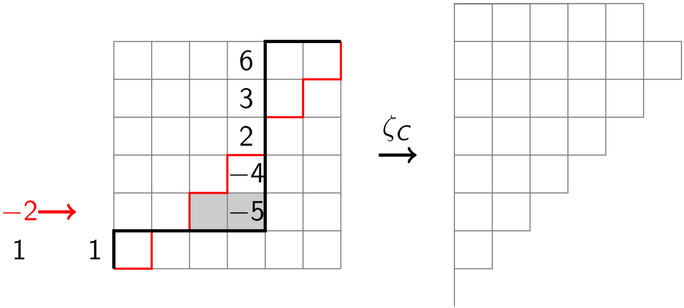

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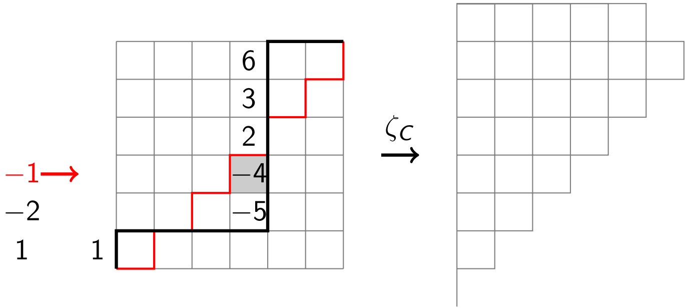

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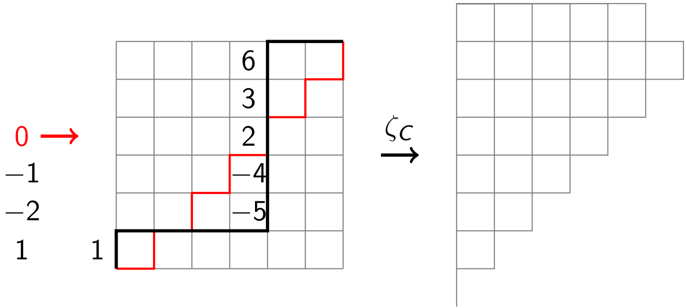

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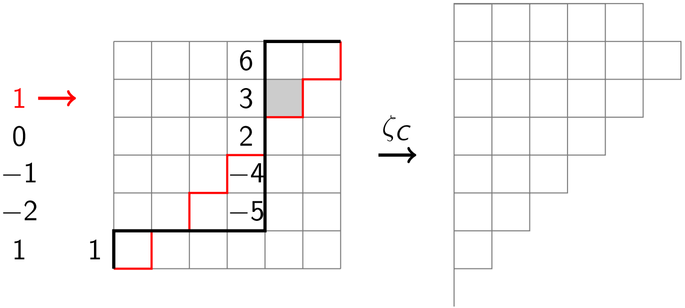

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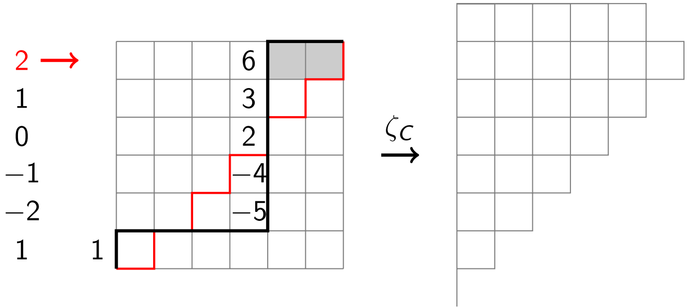

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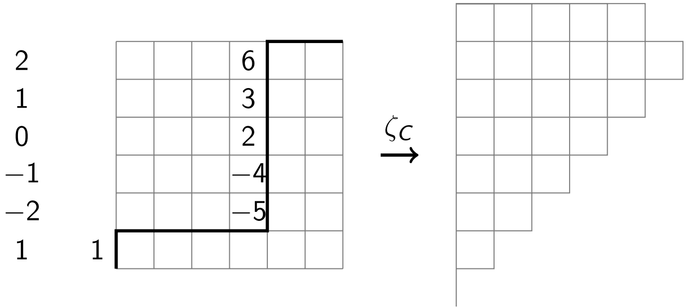

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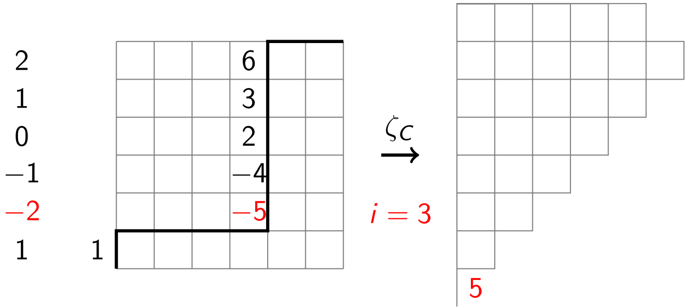

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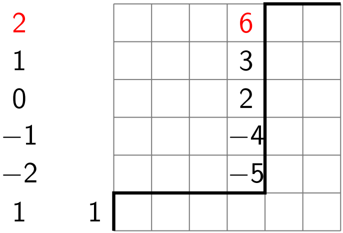
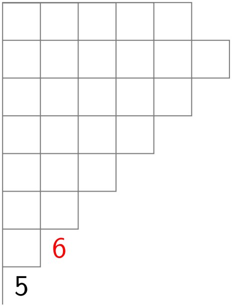


$\xrightarrow{\zeta_C}$

$i = 2$



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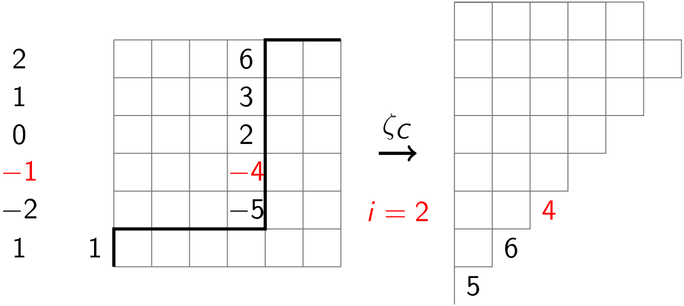

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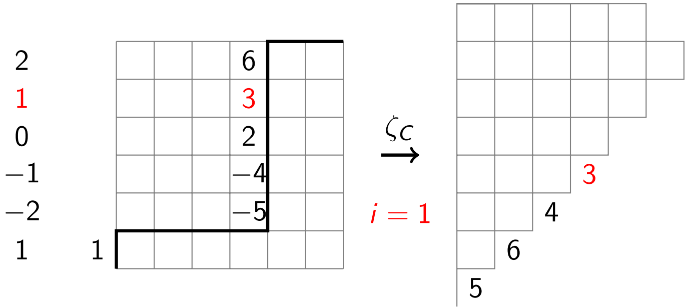

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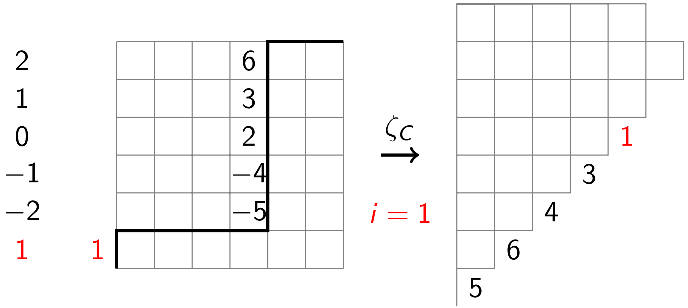

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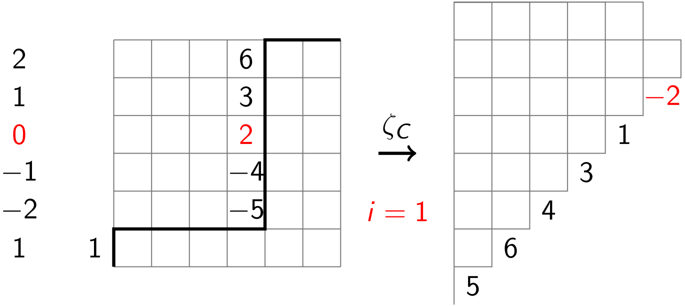

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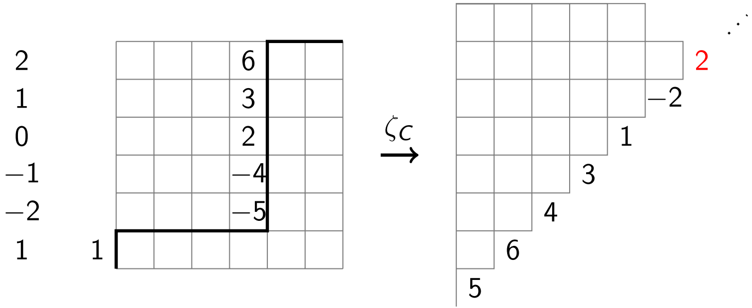

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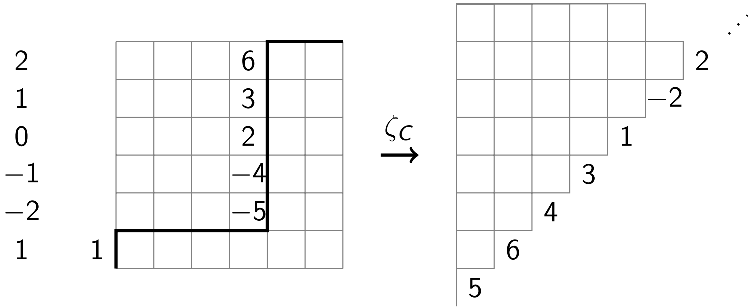

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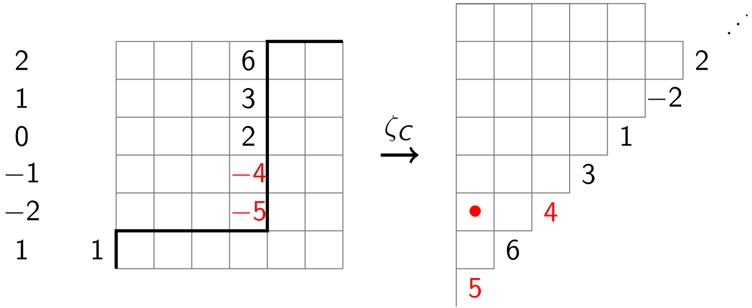

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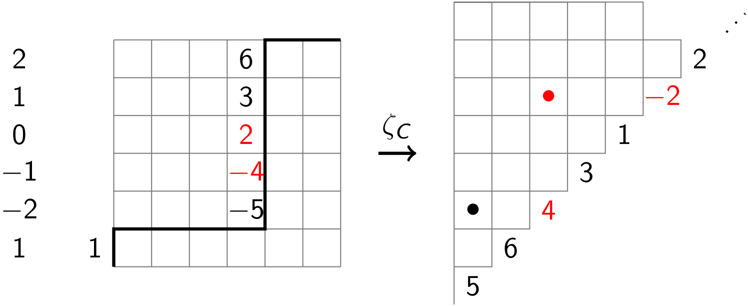

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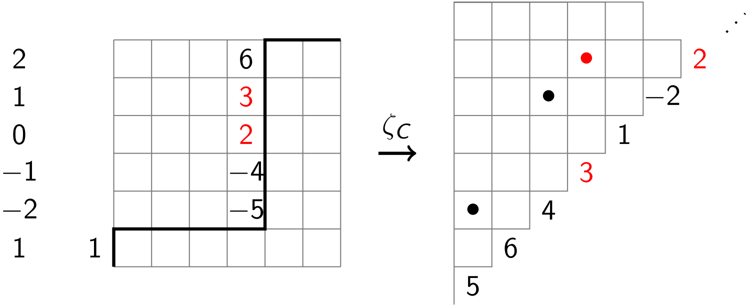

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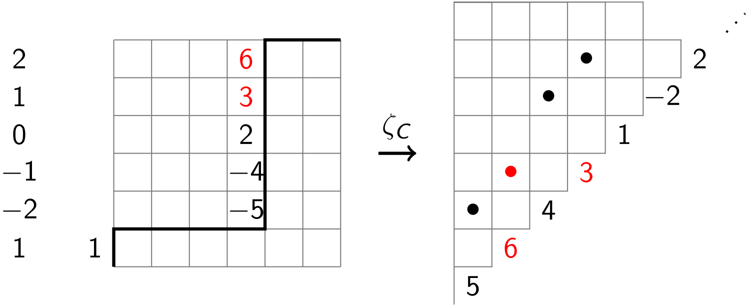

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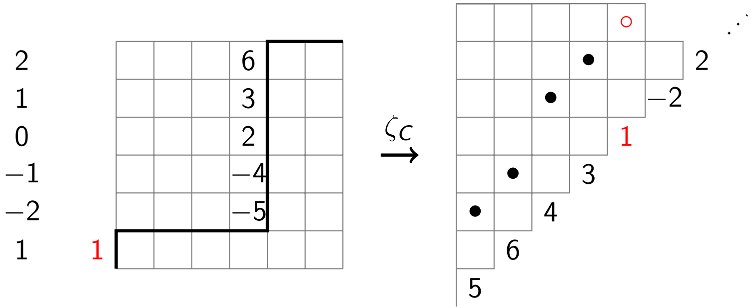

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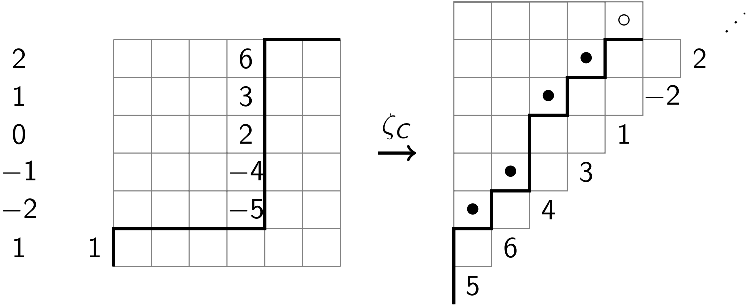

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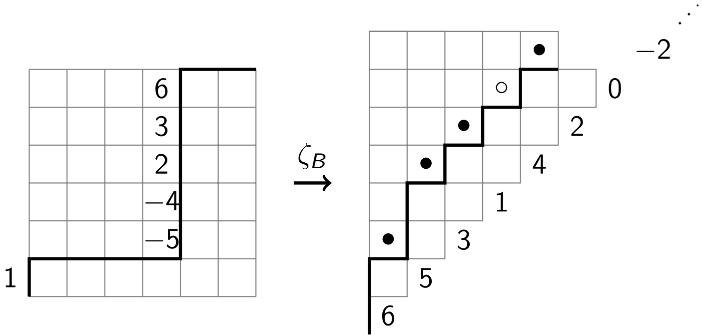

The end

Thank you!