Shifted domino tableaux

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Séminaire Lotharingien de Combinatoire
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Outline

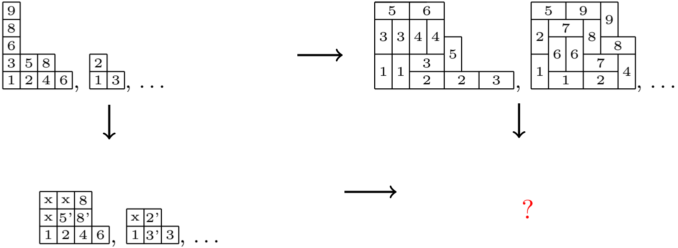
Introduction

Young tableaux: (Young)

- Schur functions
- plactic monoid (Lascoux/Schützenberger)

domino tableaux:

- product of two Schur functions
- super plactic monoid (Carré/Leclerc)



shifted Young tableaux: (Sagan/Worley)

- Q-Schur functions
- shifted plactic monoid (Serrano)

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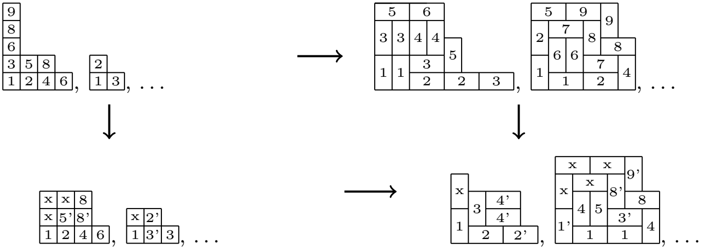

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- Q-Schur functions
- shifted plactic monoid (Serrano)

shifted domino tableaux:

- product of two Q-Schur functions
- super shifted plactic monoid

Partitions

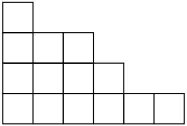
A partition λ of an integer n is:

- $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_\ell)$,
- $(\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_\ell)$,
- $\lambda_1 + \lambda_2 + \cdots + \lambda_\ell = n$.

A Young diagram is:

- a set of square cells,
- the cells are adjusted down and left,
- the i^{th} row contains λ_i cells.

The Young diagram associated to the partition (6, 4, 3, 1) is:



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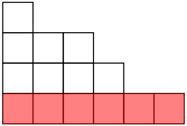

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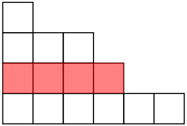

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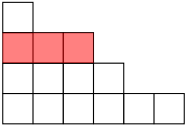

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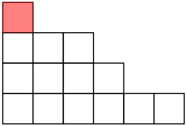
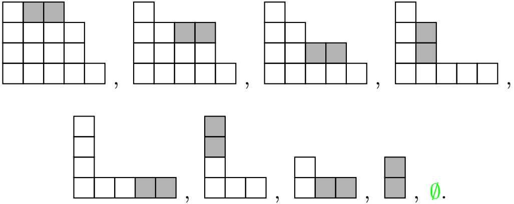
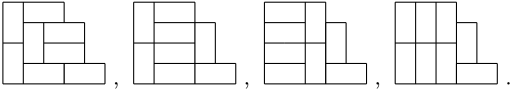


Paving of a partition by dominoes

Two adjacents cells form a domino (*i.e.*,  or ).



Thus $(5, 4, 4, 3)$ is pavable. We give below some paving of $(5, 4, 4, 3)$:



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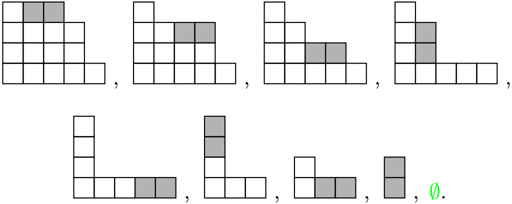

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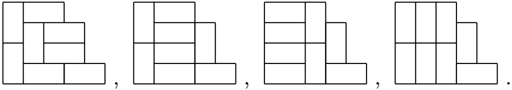
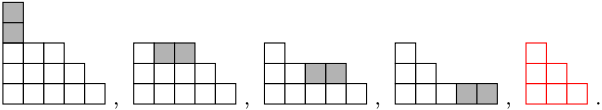
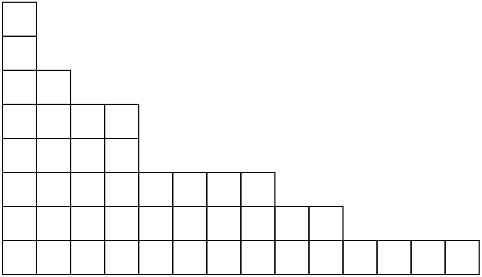


But the partition $(5, 4, 3, 1, 1)$ is not pavable.



2-quotient of a partition

The 2-quotient of a partition λ is a pair of partitions (μ, ν) obtained by:



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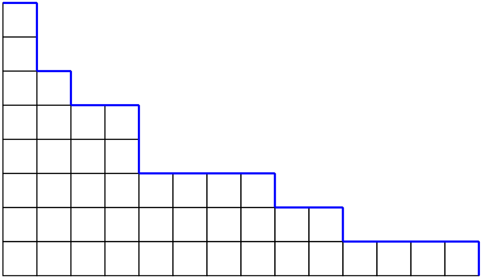
— $= 0$

| $= 1$



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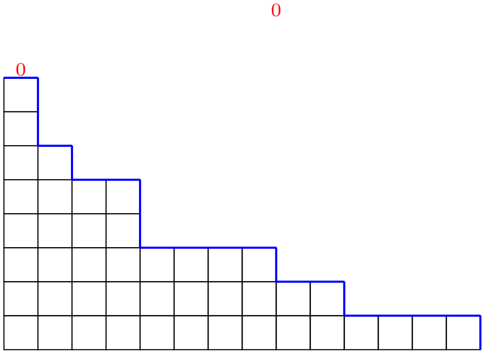

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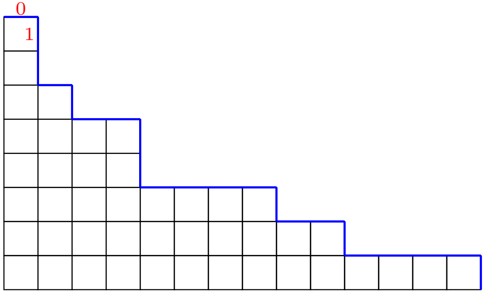

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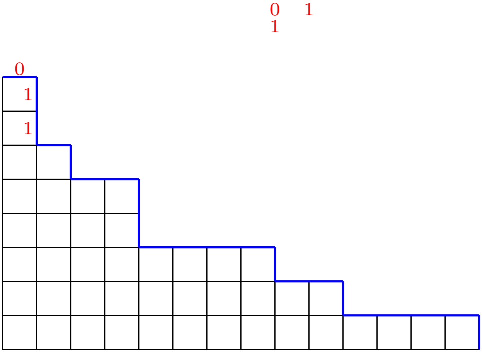

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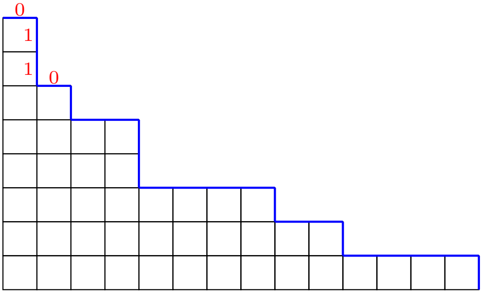

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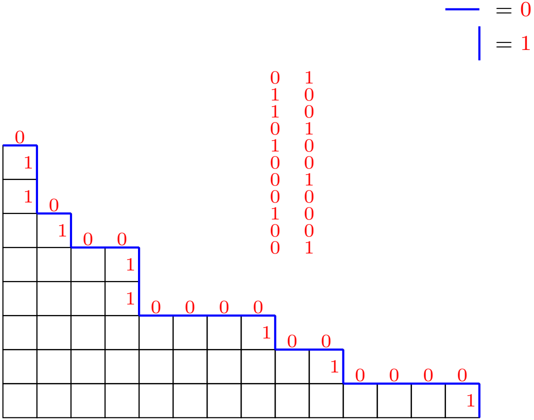

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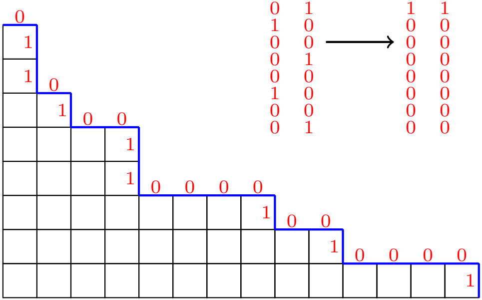
$$\begin{array}{cc}0&1\\1&0\\1&0\\0&1\\1&0\\0&0\\0&1\\0&0\\1&0\\0&0\\0&1\end{array} \longrightarrow \begin{array}{cc}1&1\\1&1\\1&1\\1&1\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\\0&0\end{array}$$



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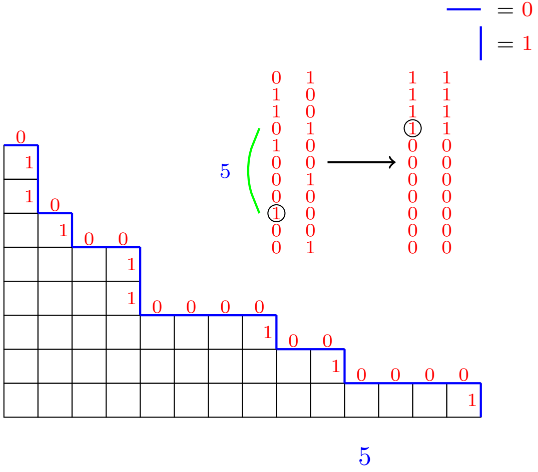

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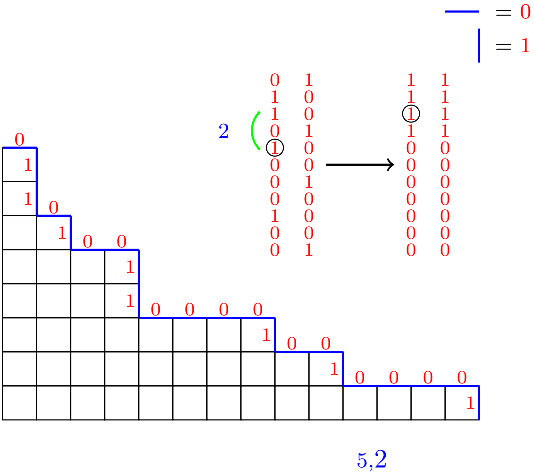

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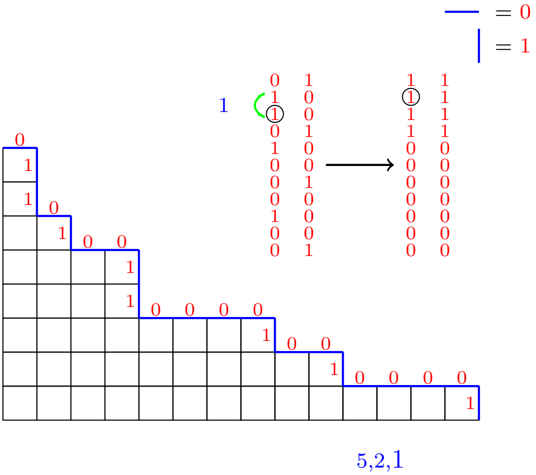

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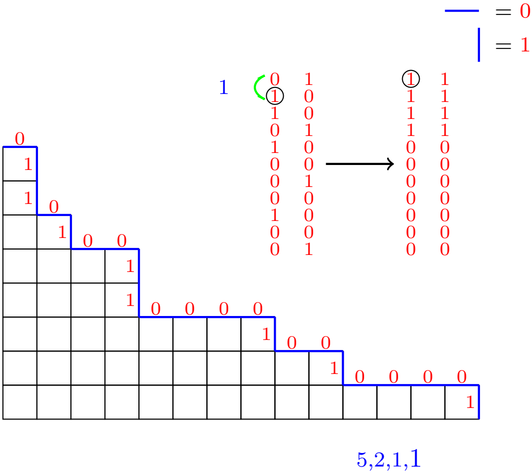

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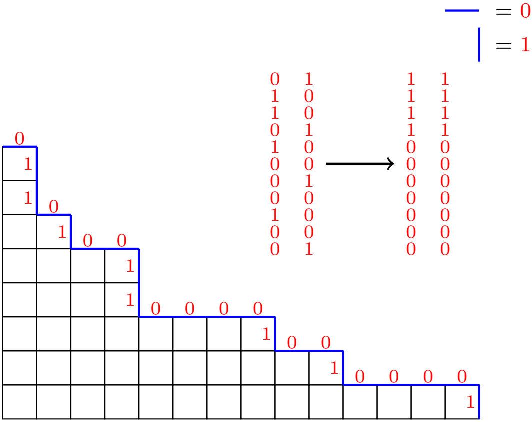

0	1		1	1
1	0		1	1
1	0		1	1
0	1		1	1
1	0		0	0
0	0	$\longrightarrow$	0	0
0	1		0	0
0	0		0	0
1	0		0	0
0	0		0	0
0	1		0	0

(5,2,1,1)

2-quotient of a partition

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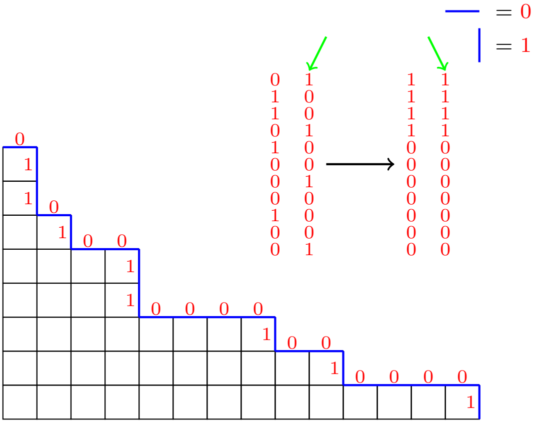

(5,2,1,1) (7,4,2)

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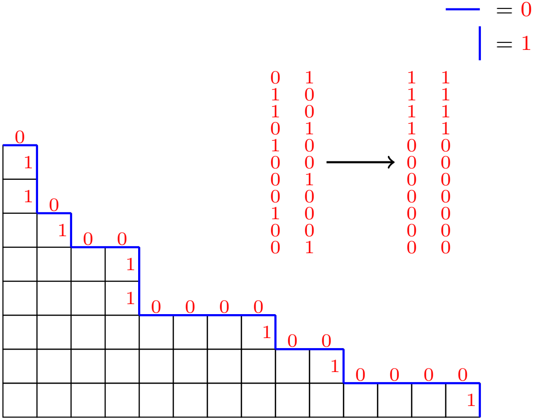

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Hence, the 2-quotient of $(14, 10, 8, 4, 4, 2, 1, 1)$ is ((5,2,1,1),(7,4,2)).

Young tableaux

A Young tableau is:

- a filling of a Young diagram with positive integers,

5					
4	9	9			
2	4	7	8		
1	3	6	6	7	8

Young tableaux

A Young tableau is:

- a filling of a Young diagram with positive integers,
- the rows are non-decreasing from left to right,
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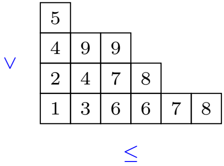

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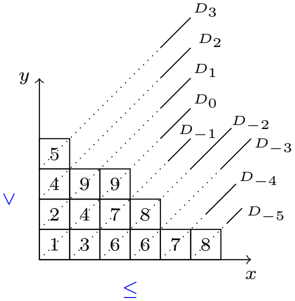

$D_k = x + k$

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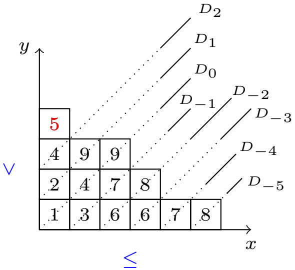

$D_k = x + k$

The diagonal reading is: 5

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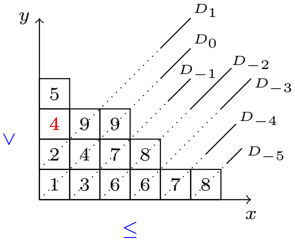

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The diagonal reading is: 5/4

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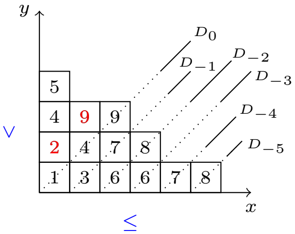

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The diagonal reading is: 5/4/2,9

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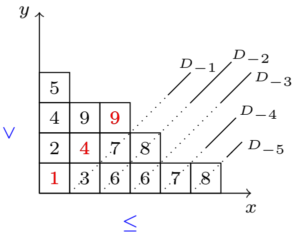

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The diagonal reading is: 5/4/2,9/1,4,9

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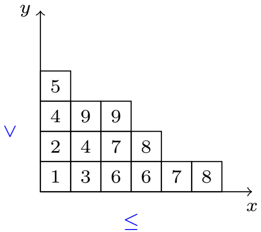

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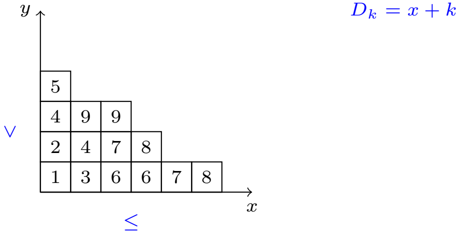


The diagonal reading is: 5/4/2,9/1,4,9/3,7/6,8/6/7/8.

Conversley, given a diagonal reading, **we can construct the associated Young tableau**.

Schur functions

Given a Young tableau t, its corresponding monomial is:

$$x^t = \prod_{i \in t} x_i.$$

For each partition λ, the Schur function s_λ is:

$$s_\lambda = \sum_t x^t$$

where the sum runs over all Young tableaux t of shape λ.

For example:

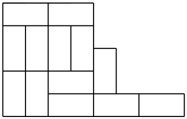
$$s_{(2,1)} = x_1^2 x_2 + x_1 x_2^2 + x_1^2 x_3 + x_1 x_3^2 + x_2^2 x_3 + x_2 x_3^2 + 2x_1 x_2 x_3,$$

where the monomial $x_1 x_2 x_3$ is obtained from the two following Young tableaux:

$$\begin{array}{|c|c|}\cline{1-1} 3 & \multicolumn{1}{c}{} \\ \hline 1 & 2 \\ \hline \end{array}\ , \quad \begin{array}{|c|c|}\cline{1-1} 2 & \multicolumn{1}{c}{} \\ \hline 1 & 3 \\ \hline \end{array}\ .$$

Domino tableaux

Given a paved partition λ, a domino tableau is:



Domino tableaux

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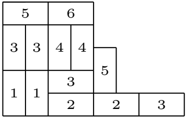

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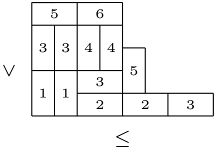

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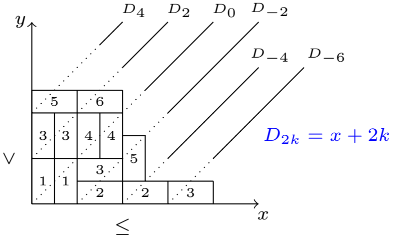
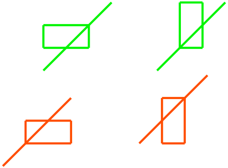


$D_{2k} = x + 2k$

two types of dominoes:



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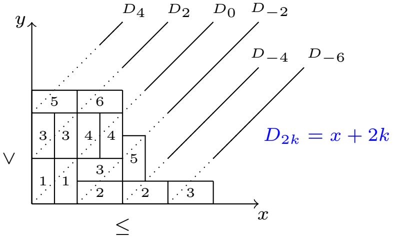

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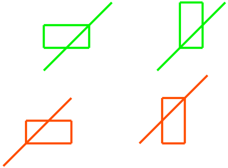
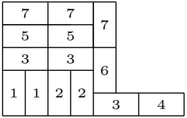


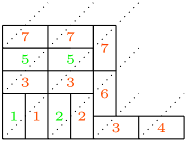
Theorem (Stanton, White 1985)

Given a pavable partition λ of 2-quotient (μ, ν), the set of domino tableaux of shape λ and the set of pairs of Young tableaux (t_1, t_2) of shape (μ, ν) are in bijection.

Sketch of proof [Carré, Leclerc 1993] (1/2)

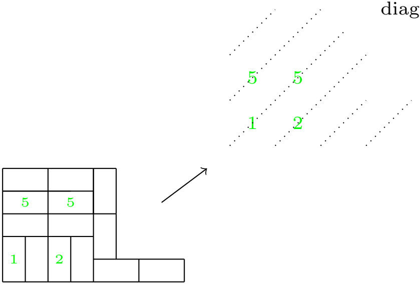


Sketch of proof [Carré, Leclerc 1993] (1/2)



Sketch of proof [Carré, Leclerc 1993] (1/2)

diagonal reading: $5/1, 5/2$



$$\longrightarrow t_1 = \begin{array}{|c|c|}\hline 5 & 5 \\ \hline 1 & 2 \\ \hline \end{array}$$

Sketch of proof [Carré, Leclerc 1993] (1/2)

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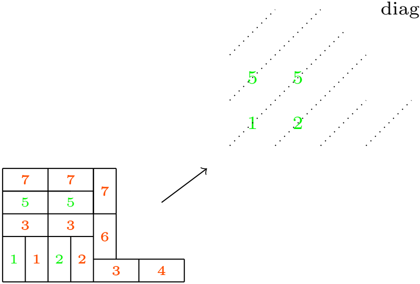

$$\longrightarrow t_1 = \begin{array}{|c|c|}\hline 5 & 5 \\ \hline 1 & 2 \\ \hline \end{array}$$

Sketch of proof [Carré, Leclerc 1993] (1/2)

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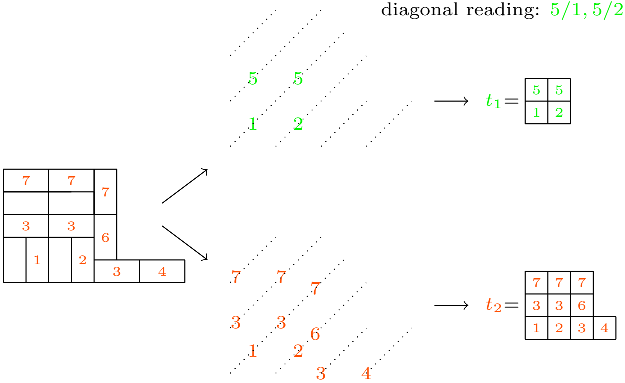

diagonal reading: $7/3, 7/1, 3, 7/2, 6/3/4$

We obtain two Young tableaux (t_1, t_2) of shape $((2, 2), (4, 3, 3))$.

Sketch of proof [Carré, Leclerc 1993] (2/2)

Given a pair of Young tableaux:

$$\left(\begin{array}{|c|c|}\hline 5 & 5 \\ \hline 1 & 2 \\ \hline \end{array} \; , \; \begin{array}{|c|c|c|c|}\cline{1-3} 7 & 7 & 7 & \multicolumn{1}{c}{} \\ \cline{1-3} 3 & 3 & 6 & \multicolumn{1}{c}{} \\ \hline 1 & 2 & 3 & 4 \\ \hline \end{array} \right)$$

Sketch of proof [Carré, Leclerc 1993] (2/2)

Given a pair of Young tableaux:

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we construct the associated domino tableau:

Sketch of proof [Carré, Leclerc 1993] (2/2)

Given a pair of Young tableaux:

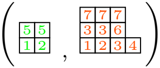
$$\left(\begin{array}{|c|c|}\hline 5 & 5 \\ \hline 1 & 2 \\ \hline \end{array} \; , \; \begin{array}{|c|c|c|c}\cline{1-3} 7 & 7 & 7 & \multicolumn{1}{c}{} \\ \hline 3 & 3 & 6 & \multicolumn{1}{|c|}{} \\ \hline 1 & 2 & 3 & \multicolumn{1}{c|}{4} \\ \hline \end{array} \right)$$

we construct the associated domino tableau:

$$\left(\boxed{1} \; , \; \boxed{1} \right) \to \begin{array}{|c|c|}\hline 1 & 1 \\ & \\ \hline \end{array}$$

Sketch of proof [Carré, Leclerc 1993] (2/2)

Given a pair of Young tableaux:



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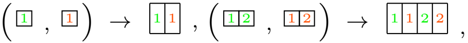

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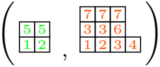

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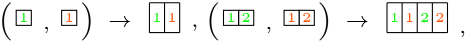

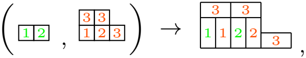

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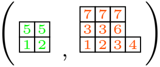

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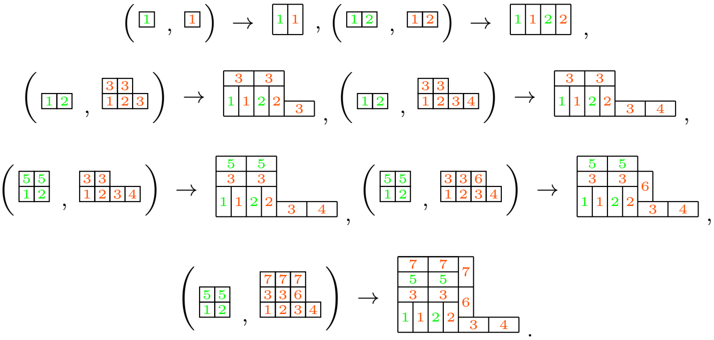


In terms of symmetric functions

Theorem (Carré, Leclerc 1993)

Let λ be a partition of 2-quotient (μ, ν). One has

$$\sum_T x^T = s_\mu s_\nu$$

where the sum runs over all domino tableaux of shape λ.

Let λ and θ be two partitions and $K^{(1)}_{\lambda\theta}$ is the number of domino tableaux of shape λ and evaluation θ.

Corollary (Carré, Leclerc 1993)

Let λ be a partition. Then

$$\sum_T x^T = \sum_\theta K^{(1)}_{\lambda\theta} m_\theta$$

where the first sum runs over all domino tableaux of shape λ and the second sum runs over all partitions θ.

The numbers $K^{(1)}_{\lambda\theta}$ are the domino analogues of the Kostka numbers.

Super plactic monoid

Given the totally ordered infinite alphabets $A_1 := \{a_1^1 < a_2^1 < a_3^1 < \cdots\}$ and $A_2 := \{a_1^2 < a_2^2 < a_3^2 < \cdots\}$. The Super Plactic monoid is the quotient of the free monoid $(A_1 \cup A_2)^*$ by the relations:

$$a_j^\epsilon a_i^\epsilon a_k^\epsilon \equiv a_j^\epsilon a_k^\epsilon a_i^\epsilon \text{ for } i < j \leq k \text{ and } \epsilon \in \{1,2\},$$

$$a_i^\epsilon a_k^\epsilon a_j^\epsilon \equiv a_k^\epsilon a_i^\epsilon a_j^\epsilon \text{ for } i \leq j < k \text{ and } \epsilon \in \{1,2\},$$

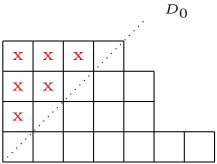
$$a_i^1 a_j^2 \equiv a_j^2 a_i^1 \quad \text{for any positive integers } i \text{ and } j.$$

Carré and Leclerc proved that **each super plactic class is represented by a unique domino tableau**.

Shifted Young tableaux

Given a partition λ of length ℓ satisfying $\lambda_\ell \geq \ell$, a shifted Young tableau is:

- a filling of the cells above D_0 by x,



Shifted Young tableaux

Given a partition λ of length ℓ satisfying $\lambda_\ell \geq \ell$, a shifted Young tableau is:

- a filling of the cells above D_0 by x,
- the remaining cells are filled by letters from $A' = \{1' < 1 < 2' < 2 < \cdots\}$,
- such that:

x	x	x	8			
x	x	7	$8'$	9		
x	$2'$	3	5	6		
1	$2'$	$3'$	3	4	5	5

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$\leq$

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$\leq$

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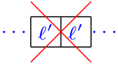

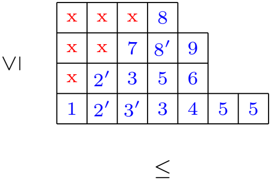

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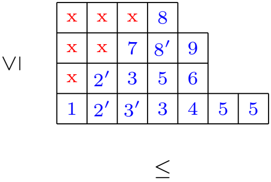
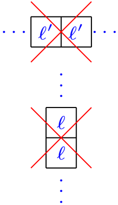


x	x	x	8			
x	x	7	8′	9		
x	2′	3	5	6		
1	2′	3′	3	4	5	5



Q-Schur functions

Given a shifted Young tableau t, its corresponding monomial is:

$$x^t = \prod_{\ell \in t} x_{|\ell|}, \text{ where } |\ell| = \ell \text{ and } |\ell'| = \ell.$$

For each partition λ, the Q-Schur function is:

$$Q_\lambda = \sum_t x^t$$

where the sum runs over all shifted Young tableaux t of shape λ.

For example:

$$Q_{(2,1)} = 4x_1^2x_2 + 4x_1x_2^2 + 4x_1^2x_3 + 4x_1x_3^2 + 4x_2^2x_3 + 4x_2x_3^2 + 8x_1x_2x_3,$$

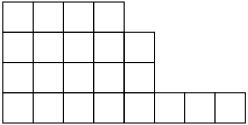
where the monomial $x_1^2x_2$ is obtained from the following four shifted Young tableaux:

$$\begin{array}{|c|c|}\hline \text{X} & 2 \\ \hline 1 & 1 \\ \hline \end{array}\,, \quad \begin{array}{|c|c|}\hline \text{X} & 2 \\ \hline 1' & 1 \\ \hline \end{array}\,, \quad \begin{array}{|c|c|}\hline \text{X} & 2' \\ \hline 1 & 1 \\ \hline \end{array}\,, \quad \begin{array}{|c|c|}\hline \text{X} & 2' \\ \hline 1' & 1 \\ \hline \end{array}\,.$$

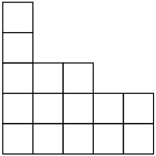
Shifted paved partition

A partition λ of 2-quotient (μ, ν) is a shifted paved partition if it satisfies the following two conditions:

- the last parts of μ and ν are greater than or equals to their lengths,



((2,2),(4,3))



((3),(3,1,1))

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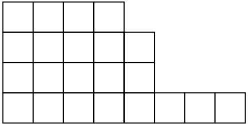

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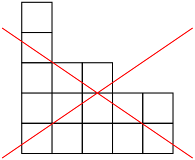

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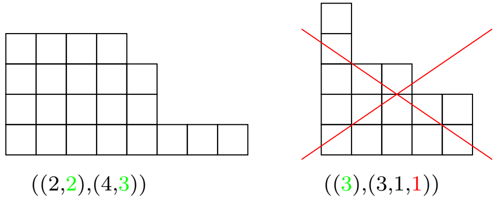
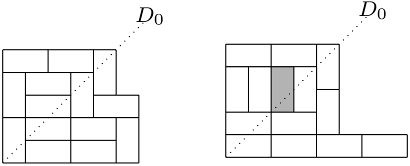

((2,2),(4,3)) ((3),(3,1,1))

- there is no vertical domino d on D_0, such that d has at its left only adjacents dominoes which are strictly above D_0,



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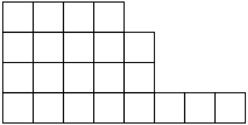

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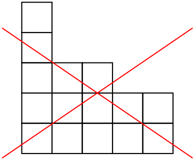

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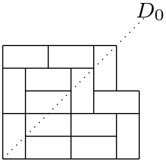
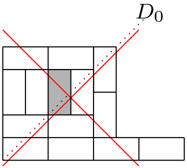
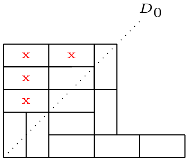




Shifted domino tableaux

Given a shifted paved partition λ, a shifted domino tableaux is:

- a filling of the dominoes above D_0 by x,



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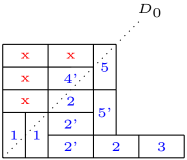

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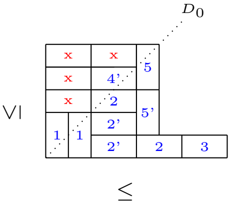

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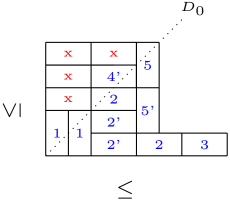

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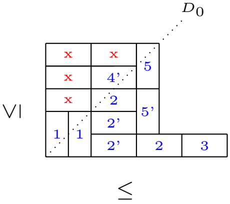
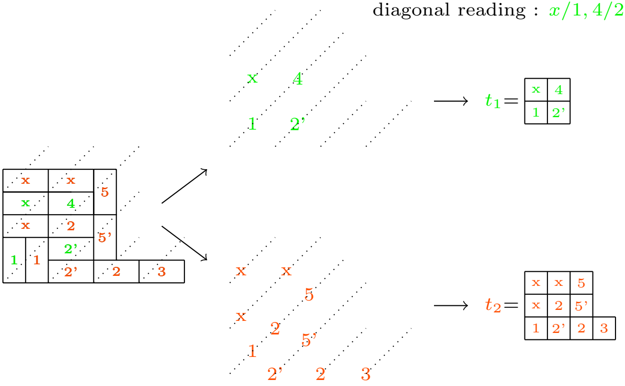


Theorem (C. 2015)

Given a valid paved partition λ of 2-quotient (μ, ν), the set of shifted domino tableaux of shape λ and the set of pairs of shifted Young tableaux (t_1, t_2) of shape (μ, ν) are in bijection.

Sketch of proof (1/2)



diagonal reading : $x/1, 4/2'$

$t_1=$

x	4
1	2'

$t_2=$

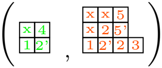
x	x	5	
x	2	5'	
1	2'	2	3

diagonal reading : $x/x, x/1, 2, 5/2', 5'/2/3$

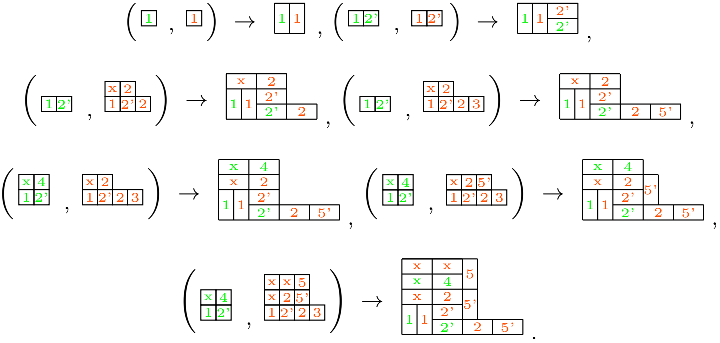
We obtain two shifted Young tableaux (t_1, t_2) of shape $((2,2),(4,3,3))$.

Sketch of proof (2/2)

Given a pair of shifted Young tableaux:



We construct the associated shifted domino tableau:



In terms of symmetric functions

Theorem (C. 2015)

Let λ be a valid paved partition of 2-quotient (μ, ν). One has

$$\sum_T x^T = Q_\mu Q_\nu$$

where the sum runs over all shifted domino tableaux of shape λ.

Let λ and θ be two partitions and $K^{(2)}_{\lambda\theta}$ is the number of shifted domino tableaux of shape λ and evaluation θ.

Corollary (C. 2015)

Let λ be a partition. Then

$$\sum_T x^T = \sum_\theta K^{(2)}_{\lambda\theta} m_\theta.$$

where the first sum runs over all shifted domino tableaux of shape λ and the second sum runs over all partitions θ.

The numbers $K^{(2)}_{\lambda\theta}$ can be seen as analogues of the Kostka numbers.

Super shifted plactic monoid

Let $A_1 := \{a_1^1 < a_2^1 < a_3^1 < \cdots\}$ and $A_2 := \{a_1^2 < a_2^2 < a_3^2 < \cdots\}$ be two totally ordered infinite alphabets. The super shifted plactic monoid is the quotient of the free monoid $(A_1 \cup A_2)^*$ by the relations:

$$
\begin{aligned}
&a_i^\epsilon a_j^\epsilon a_l^\epsilon a_k^\epsilon \equiv a_i^\epsilon a_l^\epsilon a_j^\epsilon a_k^\epsilon \text{ for } i \leq j \leq k < l \text{ and } \epsilon \in \{1,2\},\\
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&a_l^\epsilon a_j^\epsilon a_k^\epsilon a_i^\epsilon \equiv a_j^\epsilon a_l^\epsilon a_k^\epsilon a_i^\epsilon \text{ for } i < j \leq k < l \text{ and } \epsilon \in \{1,2\},\\
&a_j^\epsilon a_k^\epsilon a_l^\epsilon a_i^\epsilon \equiv a_j^\epsilon a_k^\epsilon a_i^\epsilon a_l^\epsilon \text{ for } i < j \leq k \leq l \text{ and } \epsilon \in \{1,2\},\\
&a_k^\epsilon a_i^\epsilon a_l^\epsilon a_j^\epsilon \equiv a_k^\epsilon a_l^\epsilon a_i^\epsilon a_j^\epsilon \text{ for } i \leq j < k \leq l \text{ and } \epsilon \in \{1,2\},\\
&a_i^1 a_j^2 \equiv a_j^2 a_i^1 \quad \text{for any positive integers } i \text{ and } j.
\end{aligned}
$$

Theorem (C. 2015)

Each super shifted plactic class is represented by a unique shifted domino tableau.

Perspectives

Perspectives

- express the coefficients of the shifted Littlewood-Richardson rule in terms of shifted domino tableaux,
- find an insertion algorithm for shifted domino tableaux.

Thank you