Maule

Tilings, Young and Tamari lattices under the same roof (part II)

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September 11, 2017

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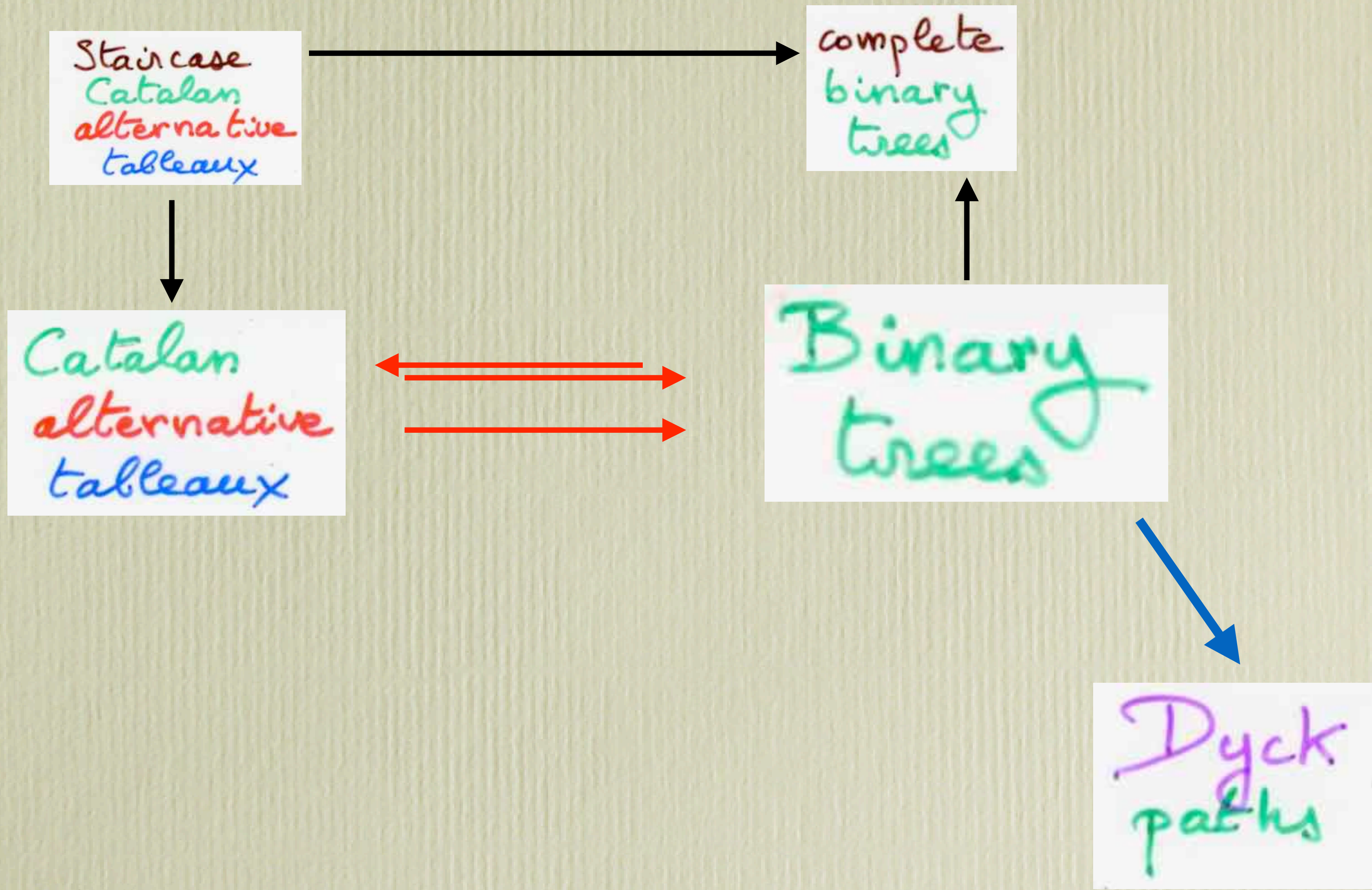
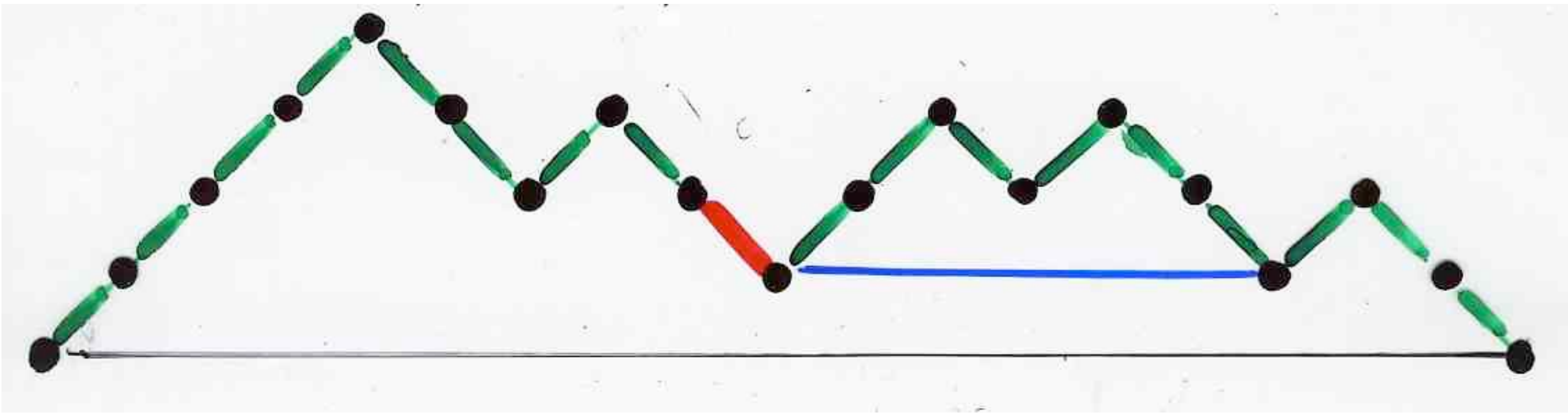
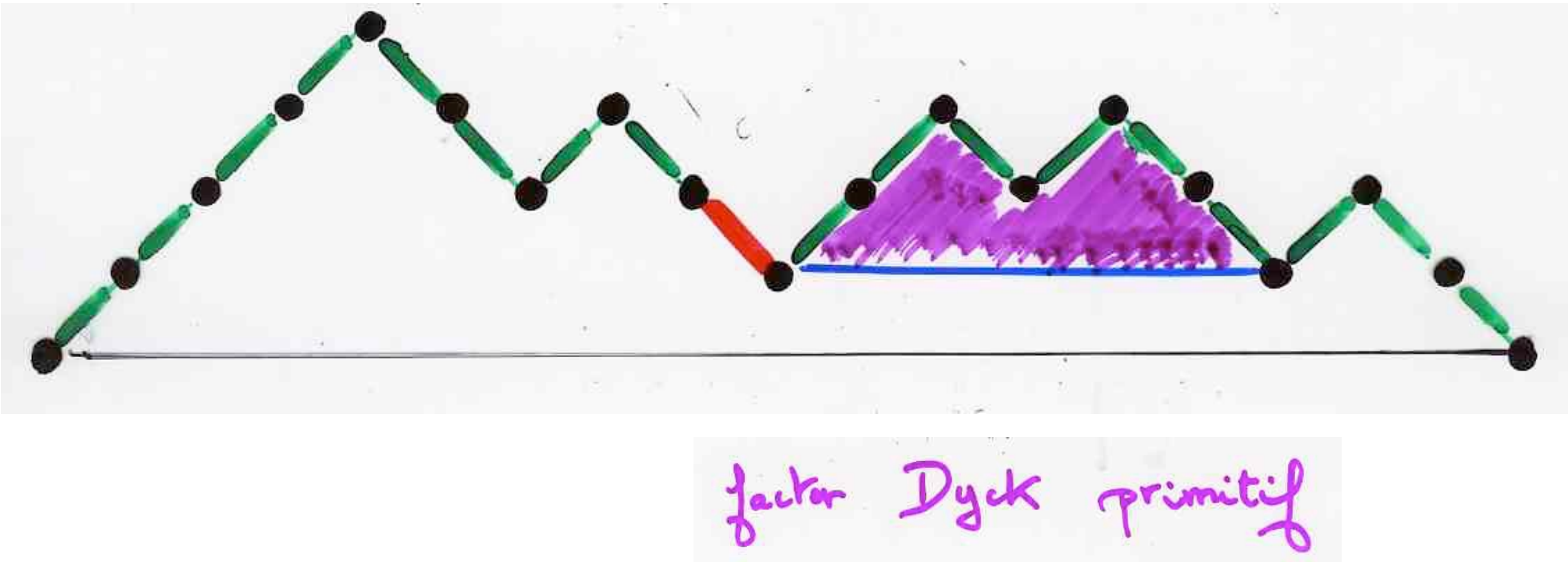
augmented set of slides with comments and references added 3 October 2017 the Tamari lattice

in term

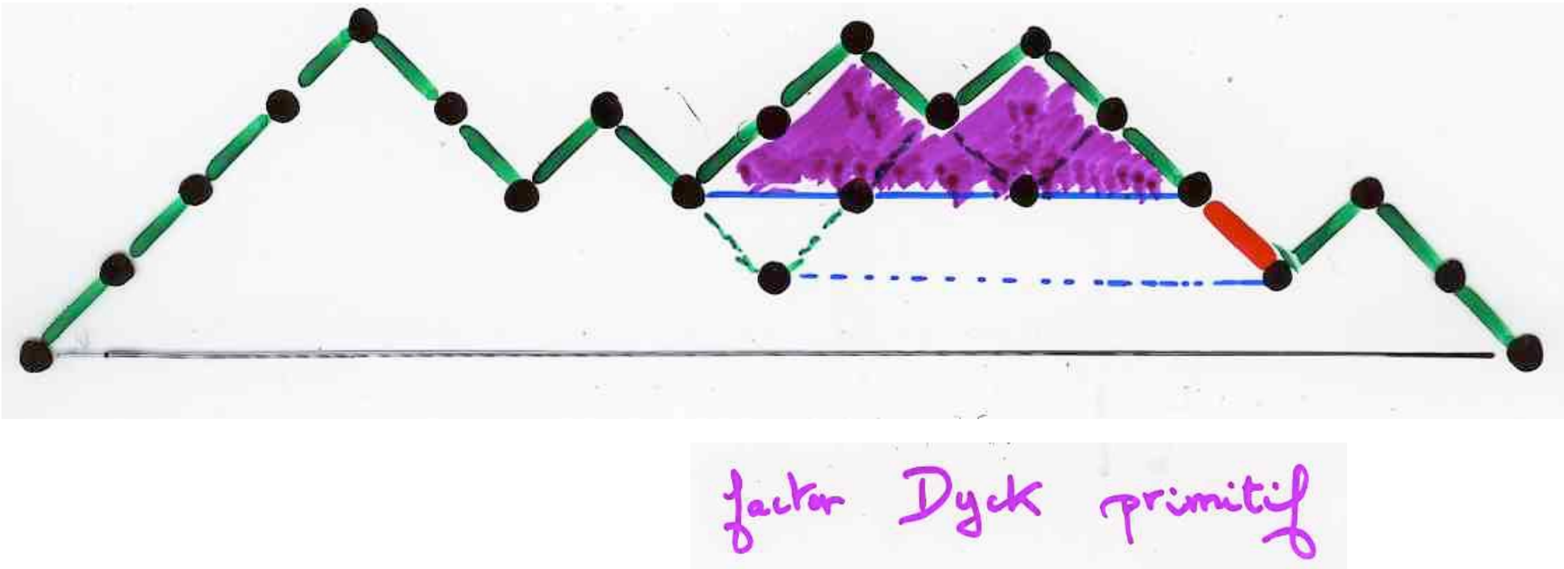
of Dyck paths


Staircase Catalan alternative tableaux
complete binary trees
Catalan alternative tableaux
Binary trees
Dyck paths



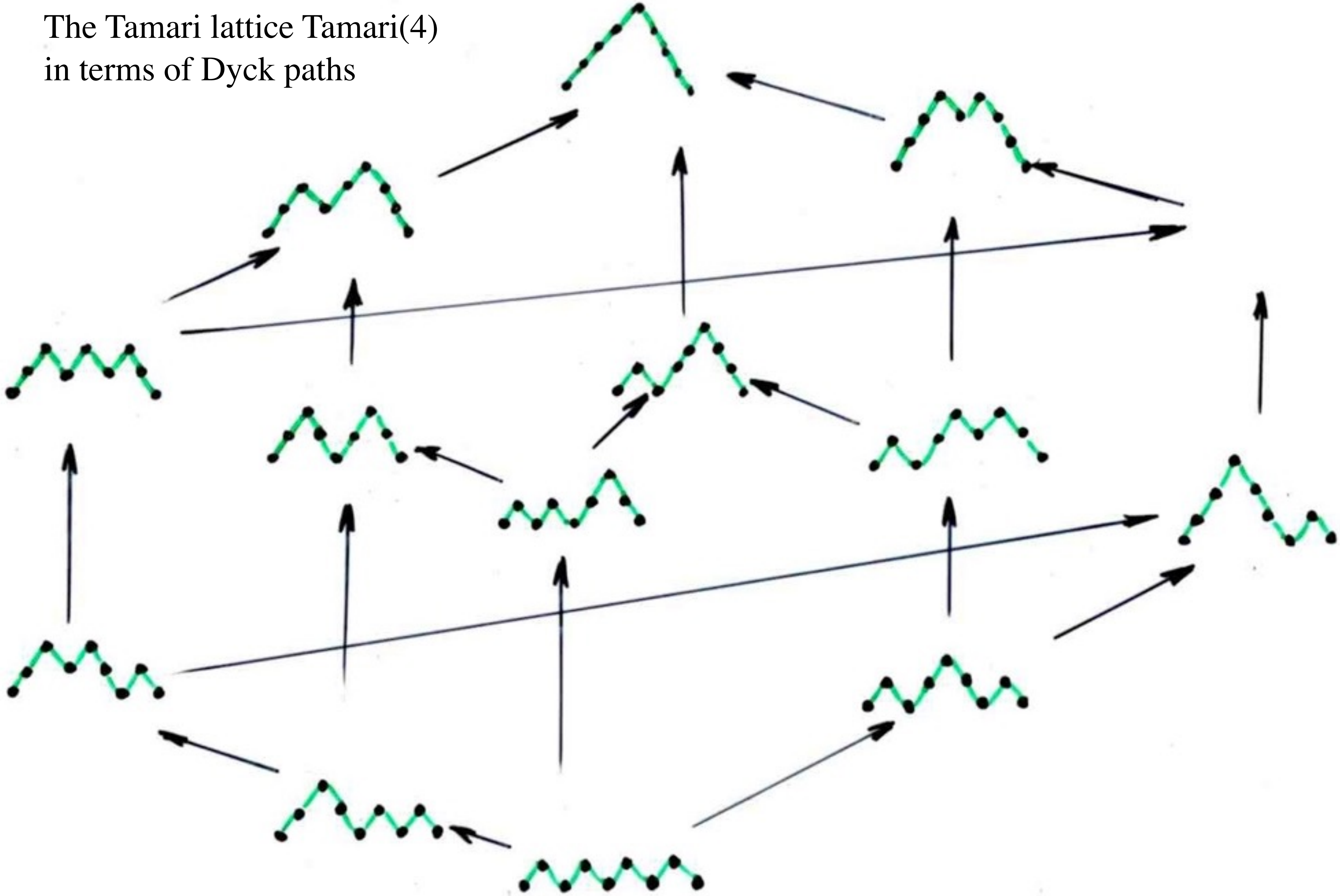


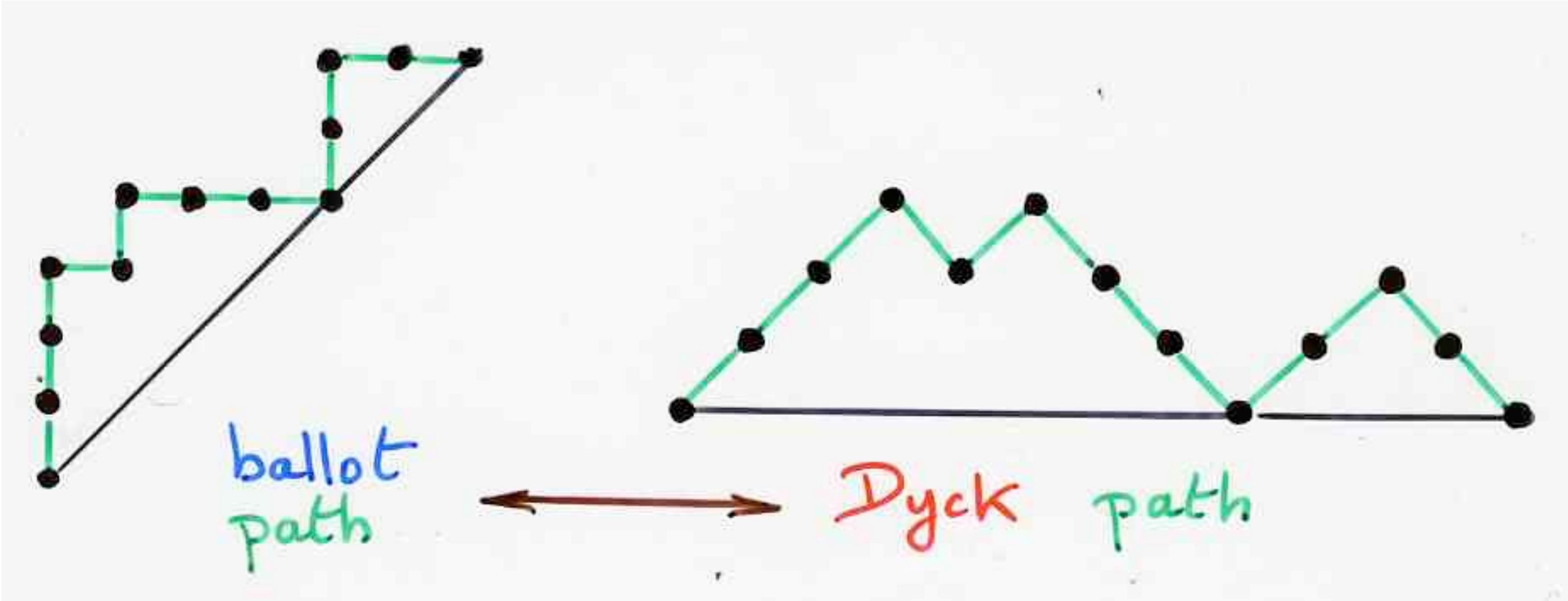
The analog of the rotation in a binary tree in term of the associated Dyck path (via the classical bijection binary trees —- Dyck paths).
An example of this bijection is given on slide 80 (part II).



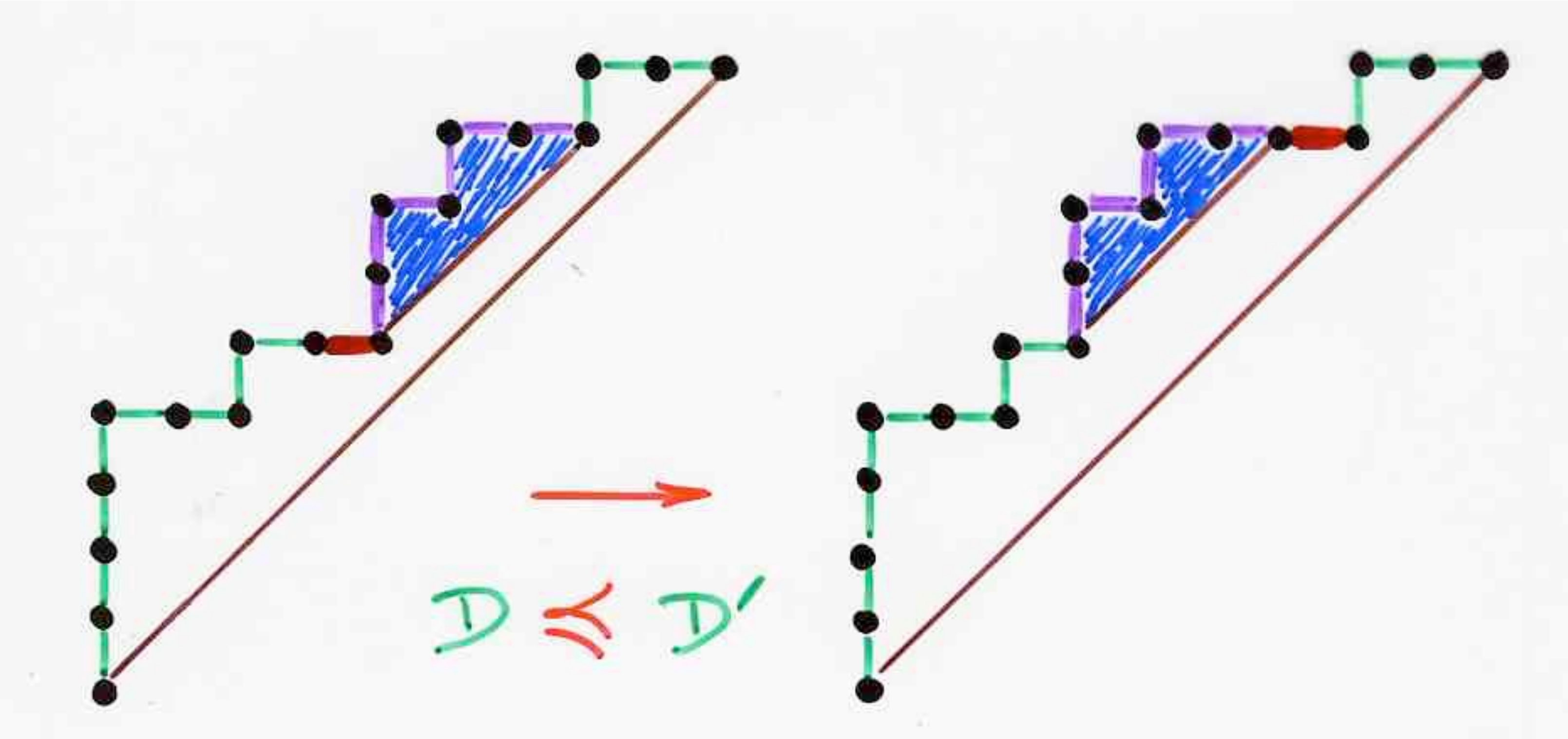
The analog of the rotation in a binary tree
in term of the associated Dyck path.

The Tamari lattice Tamari(4)
in terms of Dyck paths





vocabulary: ballot path
Dyck path



the Tamari covering relation
for ballot (Dyck) path

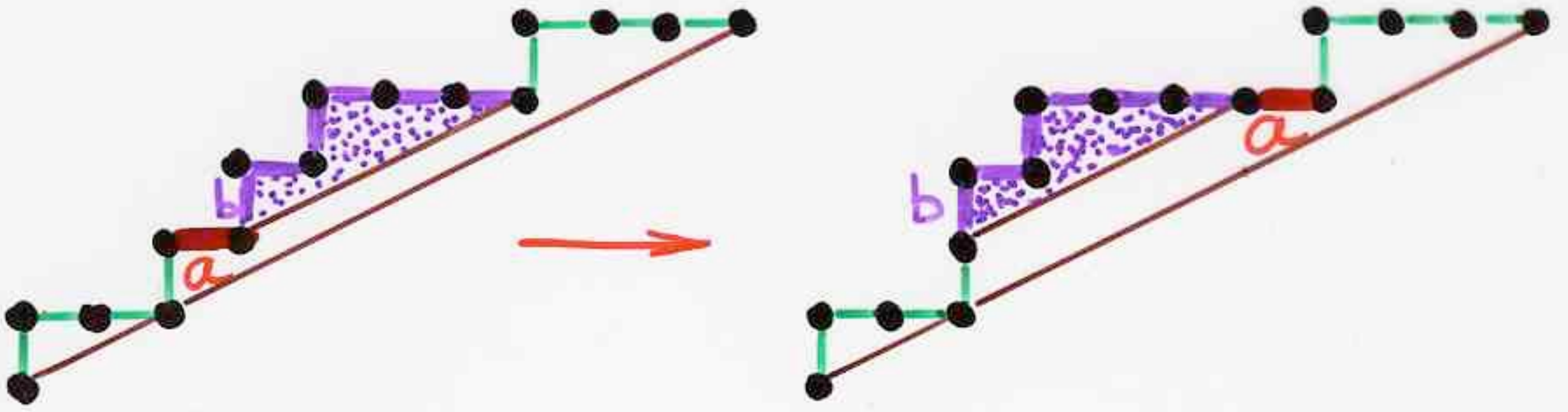
diagonal coinvariant spaces

higher diagonal coinvariant spaces

F. Bergeron (2008) introduced the m-Tamari lattice

dimension $\frac{1}{(m+1)n+1}\binom{(m+1)n+1}{mn}$

m-ballot paths



the covering relation in the m-Tamari lattice
($m = 2$)

Rational Catalan Combinatorics

Rational Catalan Combinatorics

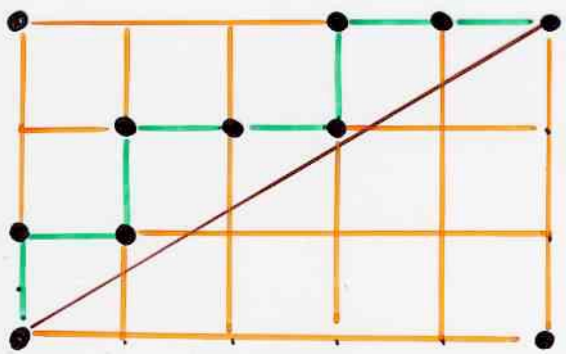
D. Armstrong

$$\mathrm{Cat}(a,b) = \frac{1}{a+b}\binom{a+b}{a,\,b}$$

number of (a,b)-ballot paths $= \mathrm{Cat}(a,b)$

Grossman (1950)
Bizley (1954)

rational ballot (Dyck) paths



$a=3$

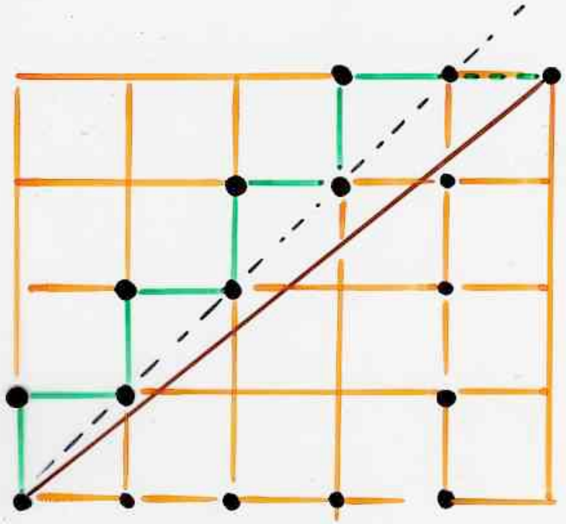
$b=5$

$(a,b)=(3,5)$

$(a, b) = (n, n+1) \longrightarrow C_n$ Catalan nb

$(a, b) = (n, mn+1) \longrightarrow \dfrac{1}{(m+1)n+1}\dbinom{(m+1)n+1}{n}$

Fuss-Catalan nb

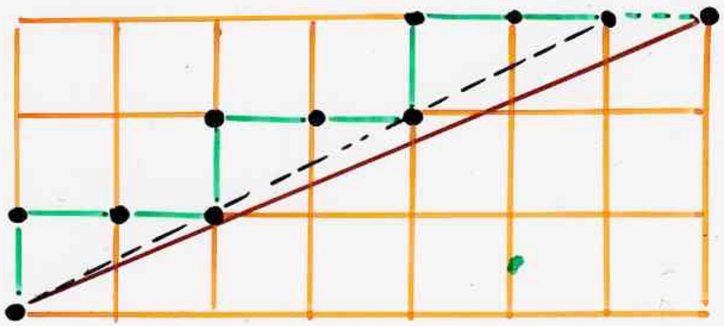


$n = 4$

$n+1 = 5$

$(a, b) = (4, 5)$

Catalan



$n = 3$

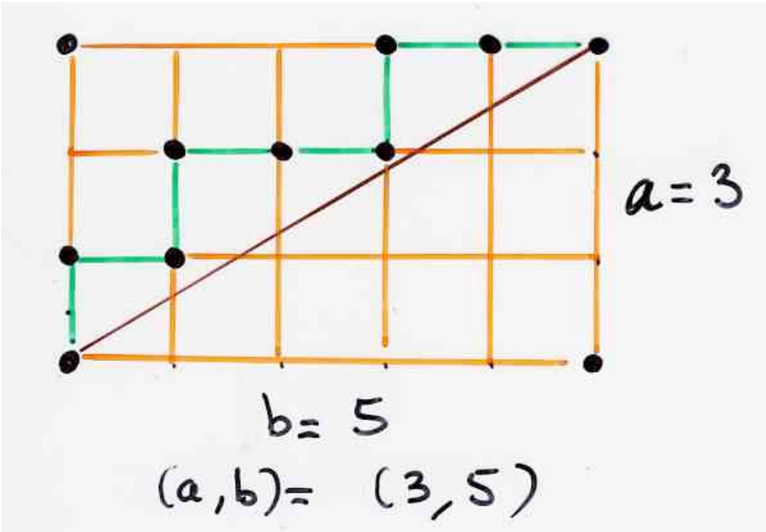
$m = 2$ $\qquad$ $mn+1 = 7$

$(a, b) = (3, 7)$

Fuss-Catalan

question:

define an (a,b)- Tamari lattice ?



Préville-Ratelle, X.V. (2015)(2017)
Tamari lattice Tamari(V)

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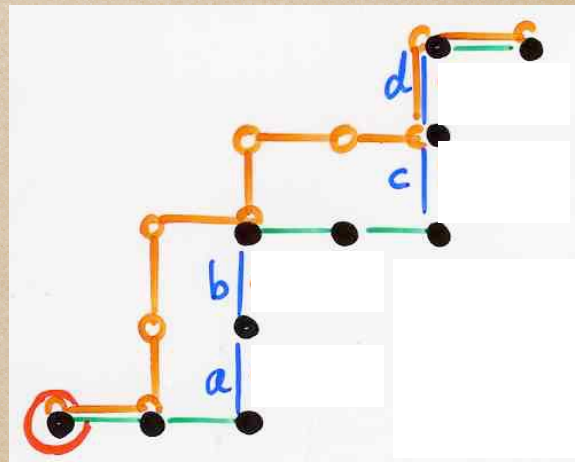
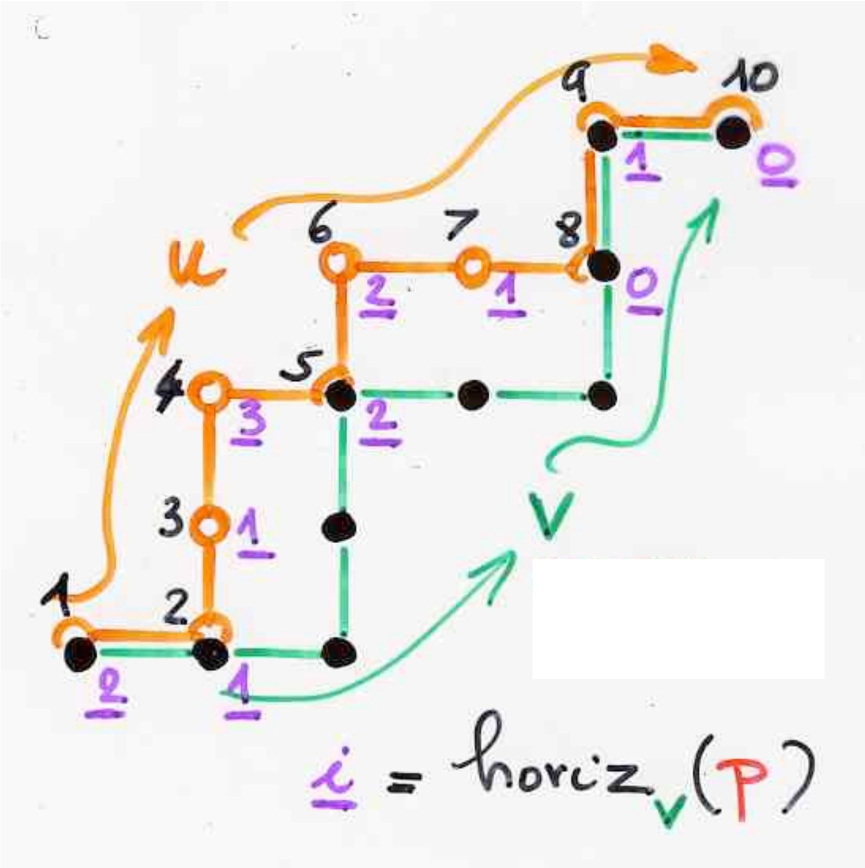
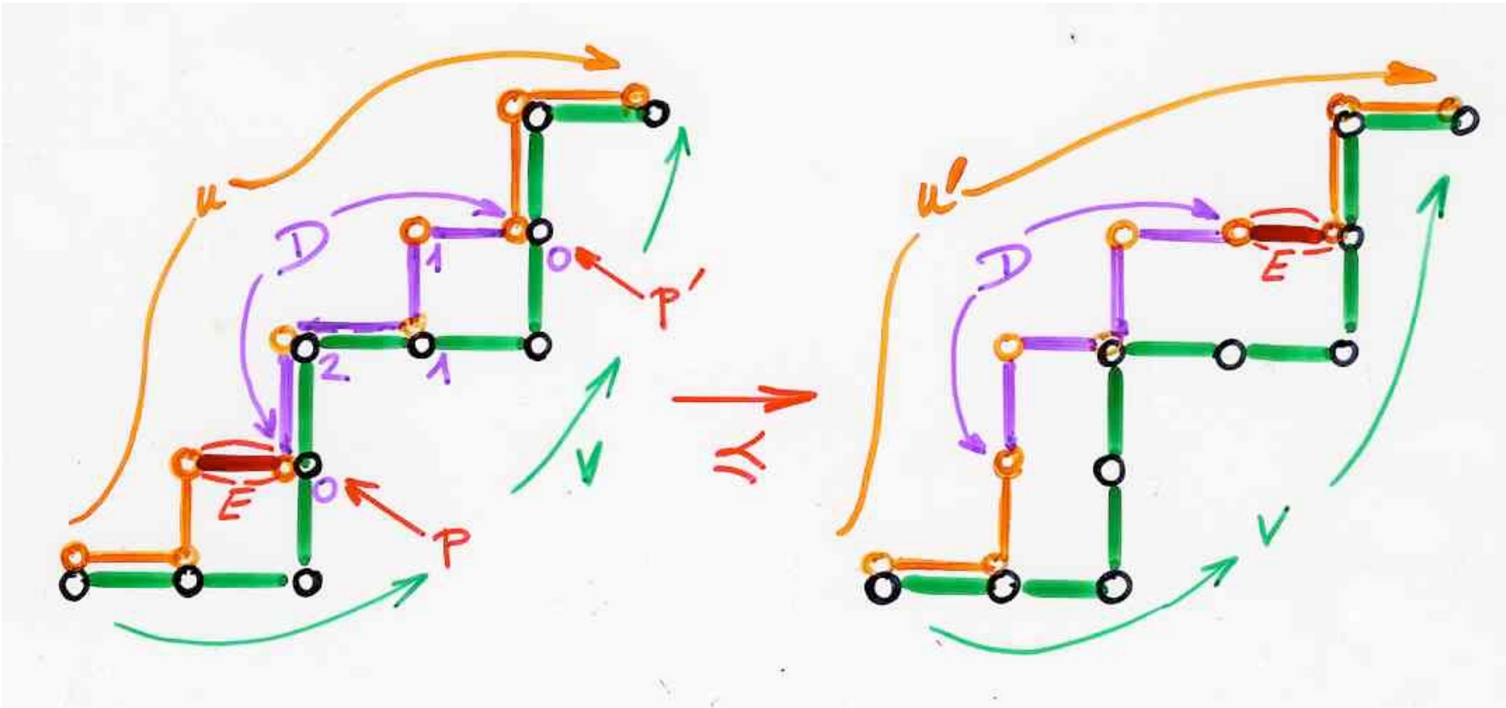




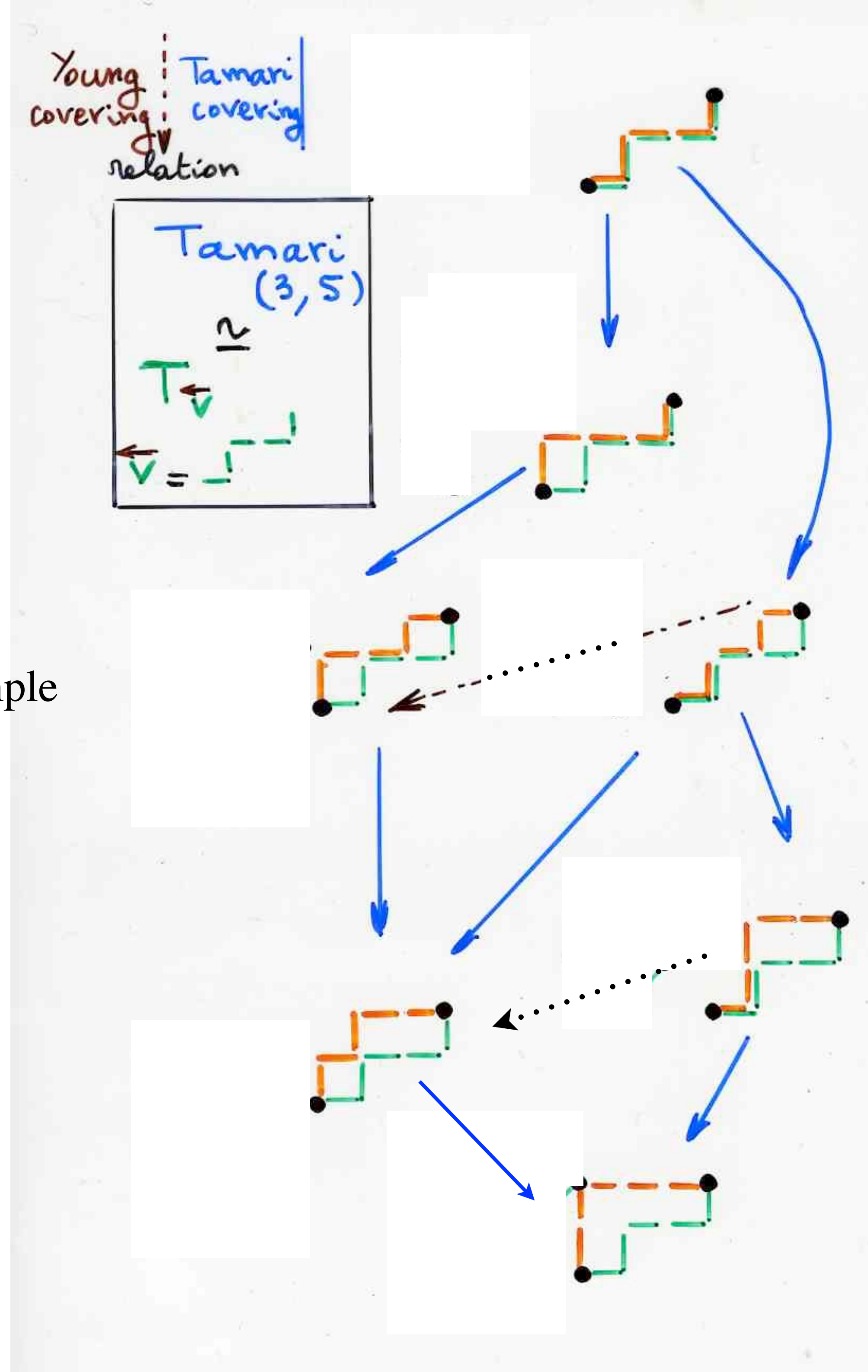
For each vertex of the path u,
we associate a number (in purple),
as the distance from this vertex
to the rightmost vertex of the path v.

a pair (u, v) of paths
with the "horizontal distance"
$\mathrm{horiz}_v(p)$

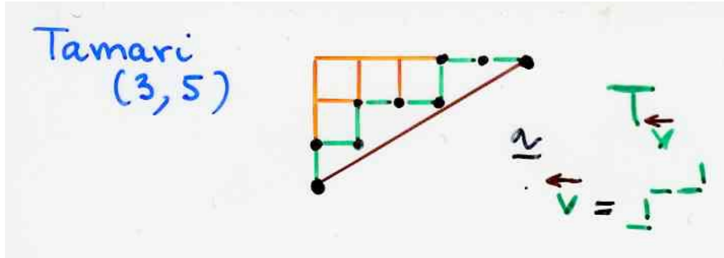
the covering relation in the poset T_v (also denoted by Tamari(v))

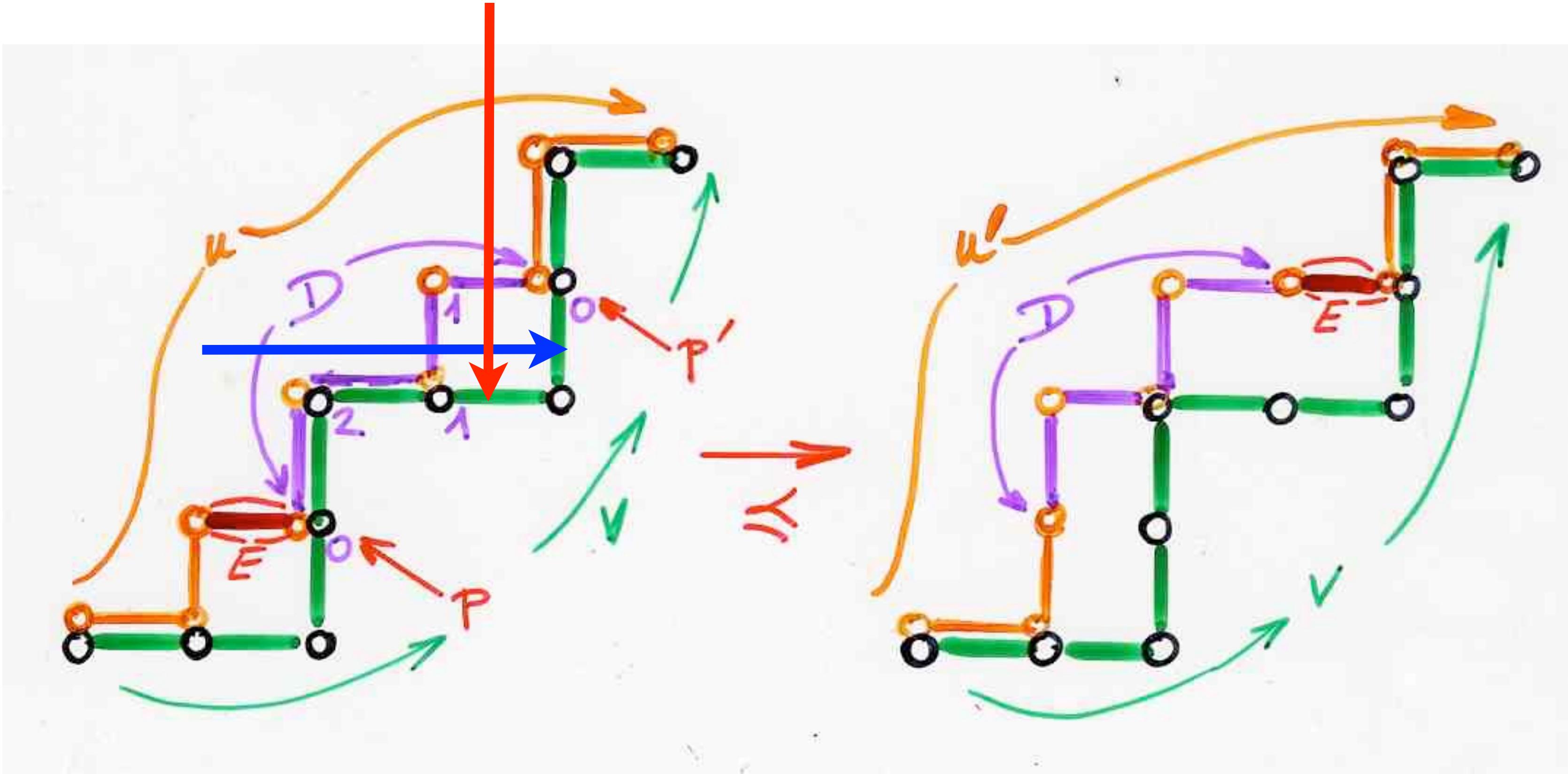


Take an East step of the path u (here in red), take the associated purple integer k associated to the vertex p at the end of the East step (here k=0). Then take the longest portion of the path u such that all the associated purple numbers are strictly bigger than k, until one get a vertex p' with purple number = k. We get the portion D of the path u (in purple on the figure). Then exchange the selected East step with the portion D.


Young covering relation
Tamari covering
Tamari (3,5)
≃
$T_{\overleftarrow{v}}$
$\overleftarrow{v}$ =

an example


Tamari (3,5)
≃
$T_{\overleftarrow{v}}$
$\overleftarrow{v}$ =

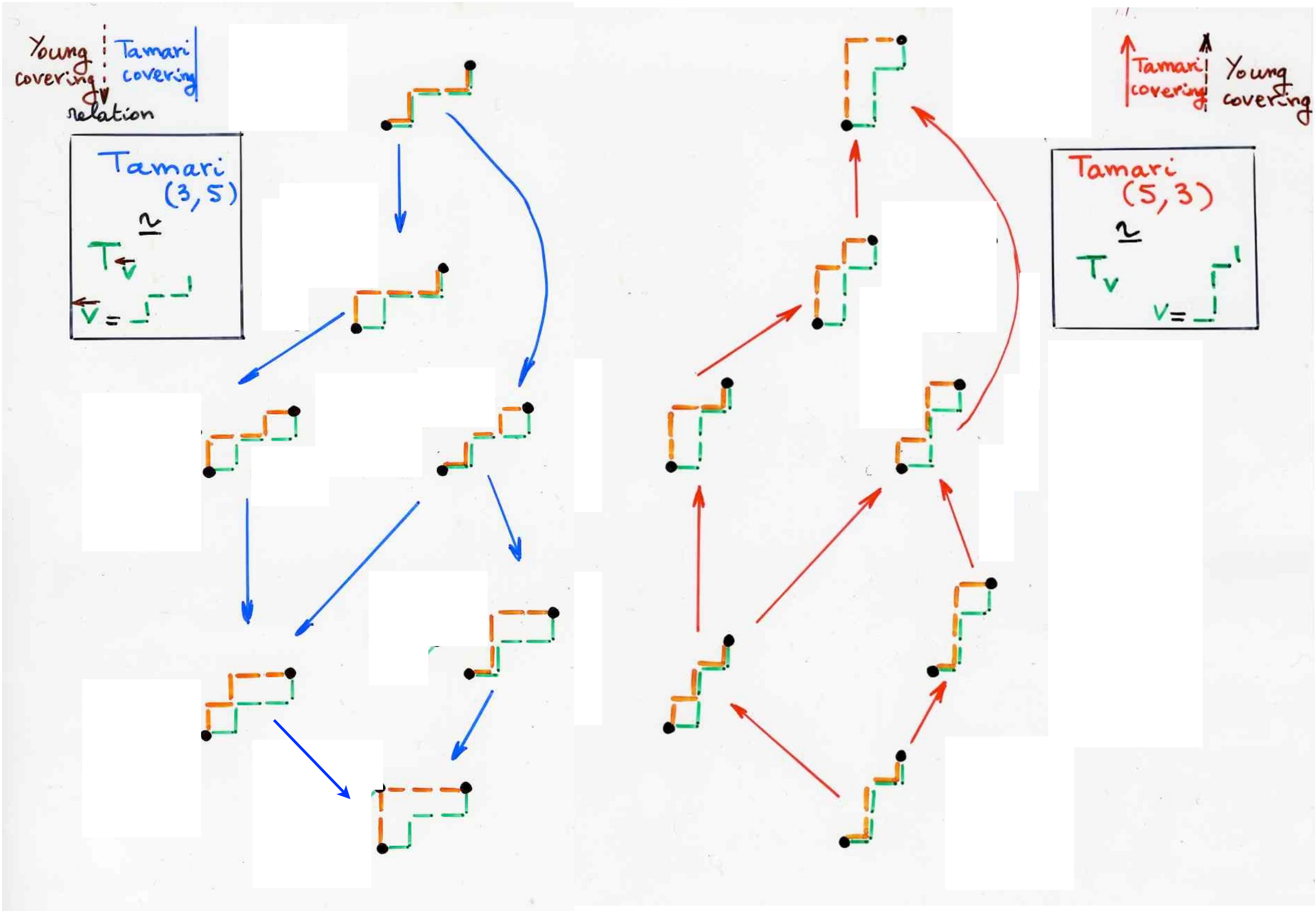


«row covering relation»

«column covering relation»

mirror image, exchange N and E

Duality $T_v \leftrightarrow T_{\overleftarrow{v}}$



Thm 1. For any path v, T_v is a lattice

Thm 2. The lattice T_v is isomorphic to the dual of $T_{\overleftarrow{v}}$

Thm 3. The usual Tamari lattice T_n can be partitioned into intervals indexed by the 2^{n-1} paths v of length $(n-1)$ with $\{E, N\}$ steps,

$$T_n \cong \biguplus_{|v|=n-1} I_v ,$$

where each $I_v \cong T_v$.

from:

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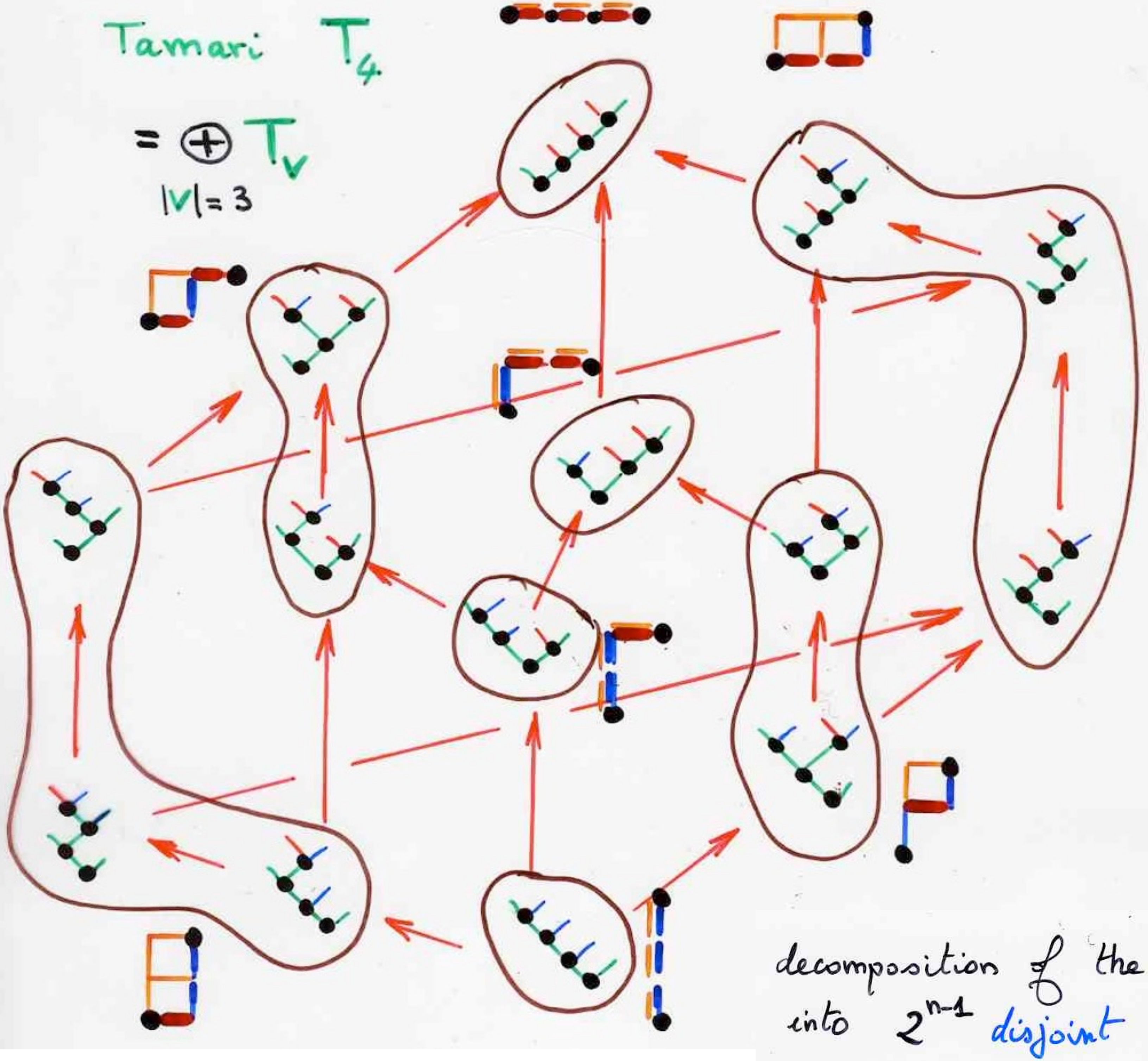
Tamari T_4

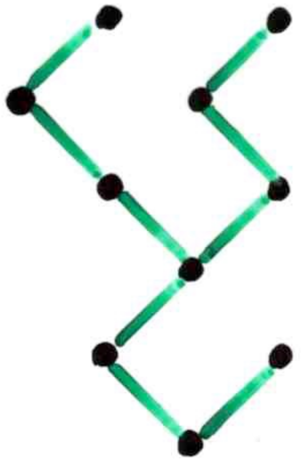
$= \bigoplus T_v$
$|v| = 3$

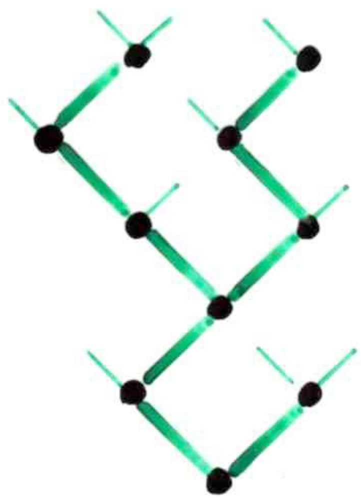


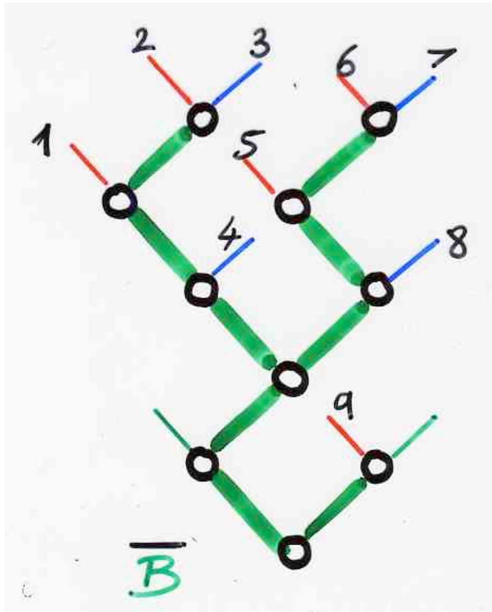
decomposition of the lattice T_n into 2^{n-1} disjoint intervals

idea of the proof of
Theorems 1,2, 3
with a bijection:

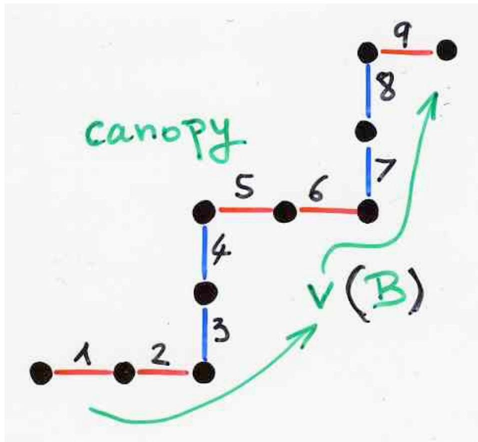
binary tree B $\longrightarrow$ pair of paths (u,v)

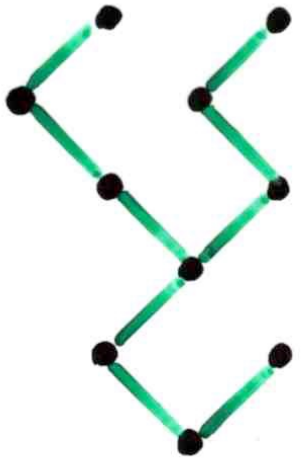


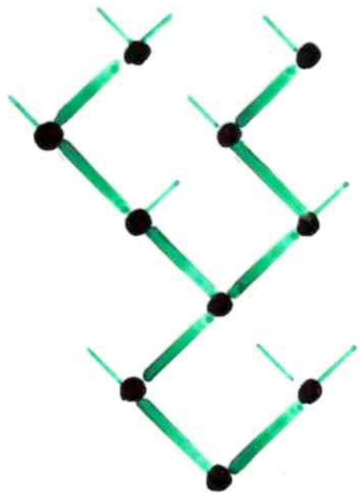


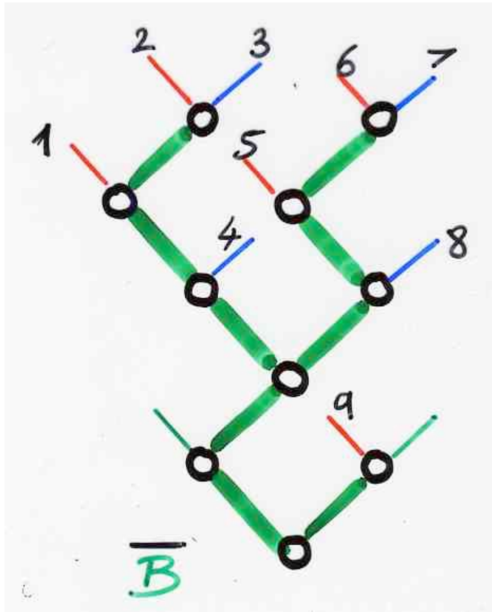


the path v is the canopy of the binary tree B

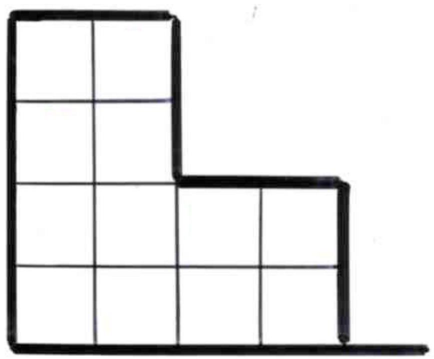


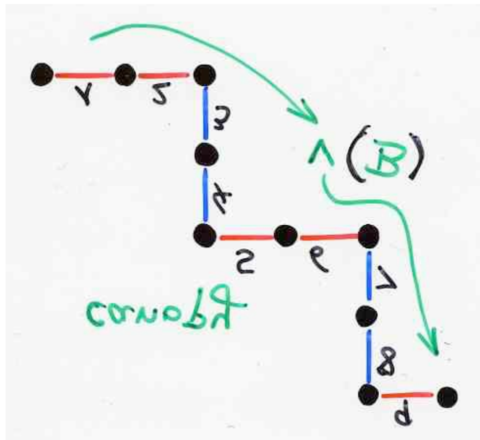


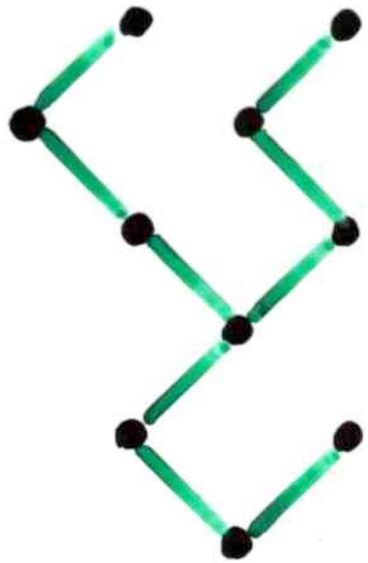


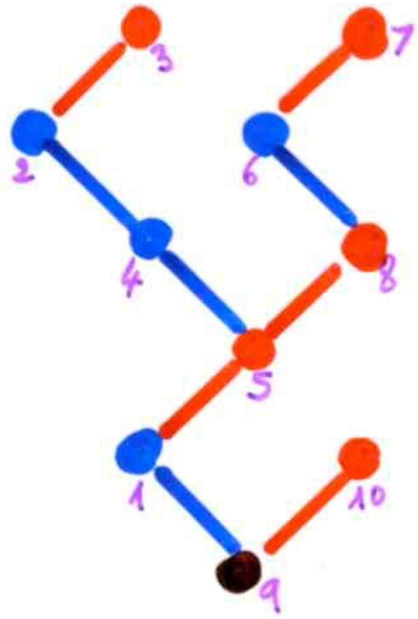


which gives a Ferrers diagram
(in french notation)



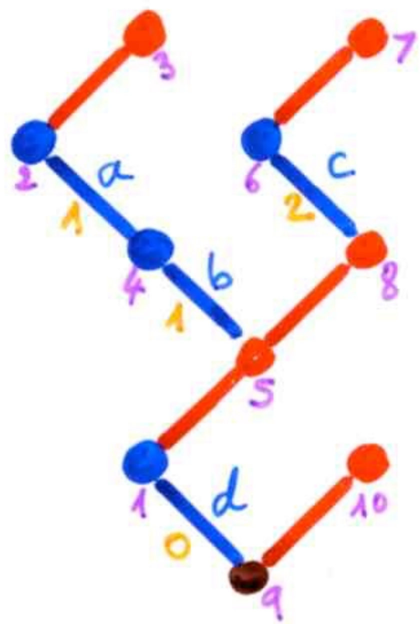






inorder
(= symmetric order)

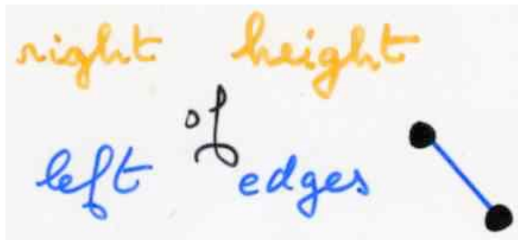
The left edges (in blue) of the binary tree are ordered according to the in-order (= symmetric order) of the first vertex of the edge.
Here the order is a, b, c, d.



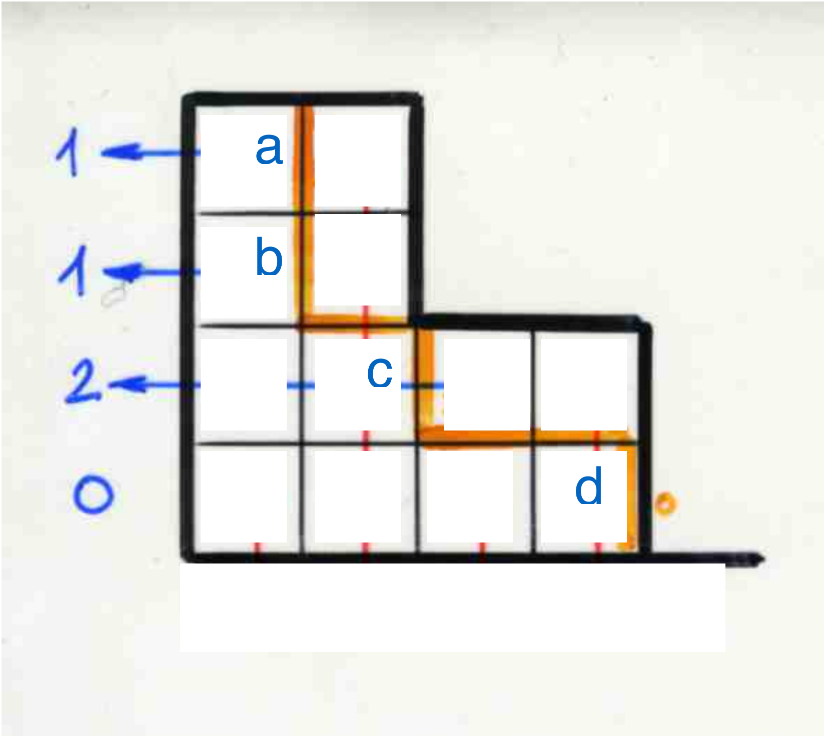
Then the right height of a left edge is the number of right edges (in red) needed to reach the vertices of that left edge.

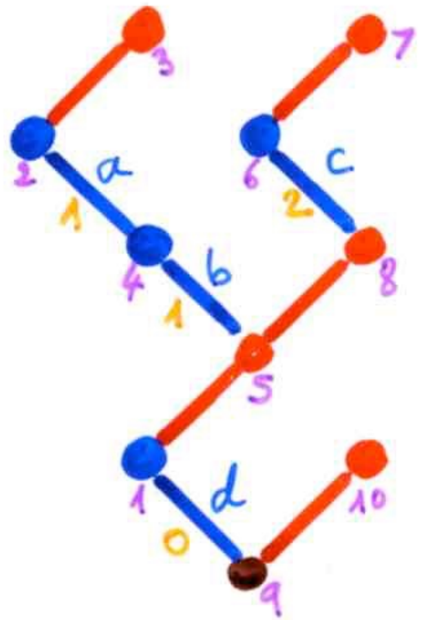
we get the vector:

a	b	c	d
1	1	2	0

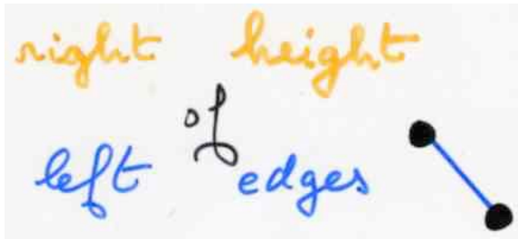


A path u (here in yellow) is uniquely defined by the following process: the South steps are ordered from top to down and associated to the order of the blue edges a,b, c, d. The distance from each North step of u to the North-East border (the path v) is given by the corresponding blue number (the right height of the left edge)





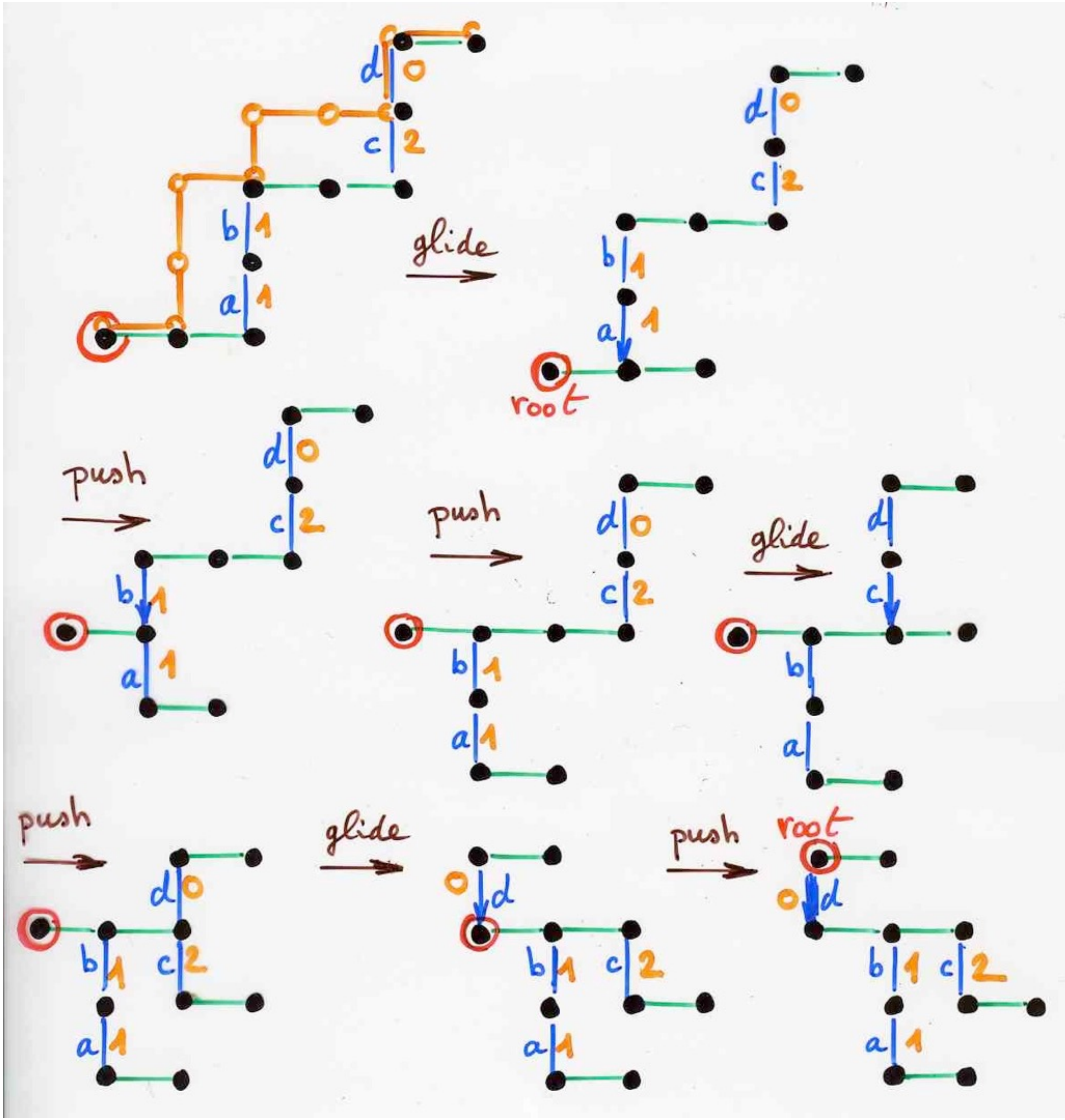
a	b	c	d
1	1	2	0



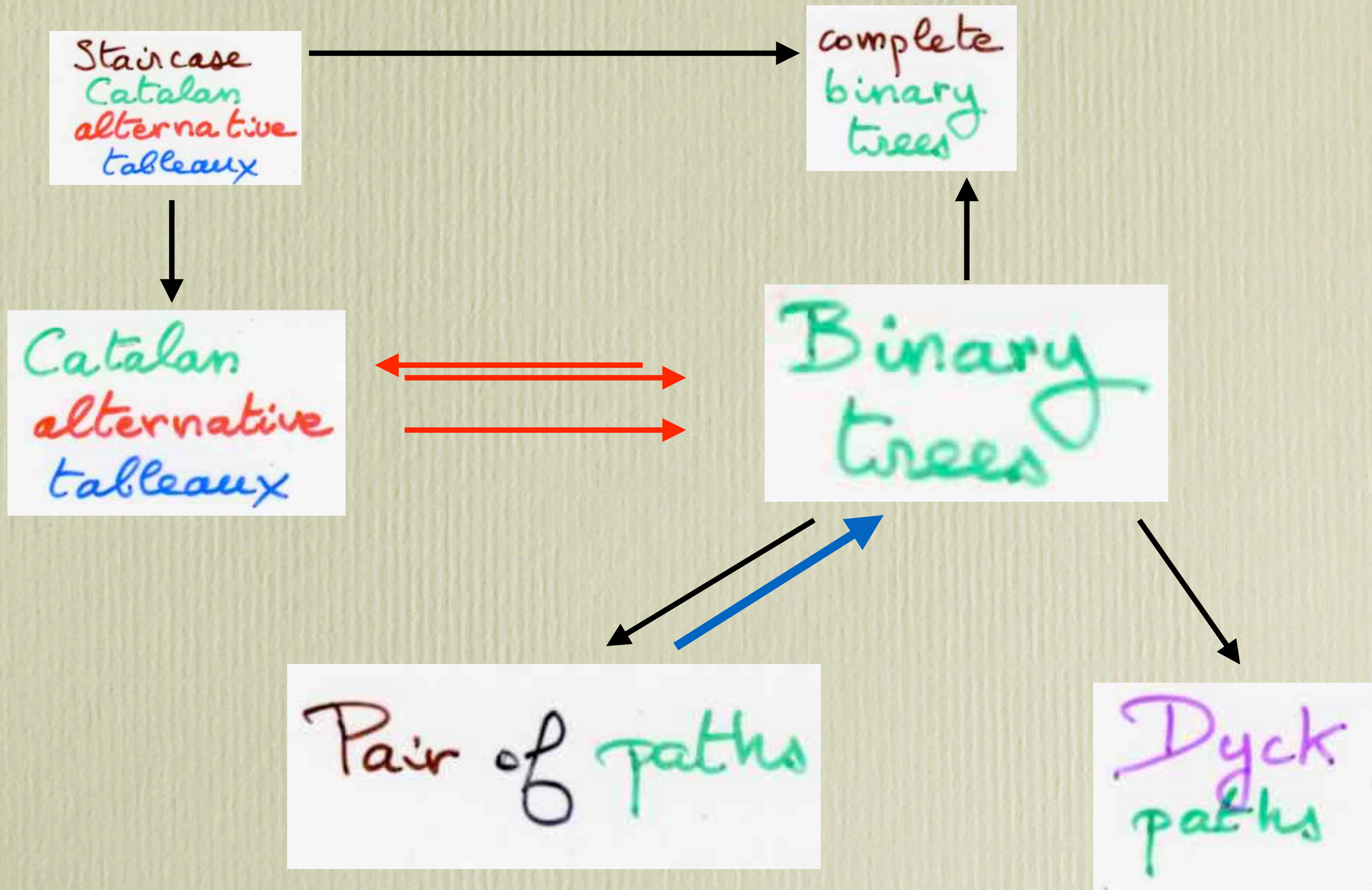
reverse bijection

pair of paths (u,v) $\longrightarrow$ binary tree B

the «push-gliding» algorithm



glide
root
push
push
glide
push
glide
push
root


Staircase Catalan alternative tableaux
complete binary trees
Catalan alternative tableaux
Binary trees
Pair of paths
Dyck paths

(idea of the) proof of Theorems 1,2 3

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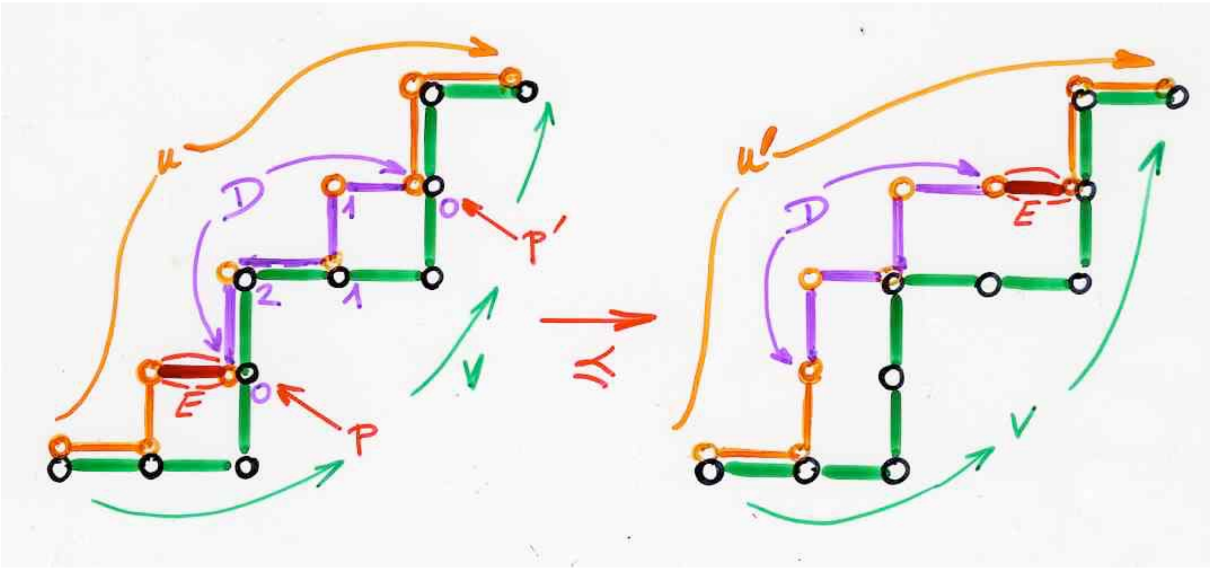
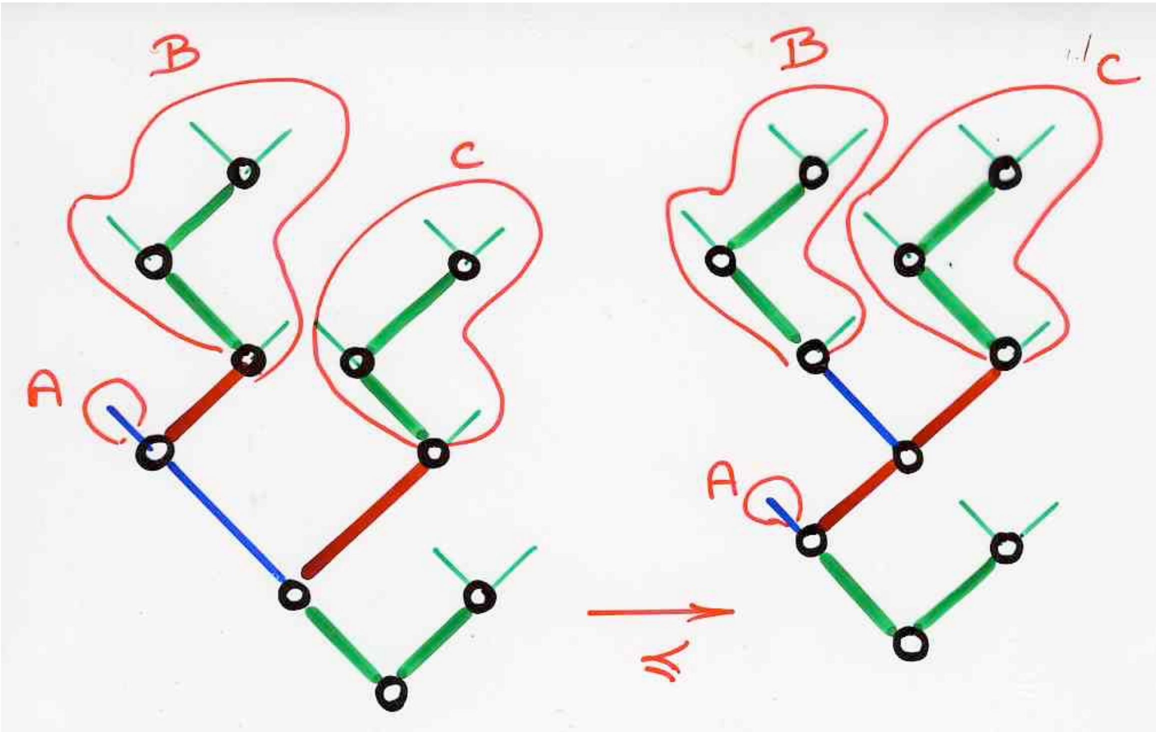
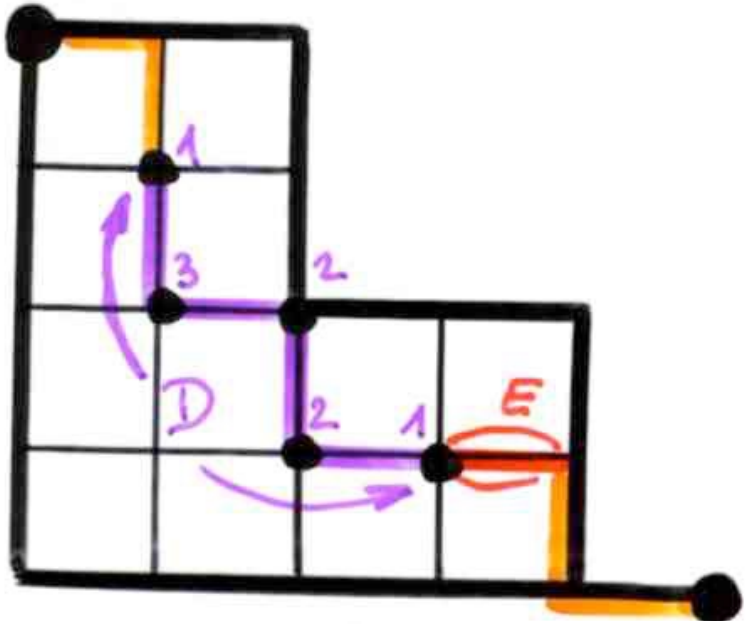
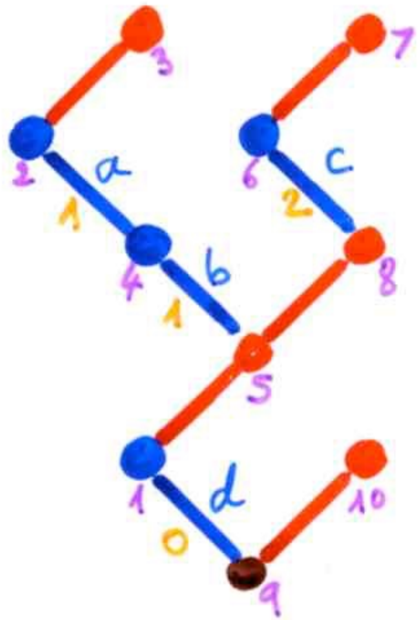


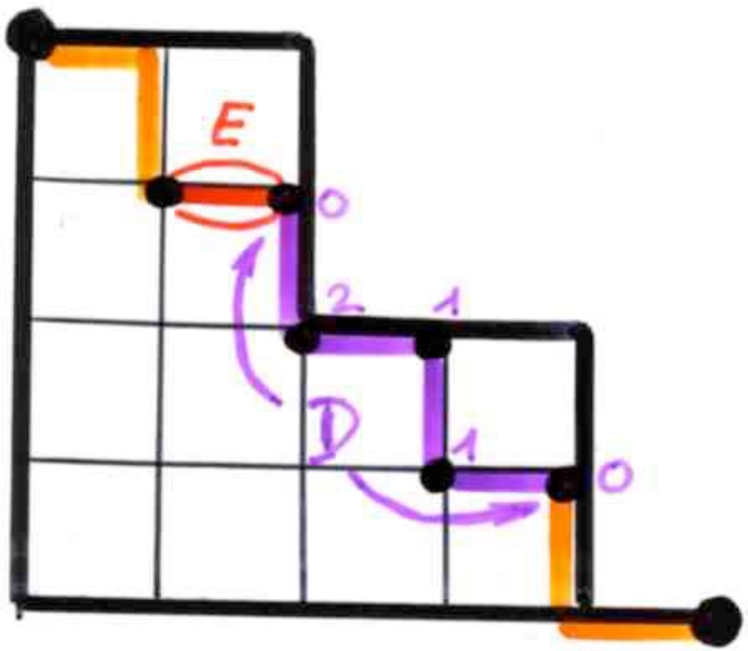


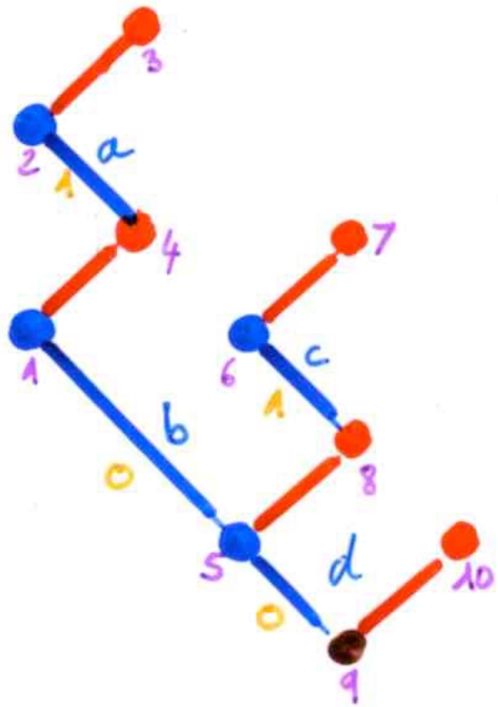
the covering relation in T_V
and the corresponding rotation
in (ordinary) T





an example



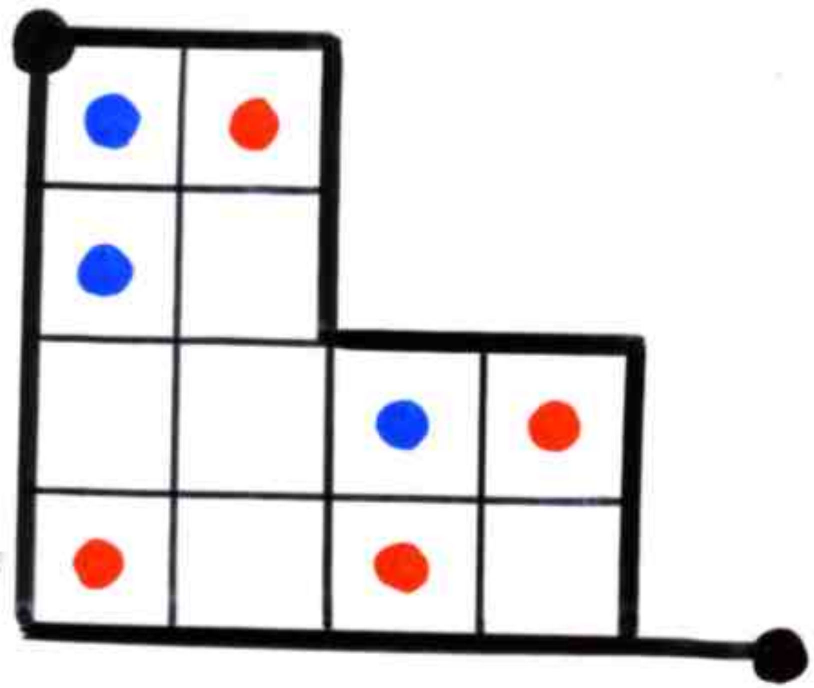


Tamari(v) lattice as a maule

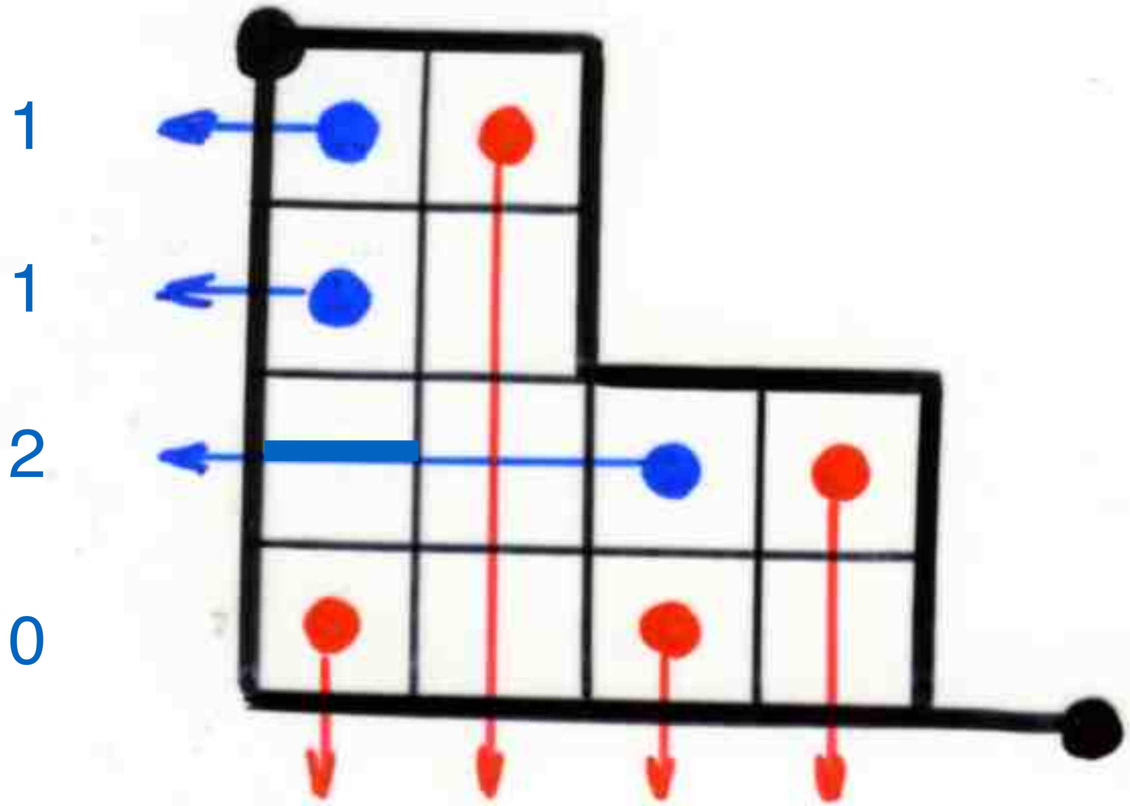
bijection

Catalan alternative tableaux

pair of paths

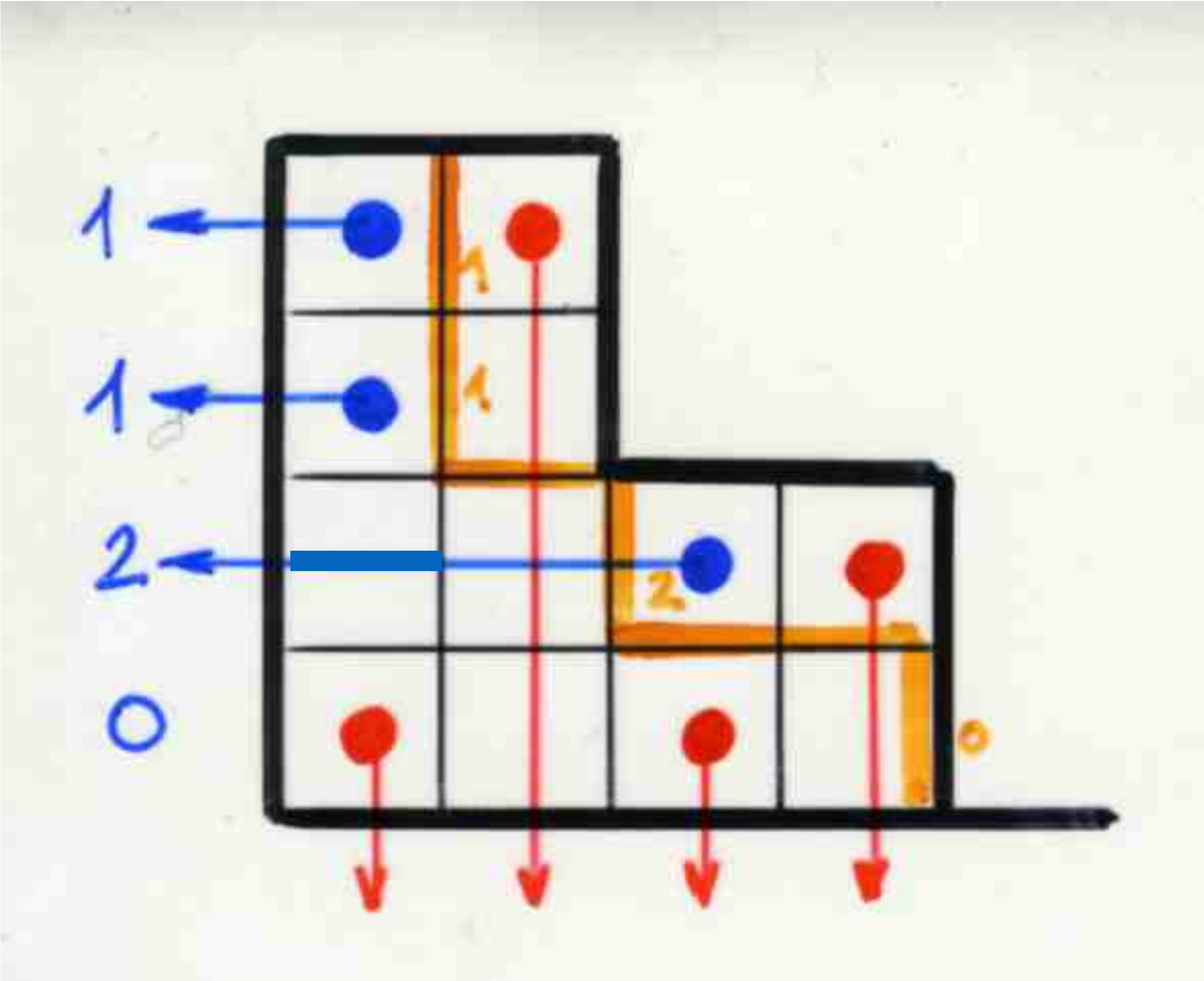


For each row of a Catalan alternative tableau we associate a blue number by the following rule:



- 0 if there are no blue point in the row
- 1 + the number of cells in the row which are of the type
  (i.e. there is a blue point at its right, but no red point above)

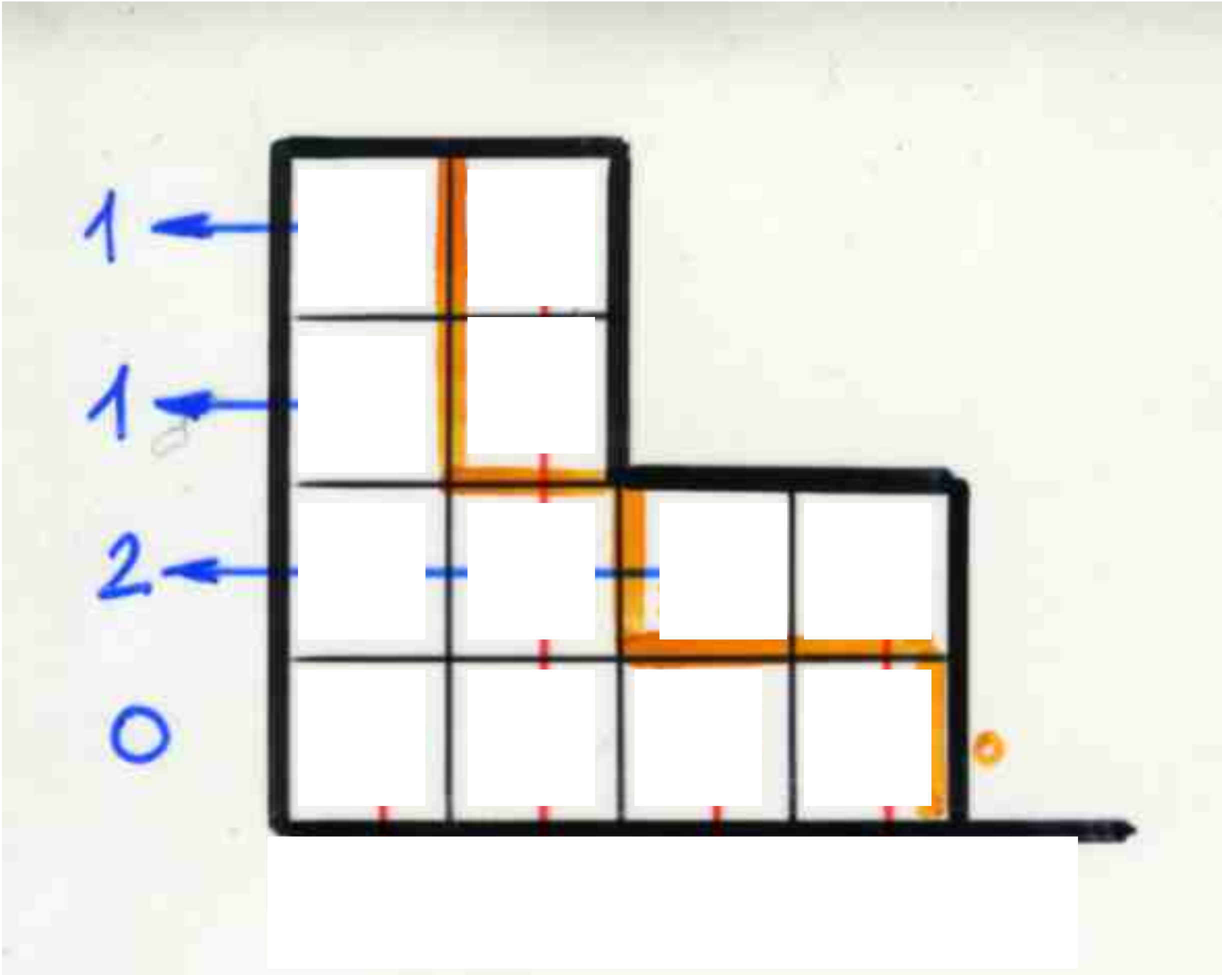
We get a vector P of blue numbers (here P= 1, 1, 2, 0), which we call the **Adela row vector** (see slides 116-119).



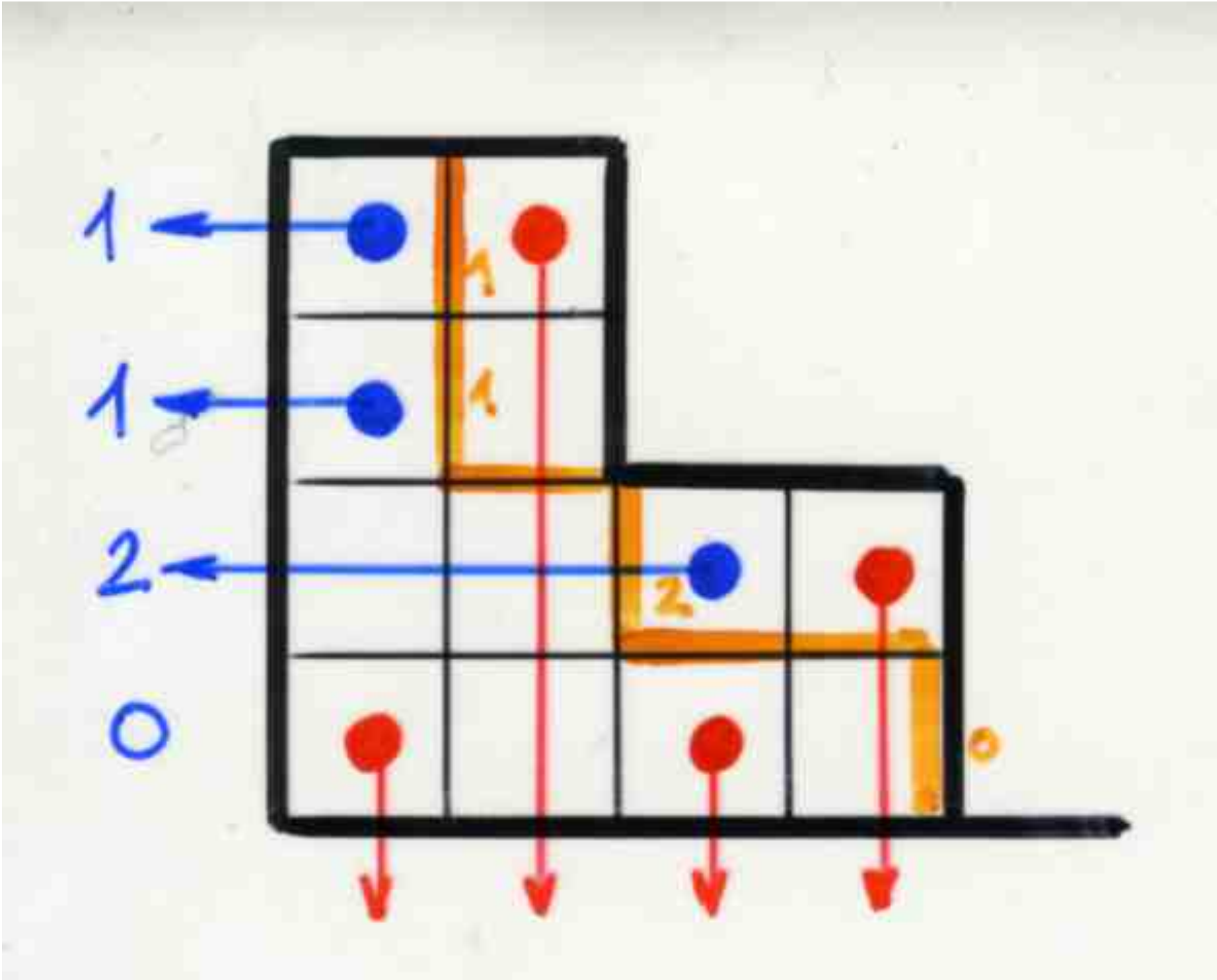
From this vector P, we define a path u (in yellow) such that the distance of each South step of u to the North-East border is given by the corresponding blue number (analog rule in slide 29) reverse bijection

pair of paths

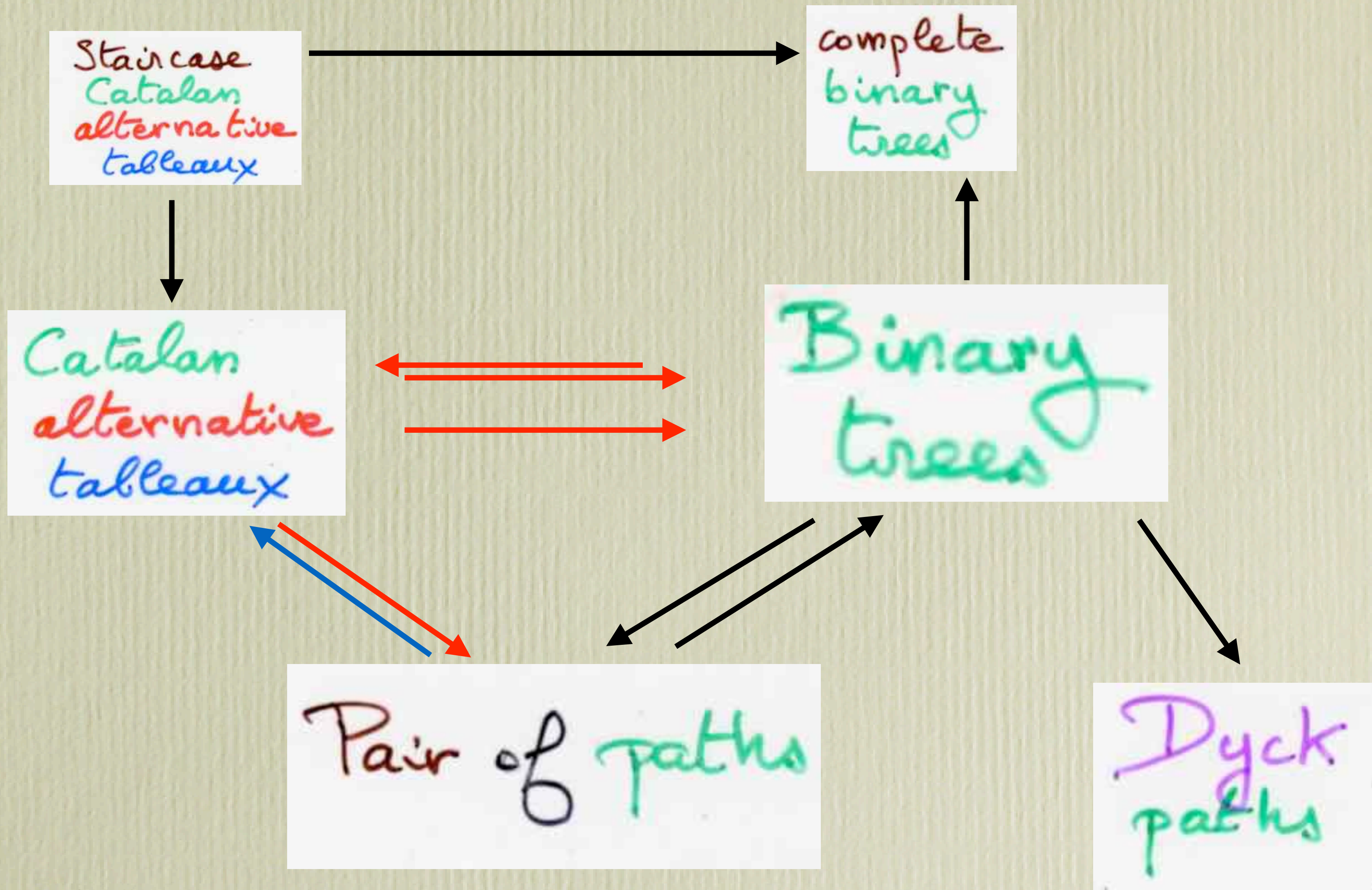
Catalan alternative tableaux



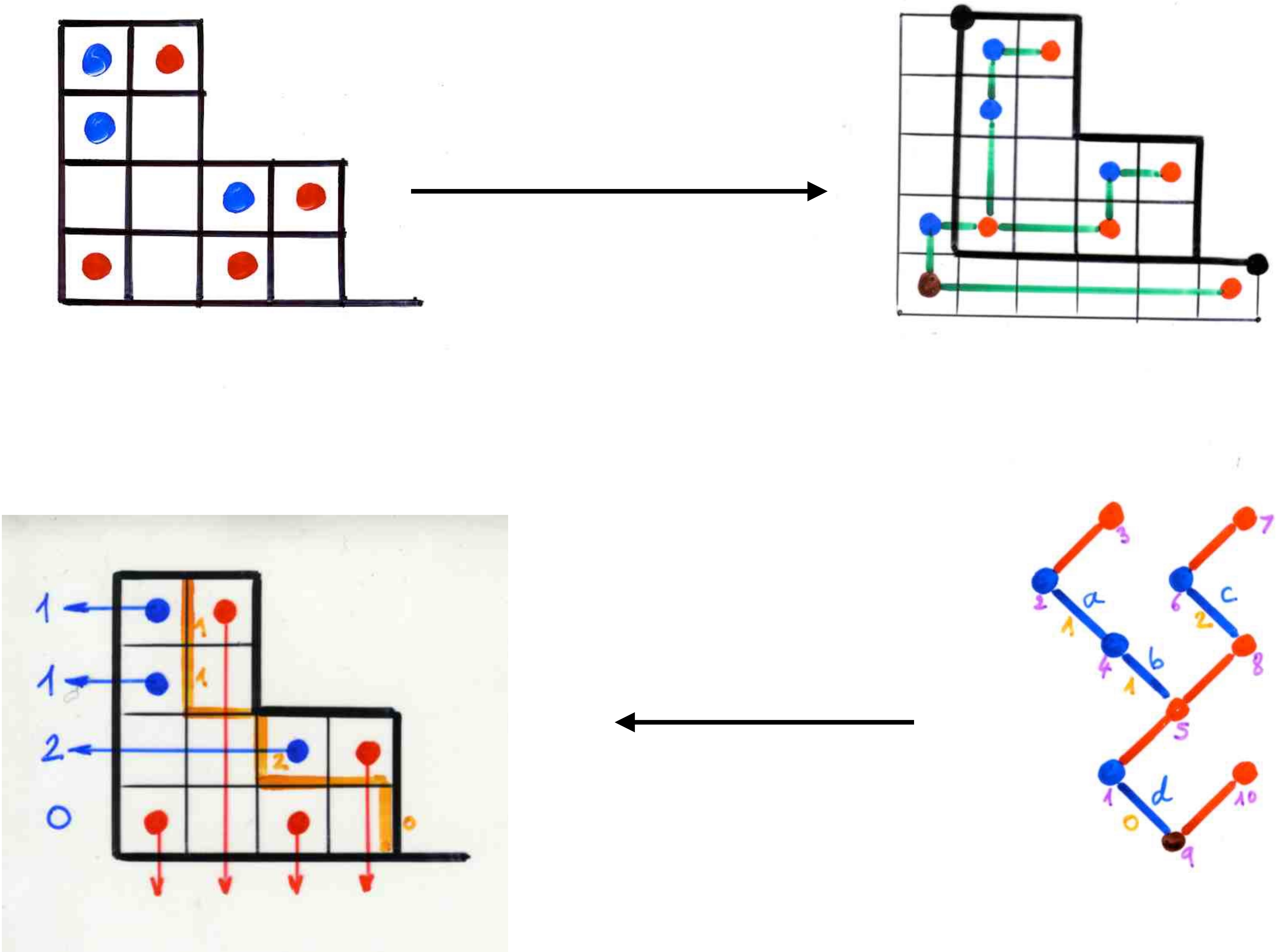
From the path u we get the blue numbers as the distance in each row of the South step of u to the border of Ferrers diagram (path v). We get a vector V (here V = 1, 1, 2, 0)



Then there is a unique Catalan alternative tableau whose Adela row vector P (see definition slide 39) is equal to V. This tableau can be obtained by filling the rows from top to down with first a (possible) blue point and then the red points in a unique way from V.



commutative diagram !

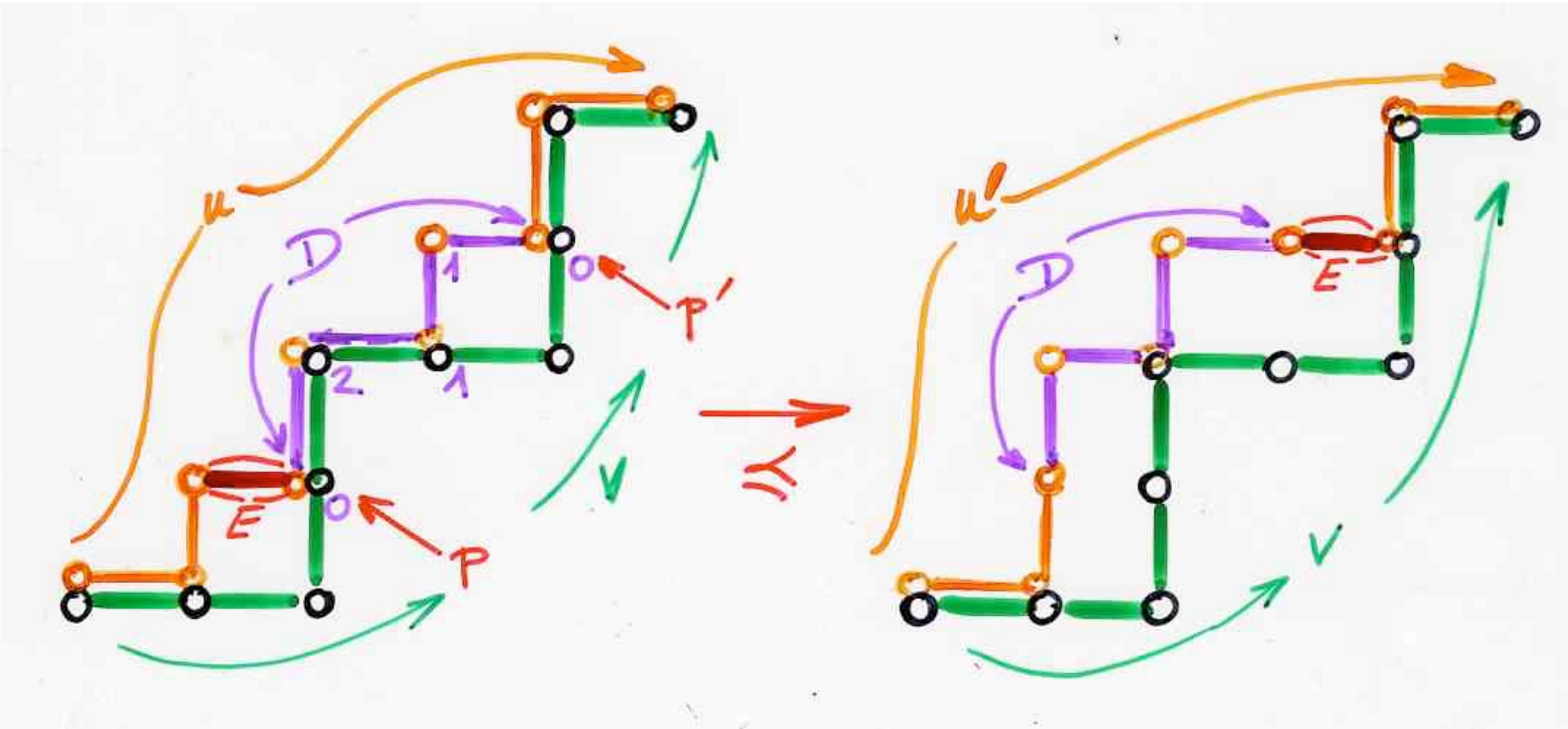


commutative diagram !

equivalence Γ-move

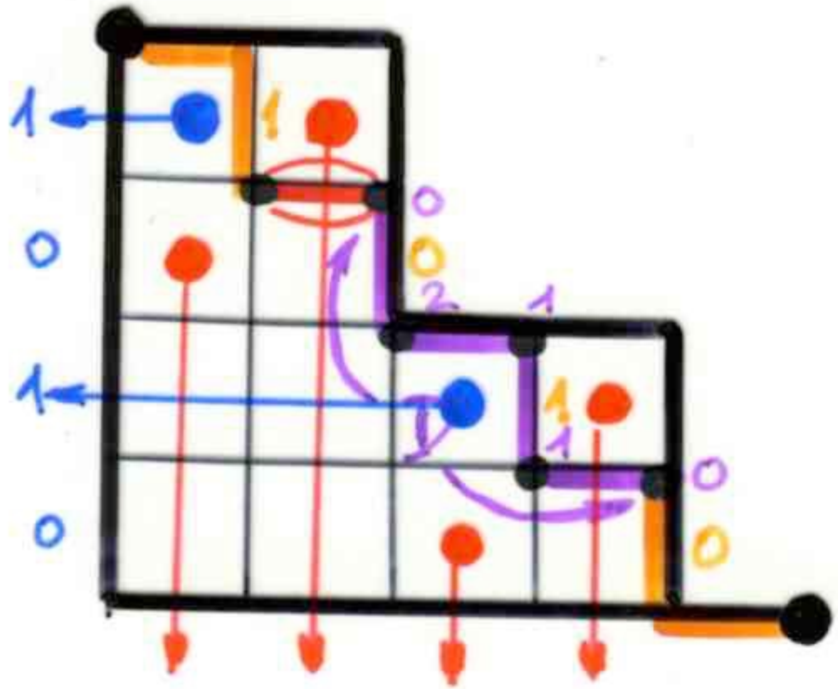
and

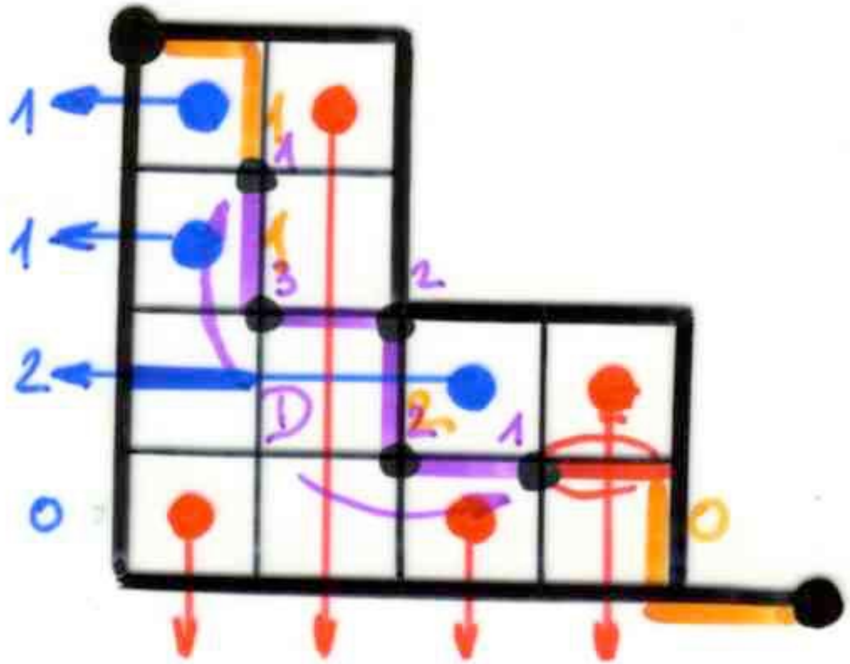
covering relation in Tamari(v)

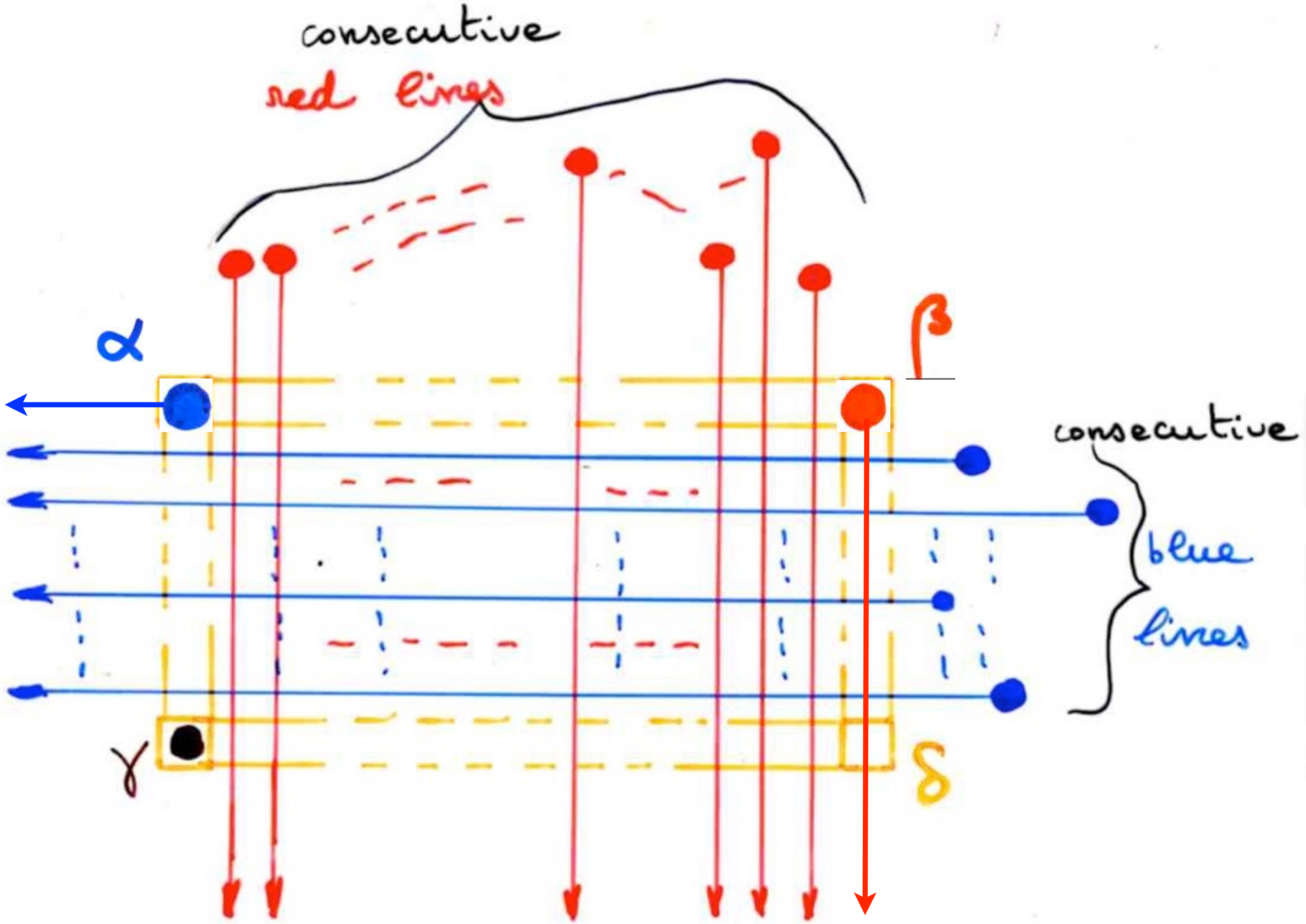


the covering relation
in the poset T_v

(also denoted by Tamari(v))

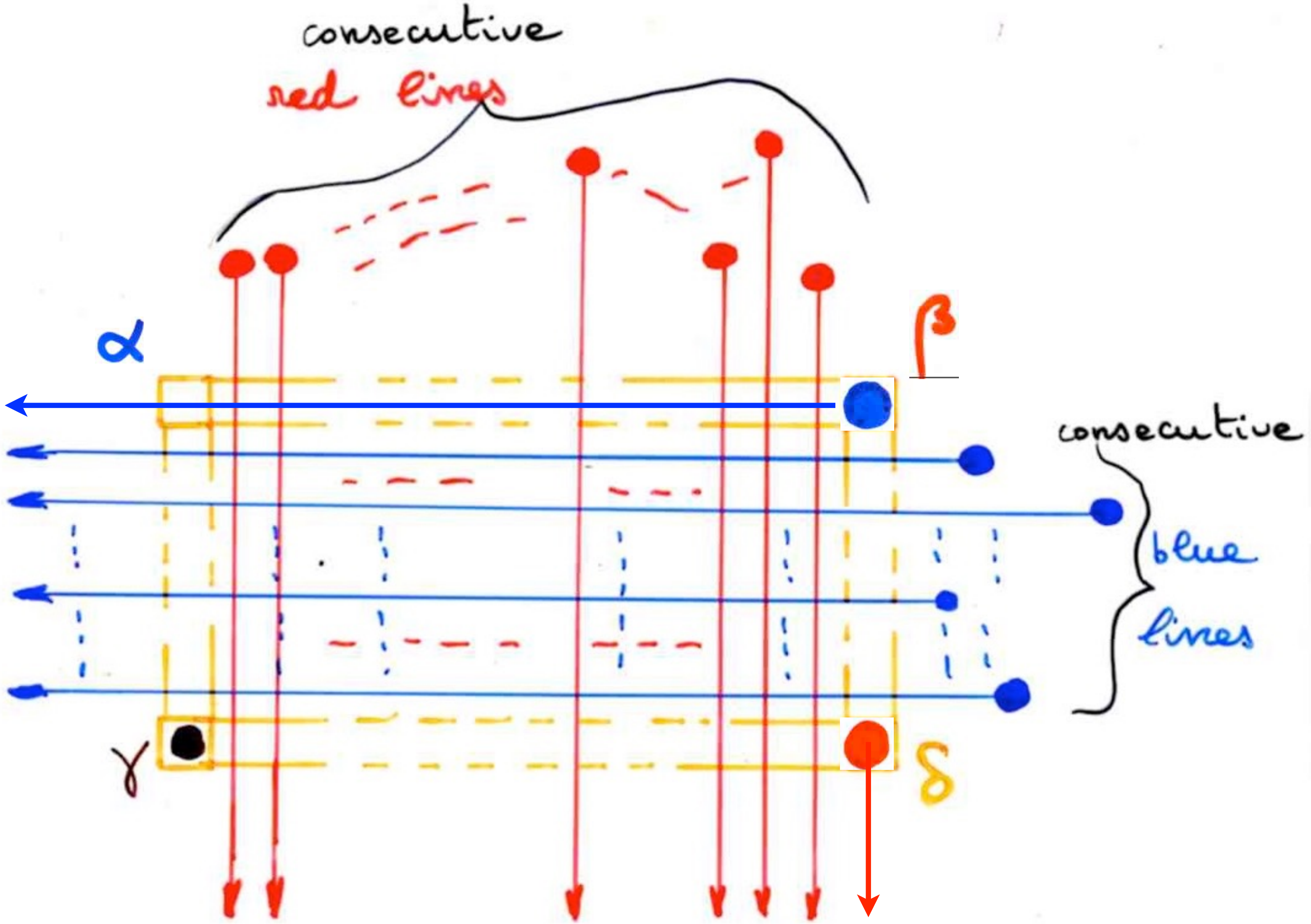




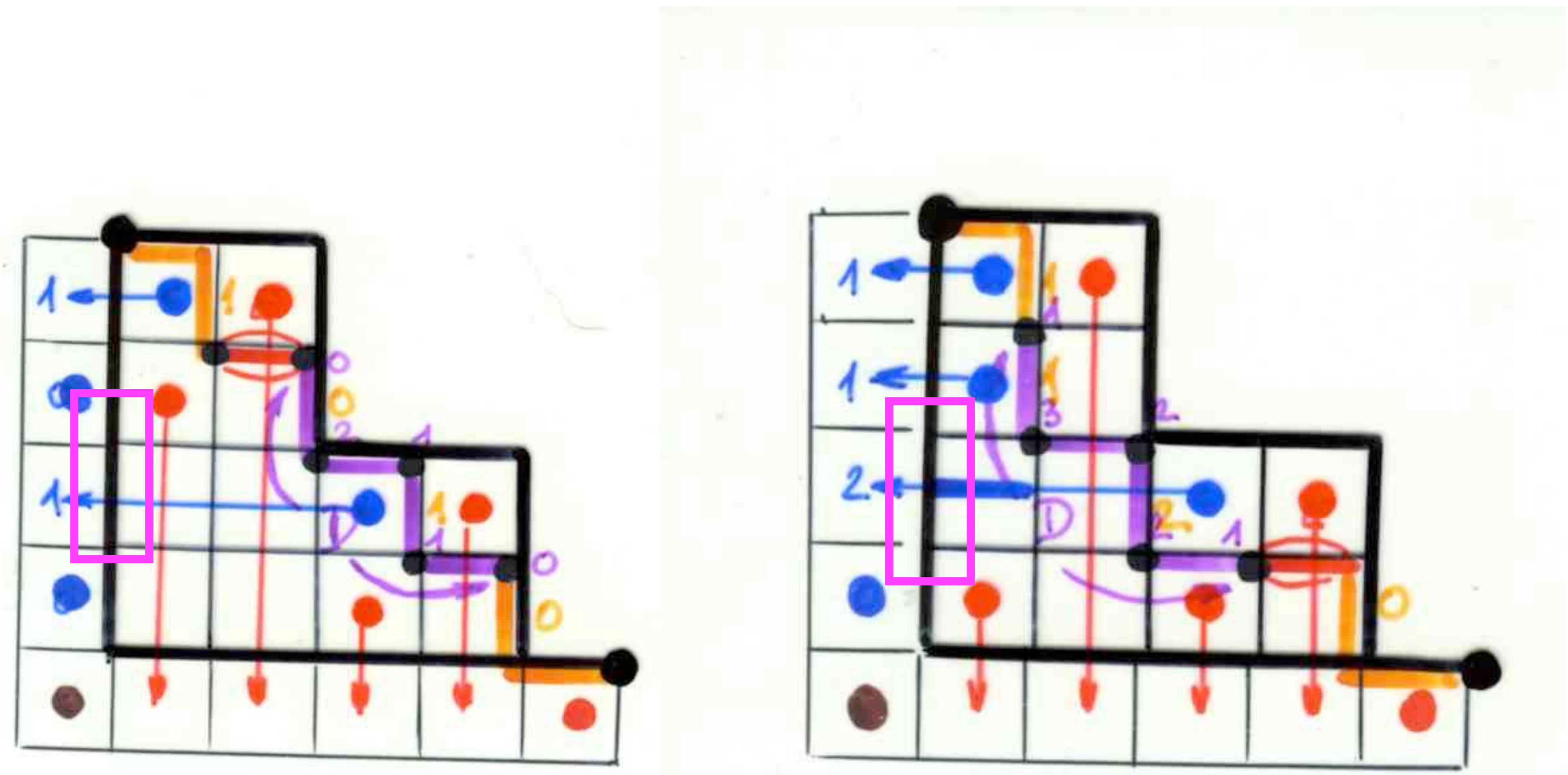


from the main Lemma, slides 121-122, part I

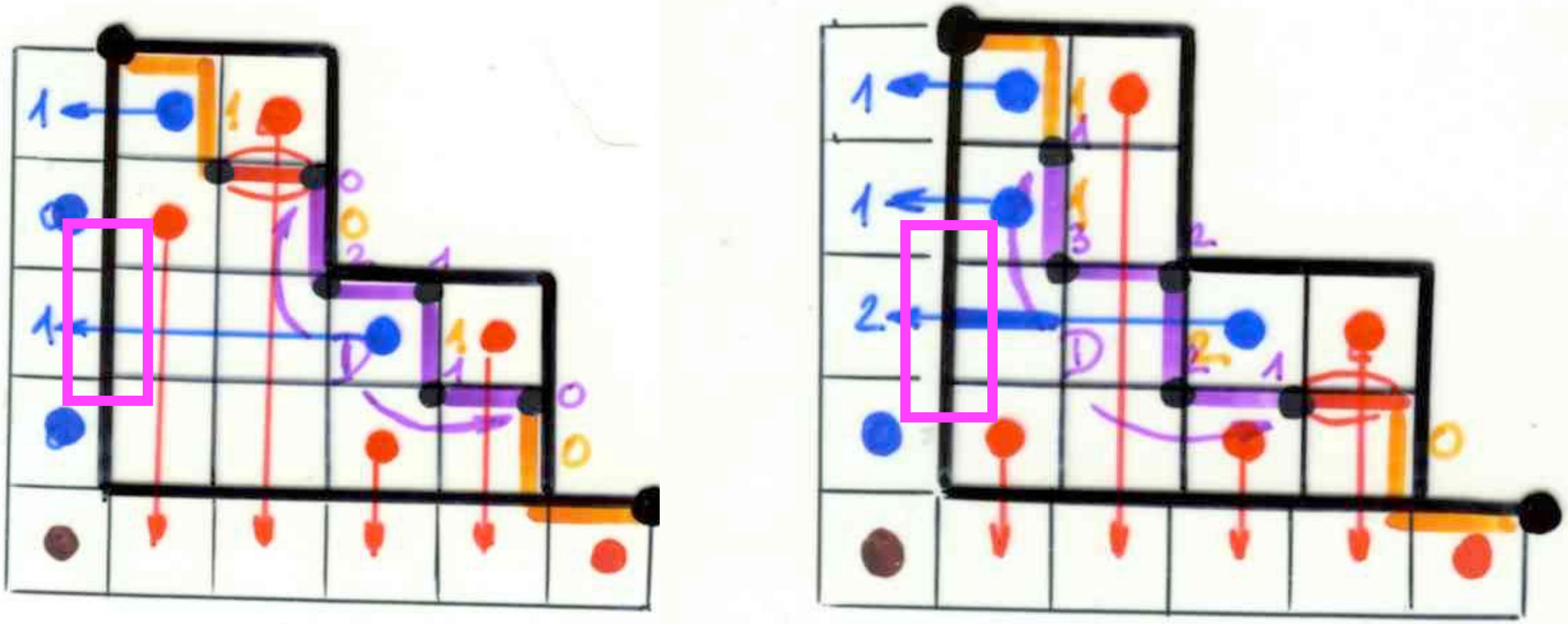
A possible Γ-move in a Catalan alternating tableau T



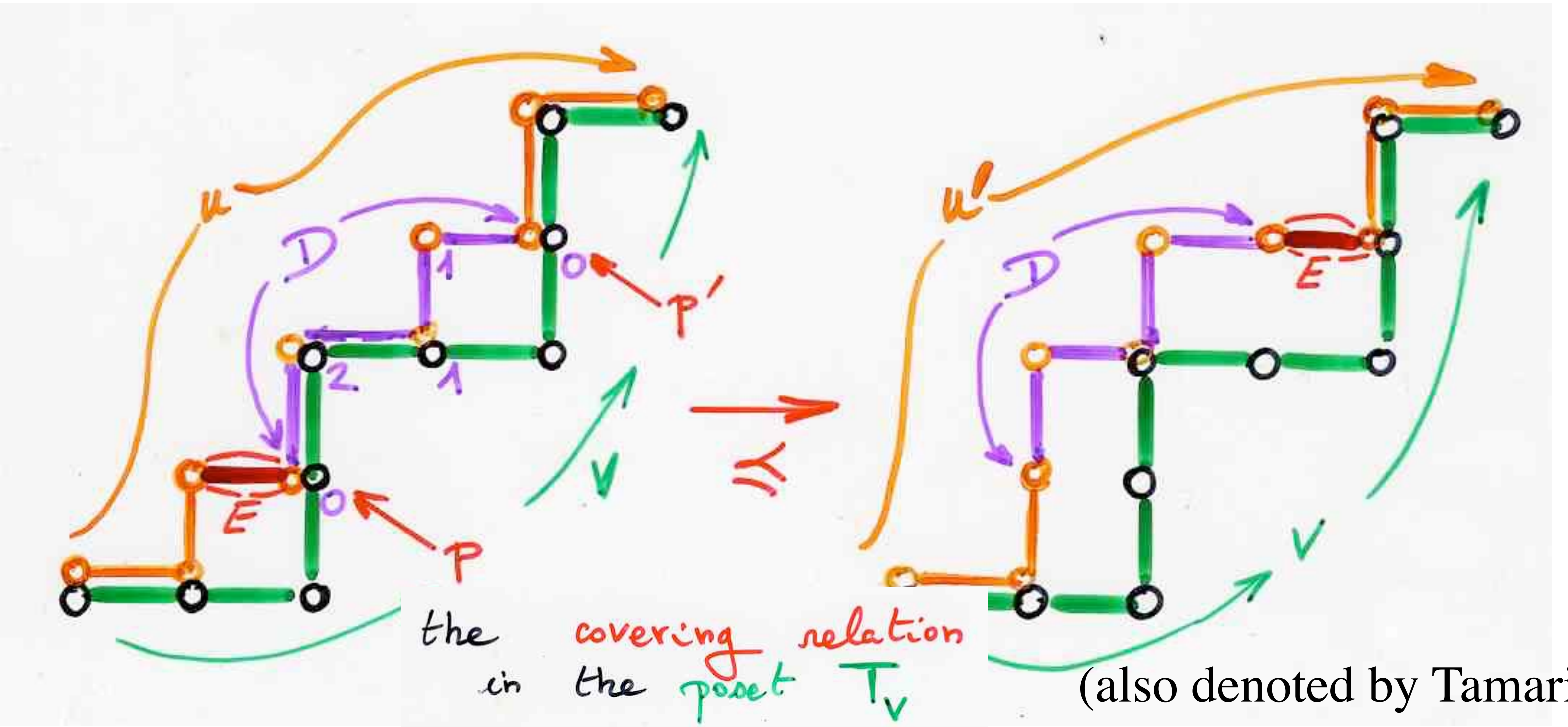
For a Γ-move in a Catalan alternating tableau T, the elements of the Adela row vector P (definition slide 39) will increase by one for all the rows of the rectangle defined by α, β, γ , δ (except the row γ δ). In all other rows, the coordinates will remain invariant.



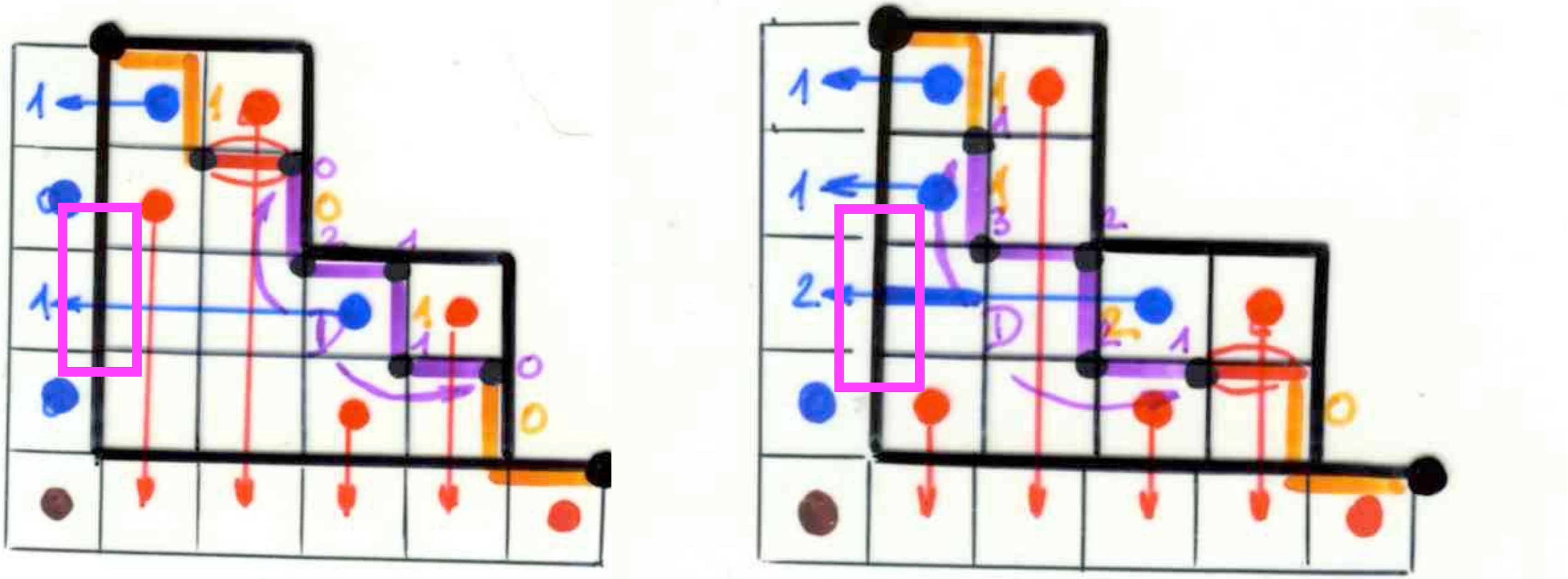
Such possible Γ-move in a Catalan alternating tableau T, related to the rectangle defined by α, β, γ , δ, corresponds exactly to a possible flip in the pair of paths (u,v). The rows of the rectangle α, β, γ , δ (except the row γ , δ) correspond to the North steps of the portion D of the path u (in purple on the figure)



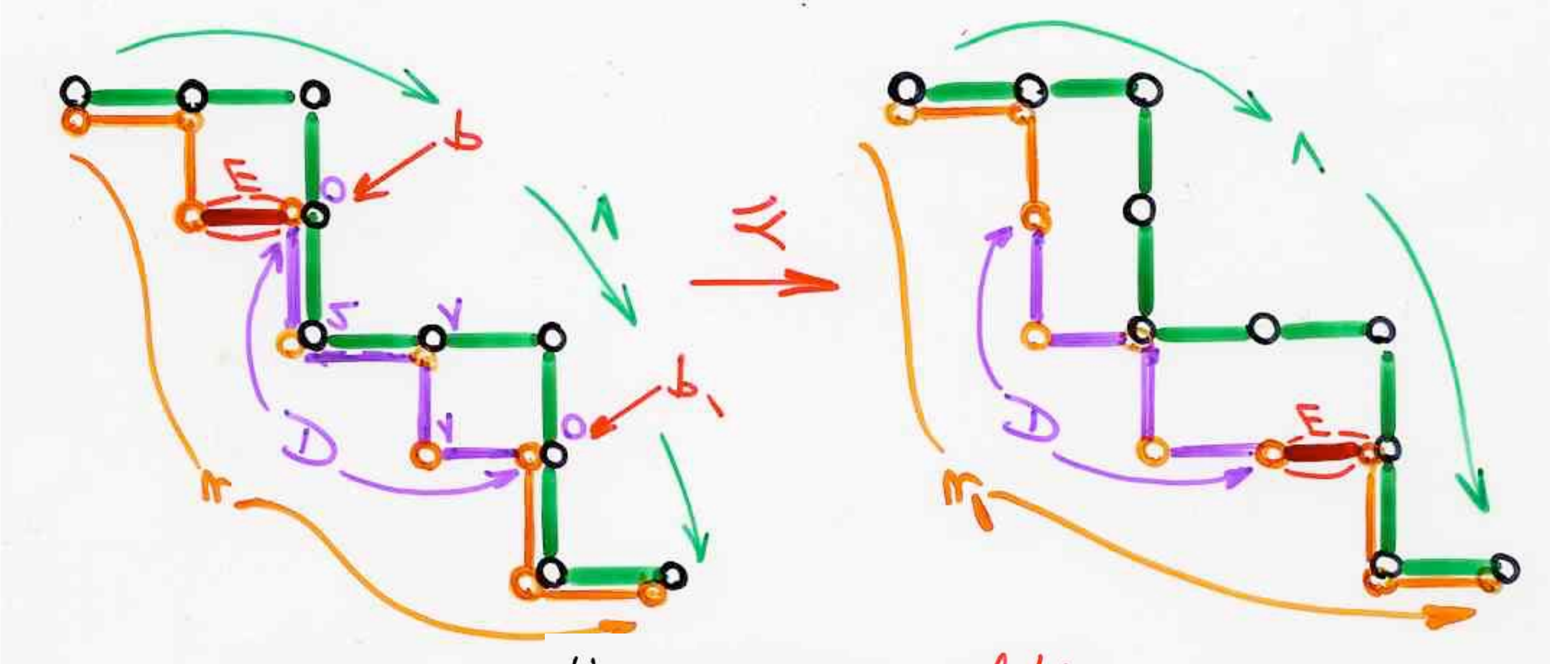
equivalence between a flip defining the covering relation of Tamari(v) and a Γ-move



(also denoted by Tamari(v))



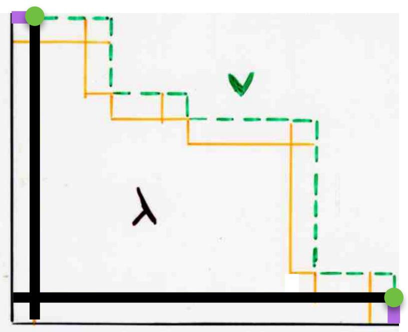
equivalence between a flip defining the covering relation of Tamari(v) and a Γ-move



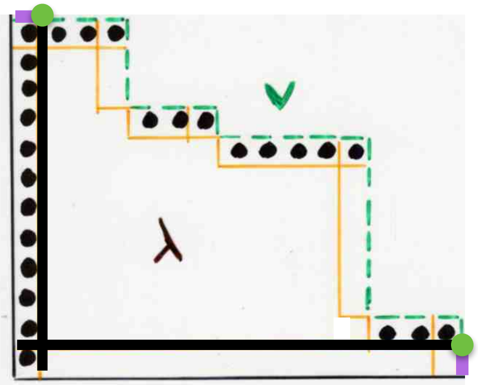
the covering relation in the poset T_v (also denoted by Tamari(v))

Main theorem

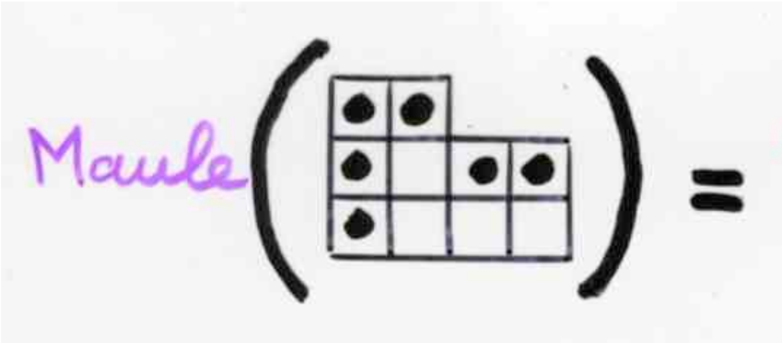
Ferrers diagram λ with profile ν

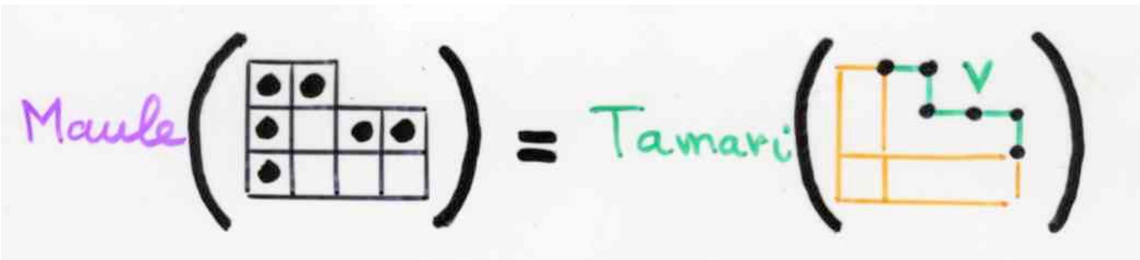


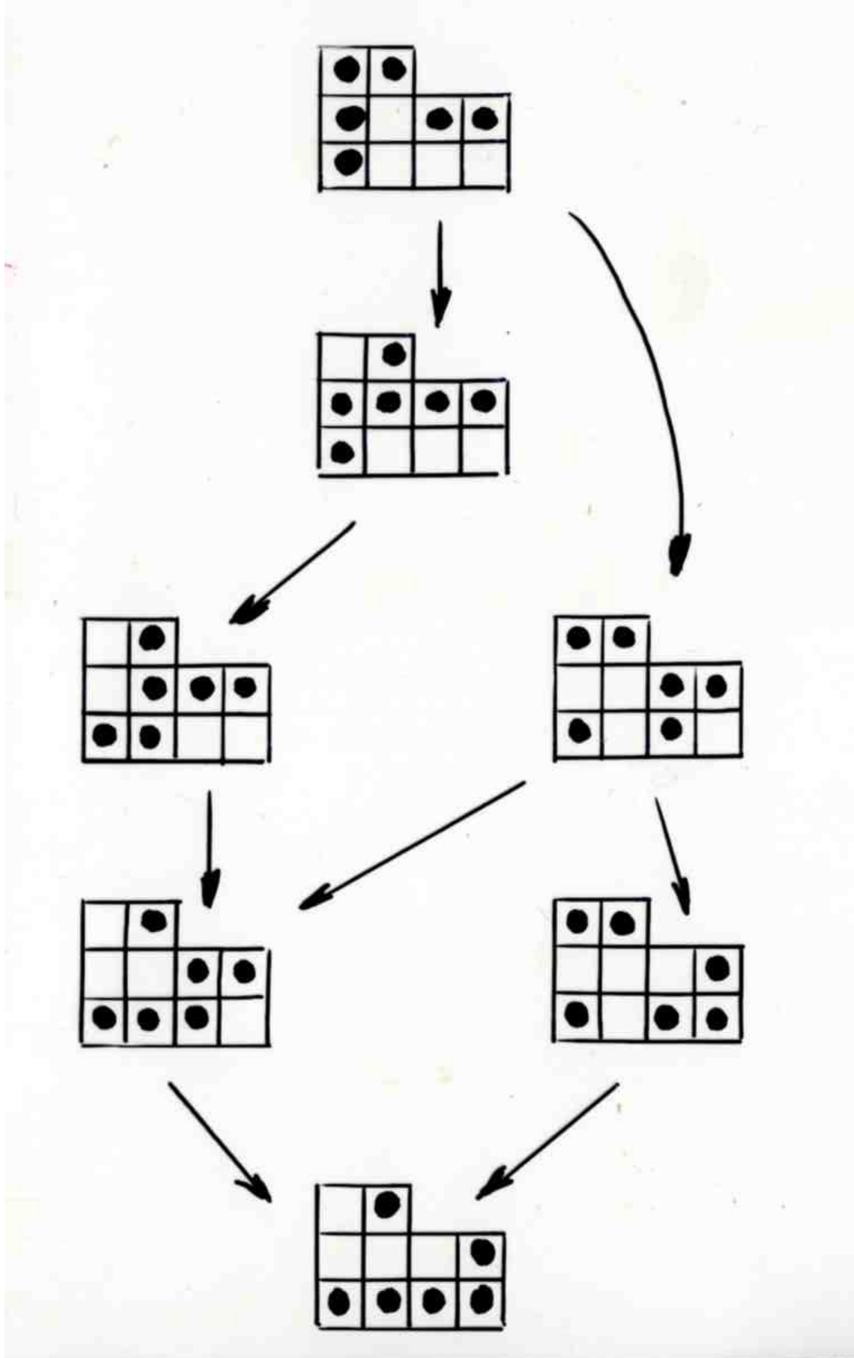
Let $X(\lambda) = X(\nu)$ be the cloud

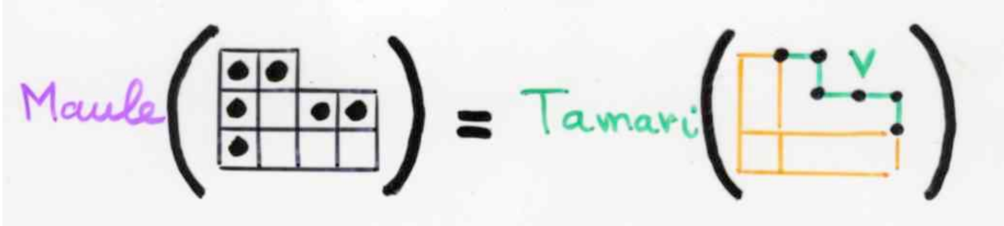


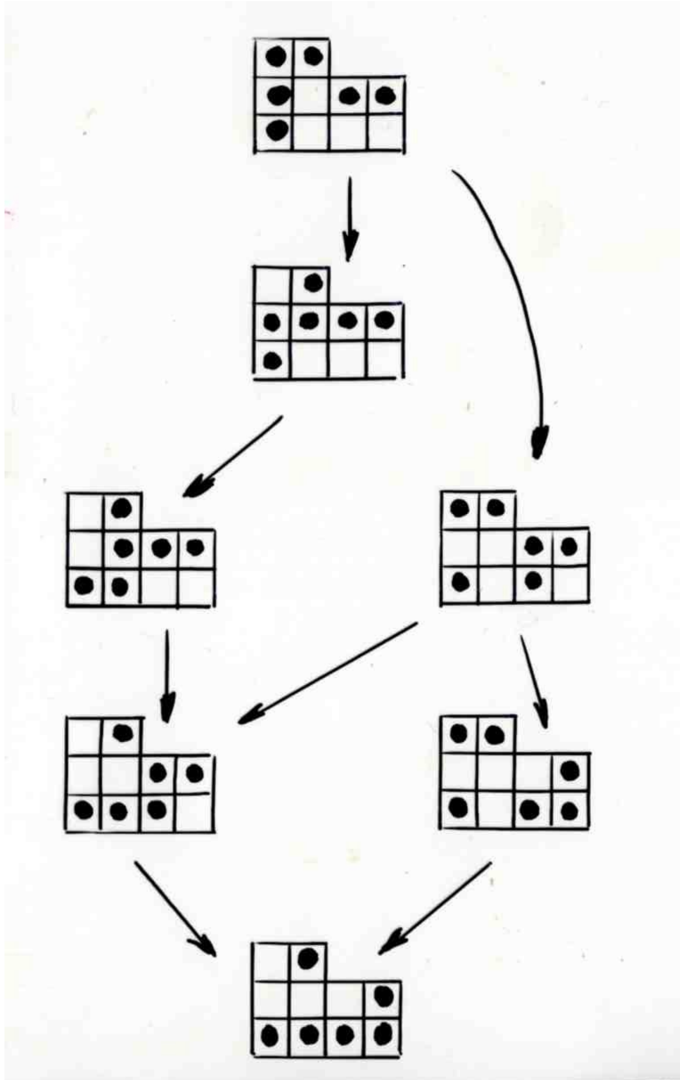
$$\mathrm{Tamari}(\nu) = \mathrm{Maule}(X(\nu))$$

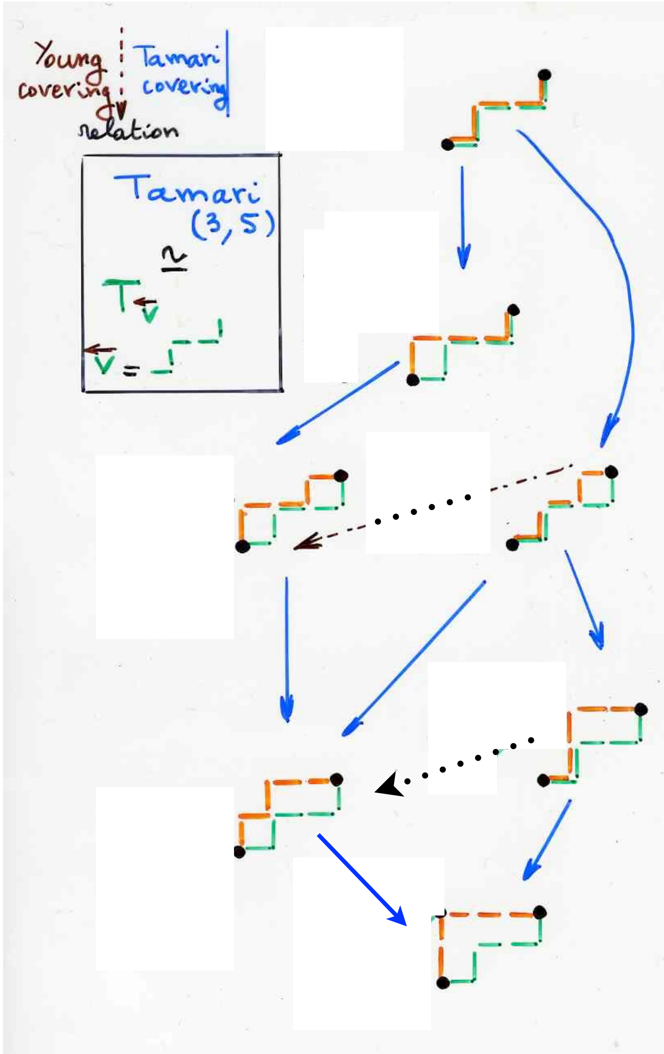

Maule
=

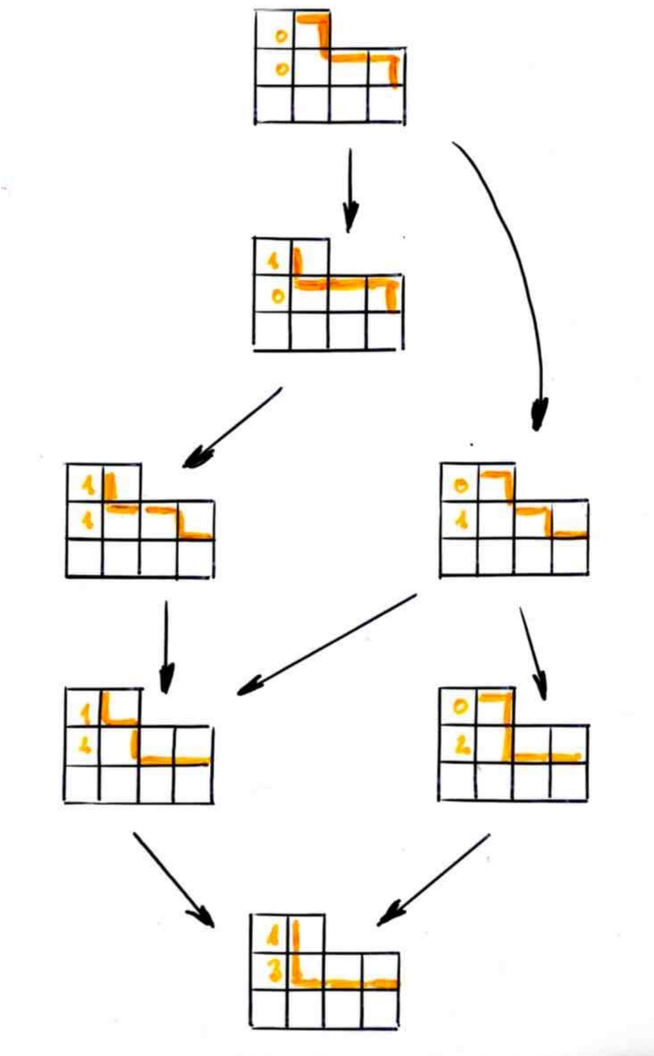

Maule
=
Tamari
v

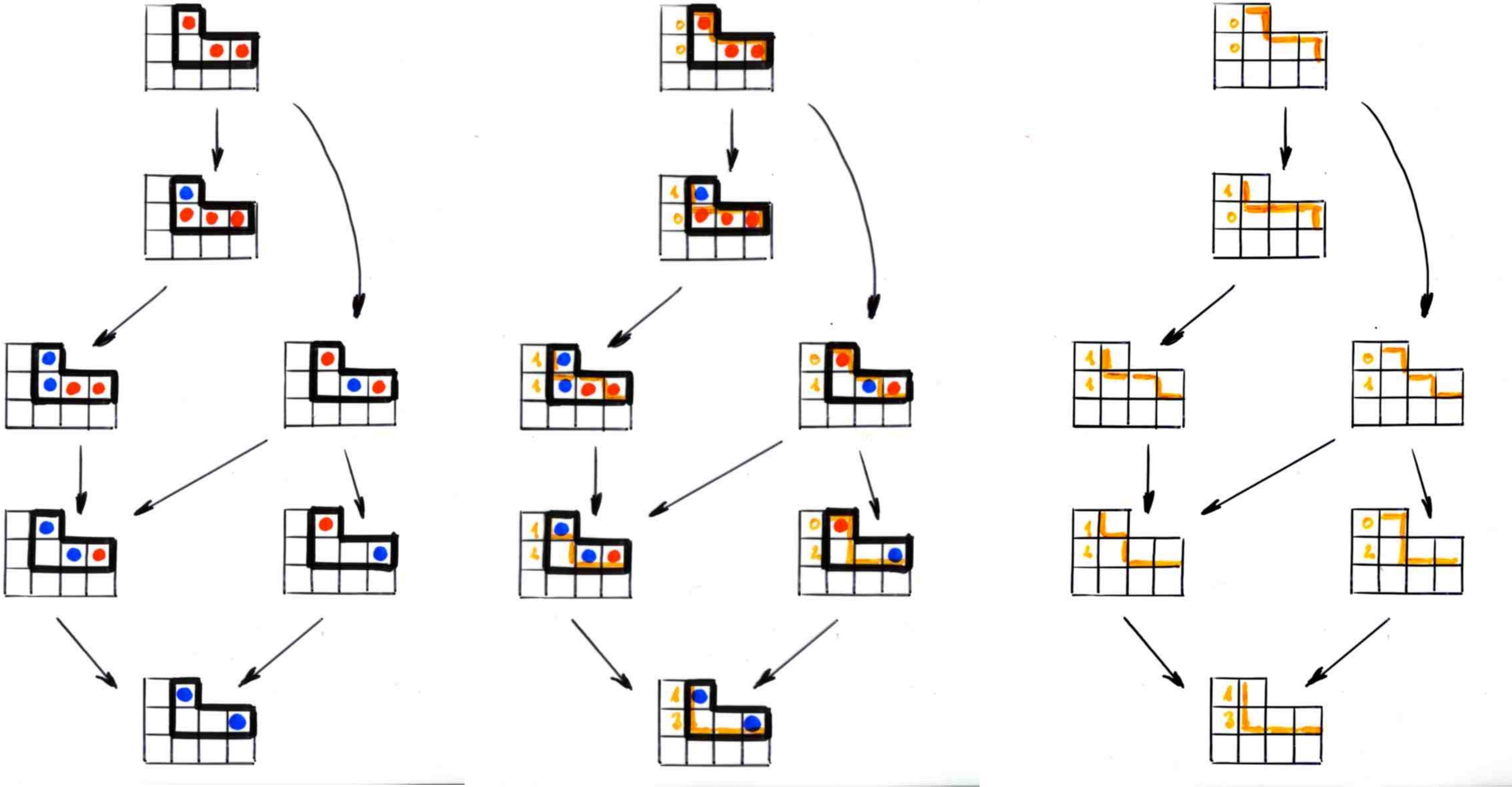



Maule
= Tamari
V

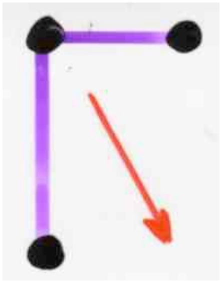



Young covering relation
Tamari covering
Tamari (3, 5)
≅
T
V =

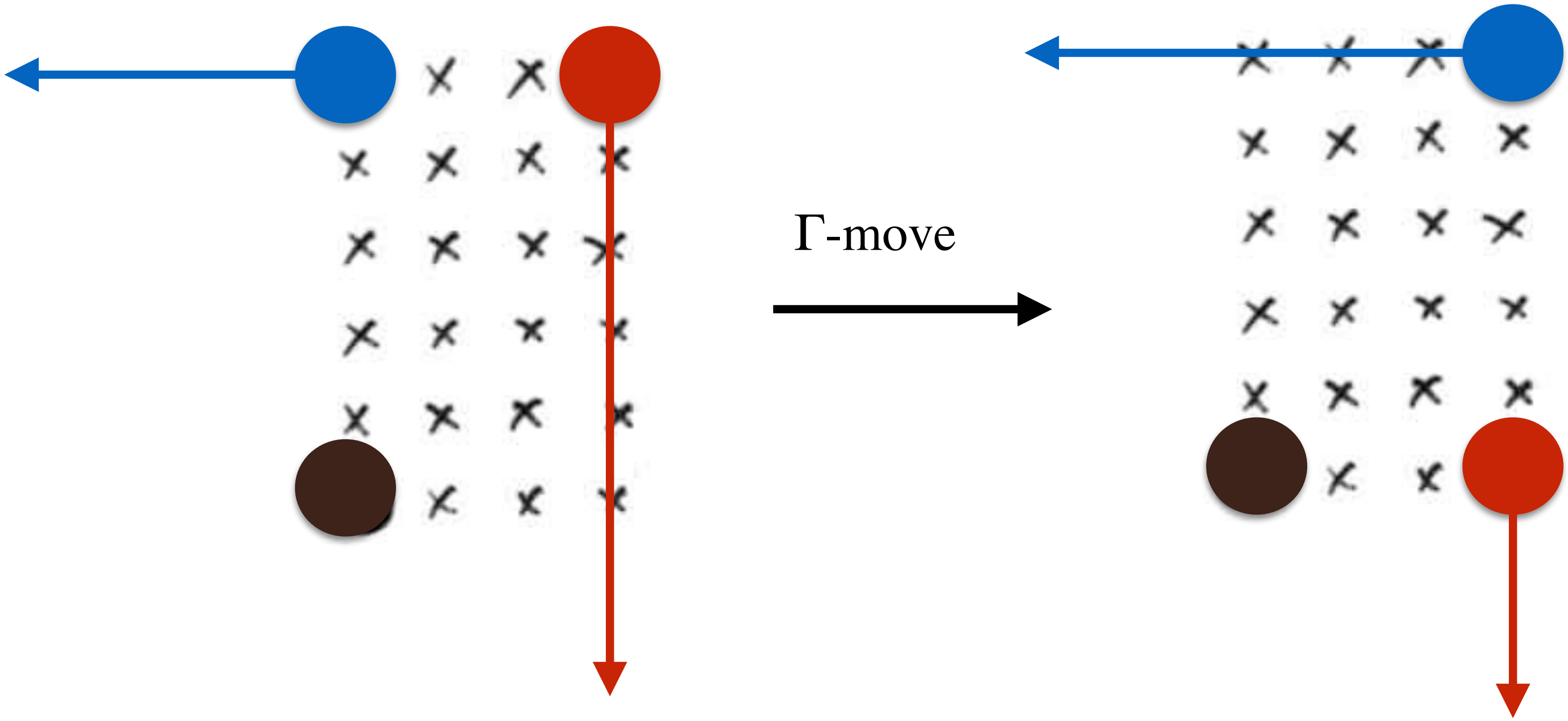




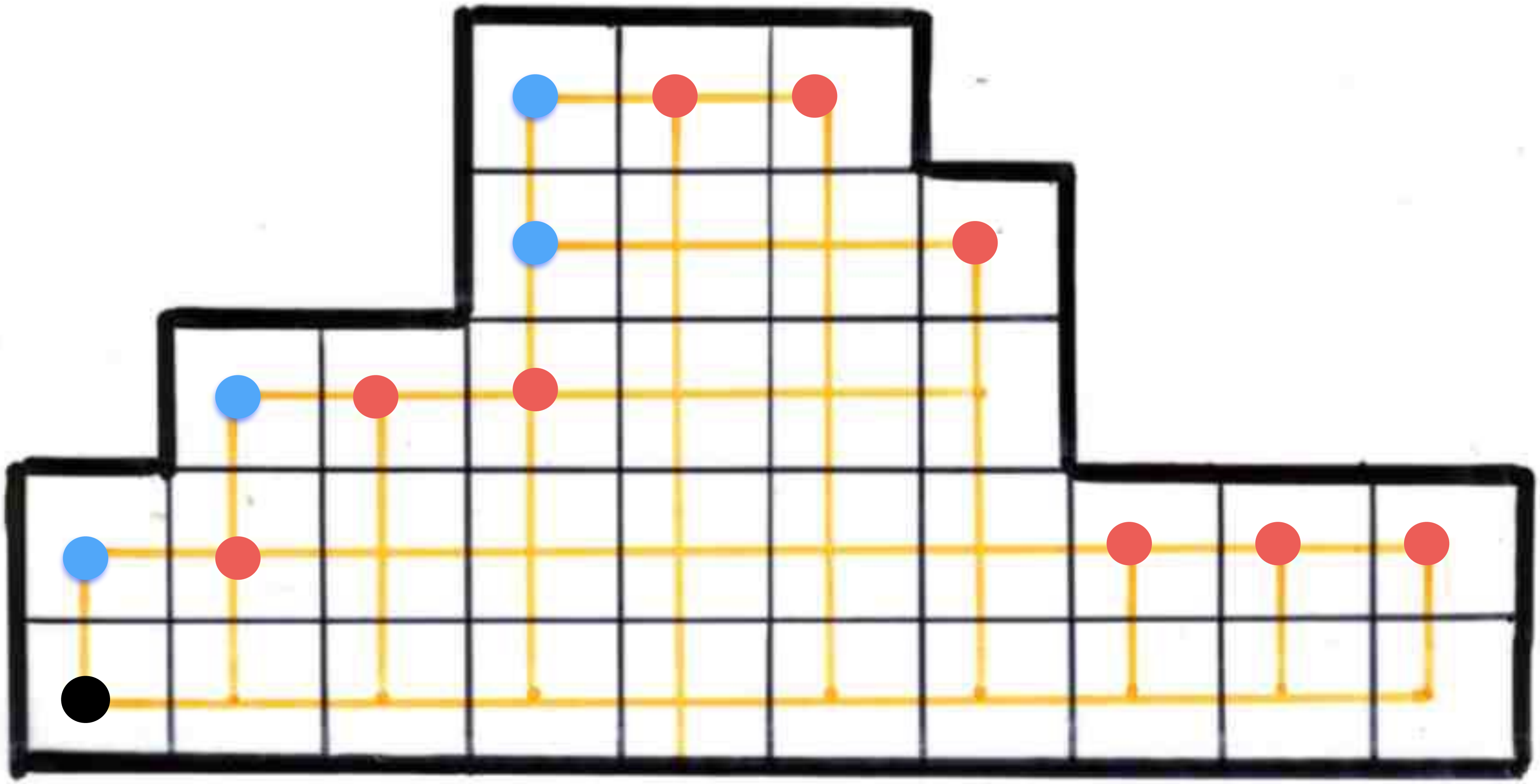
A mixture of Young Y(u) lattice
and
Tamari(v) lattice

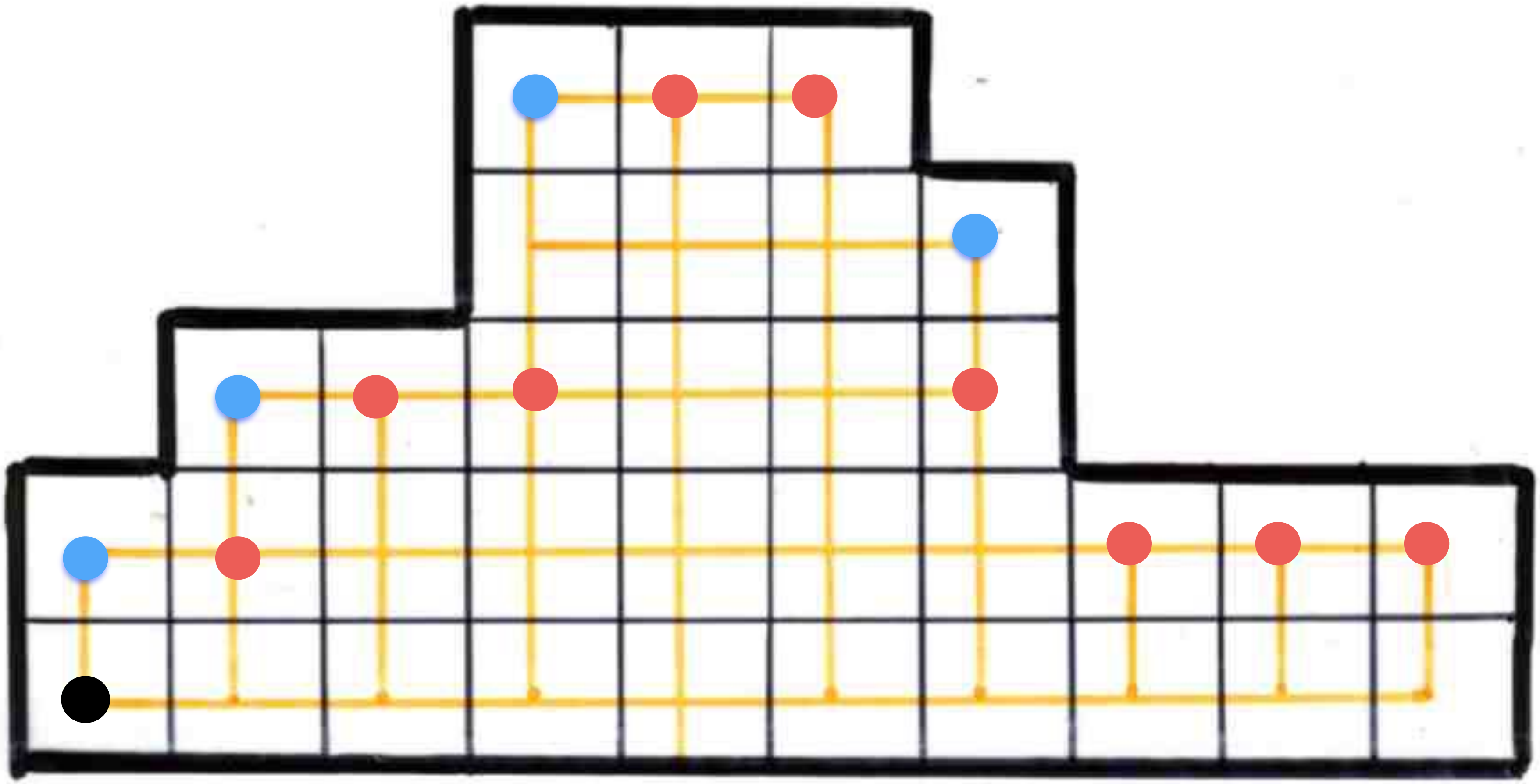


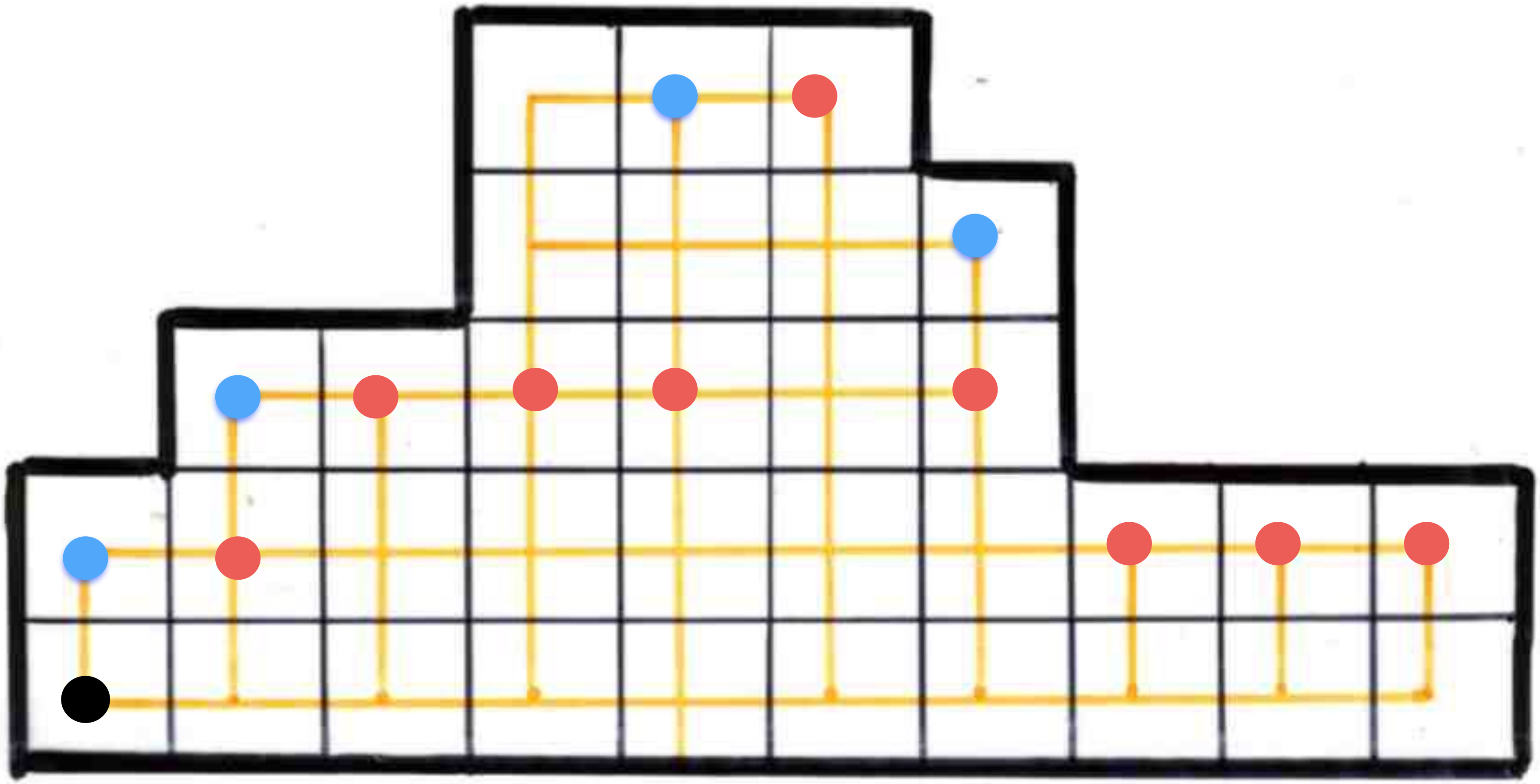
When the elements of the cloud X can be coloured in two colors blue and red satisfying the conditions defining the alternative tableaux (slide 70, part I), instead of seeing a Γ-move as the jump of a single particle, we can see it as the movement of two particles, a blue going to the right and a red going down (as on slides 122, part I and 49-50, part II)

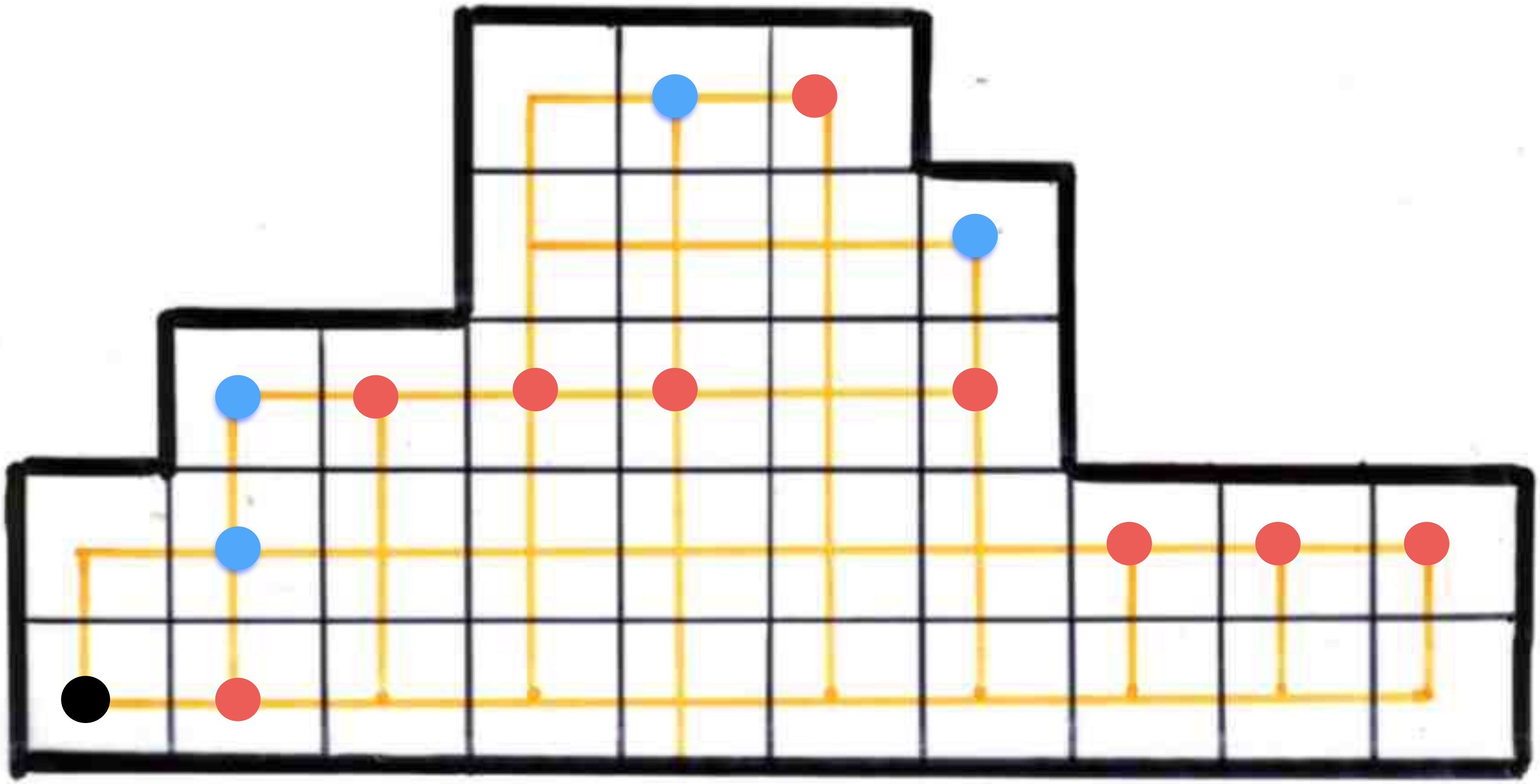


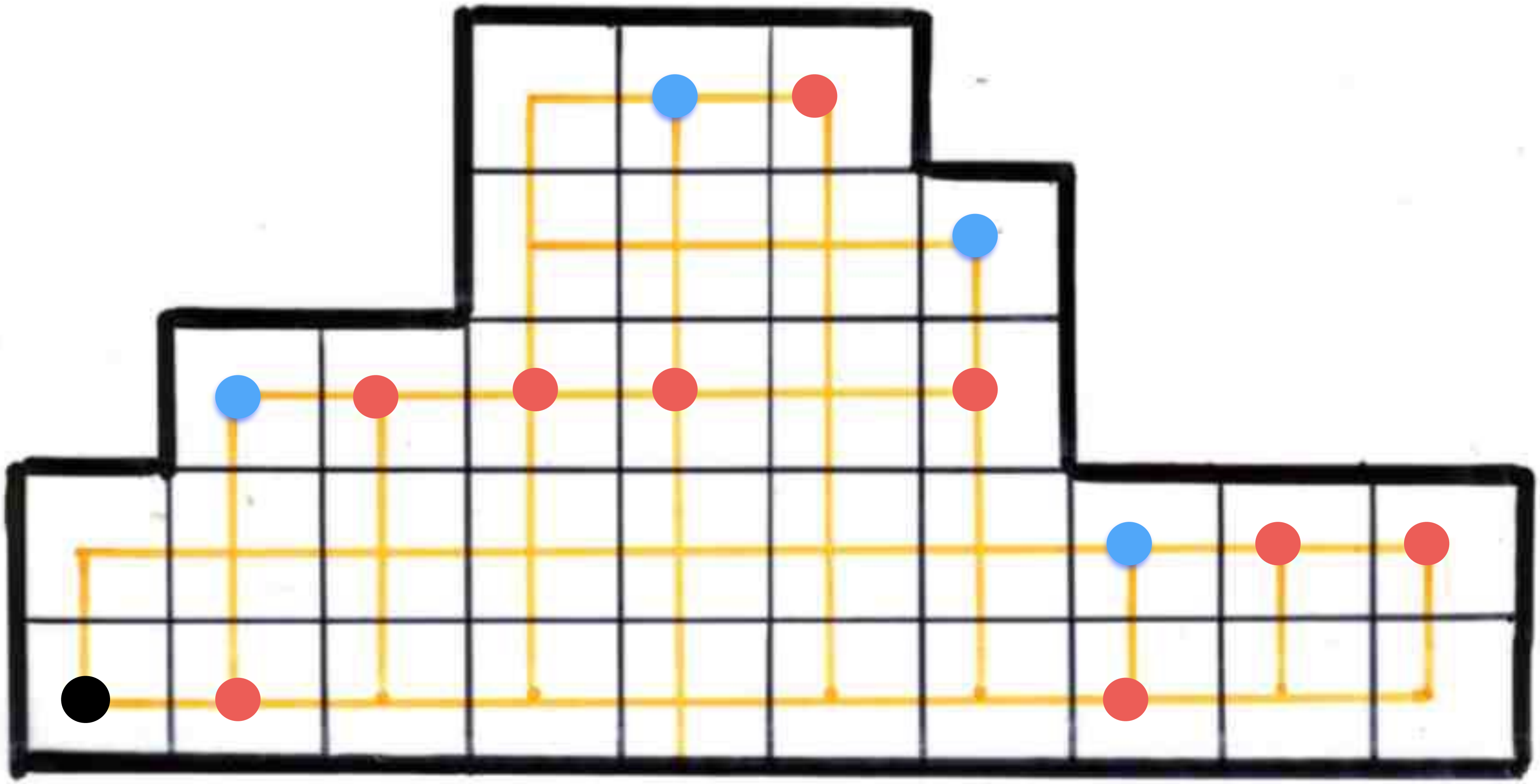
This is what we do in the following sequence of Γ-moves.

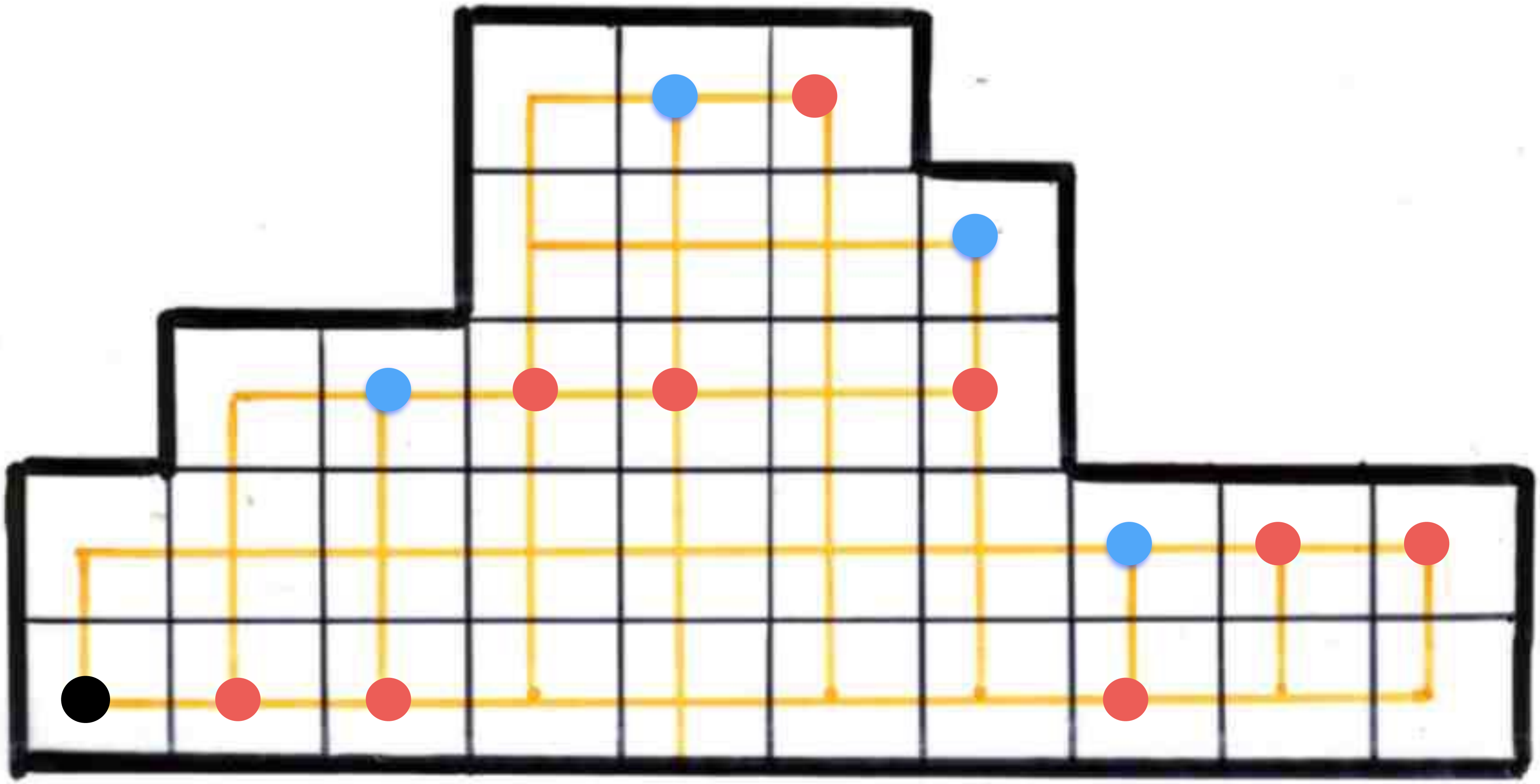


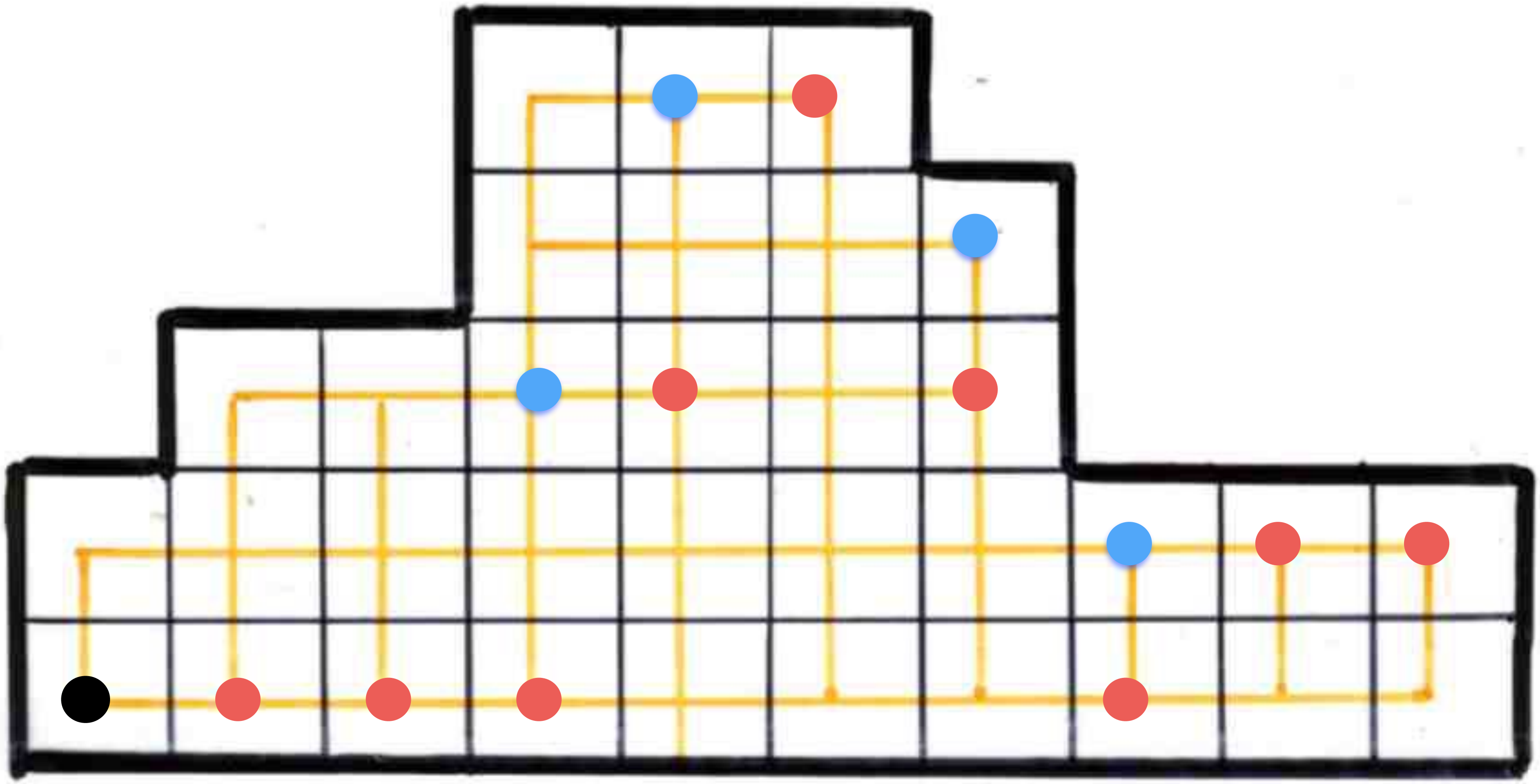


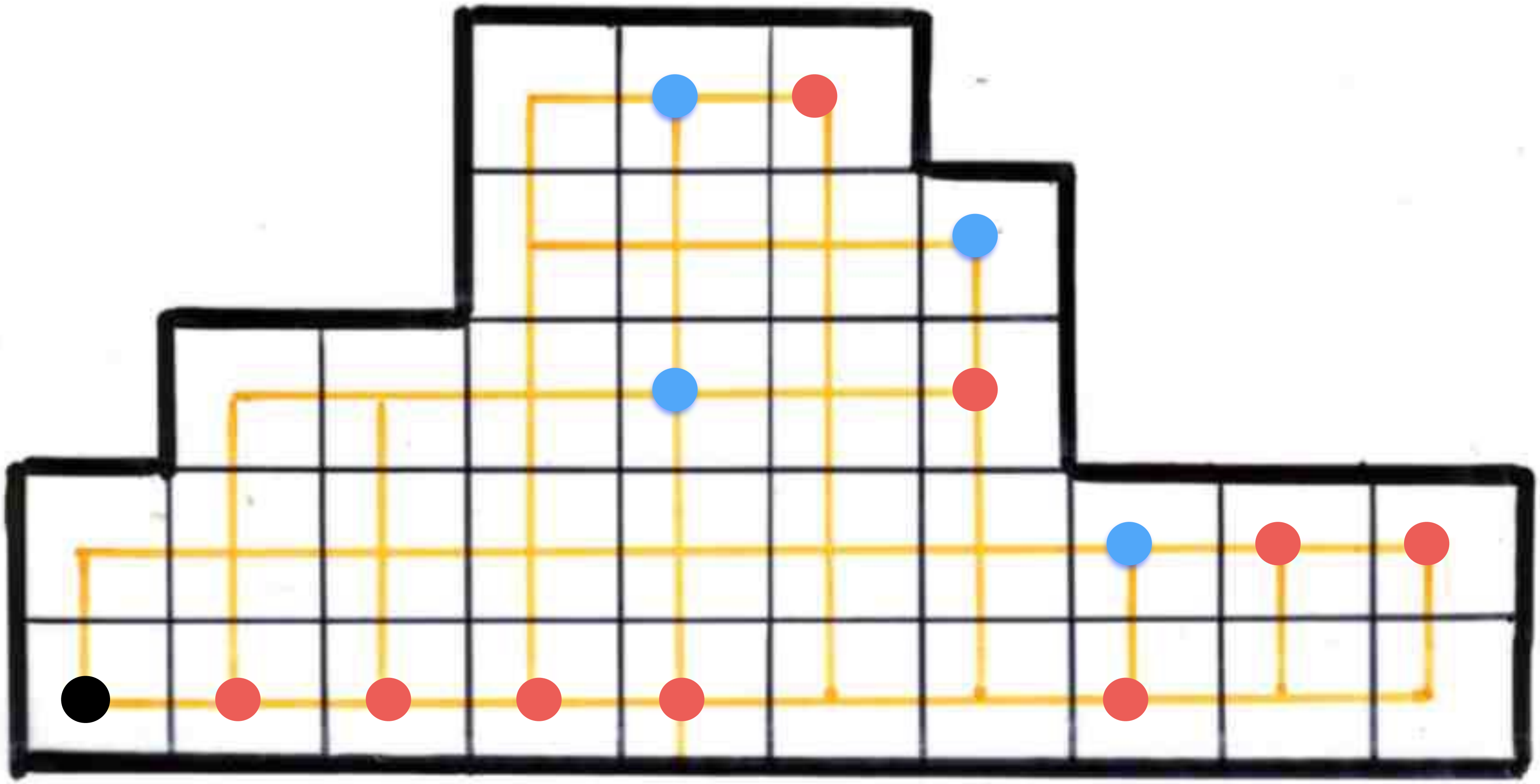


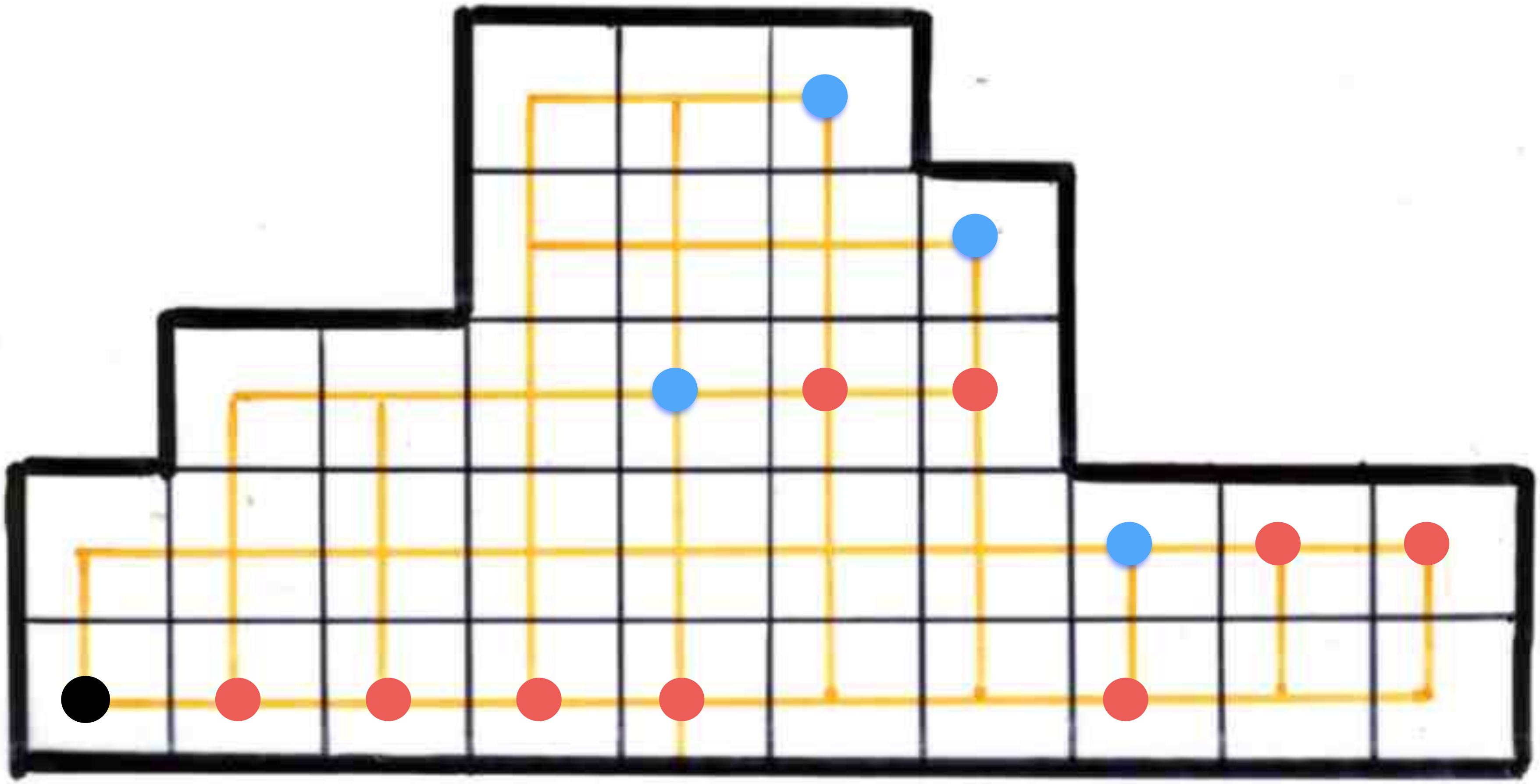


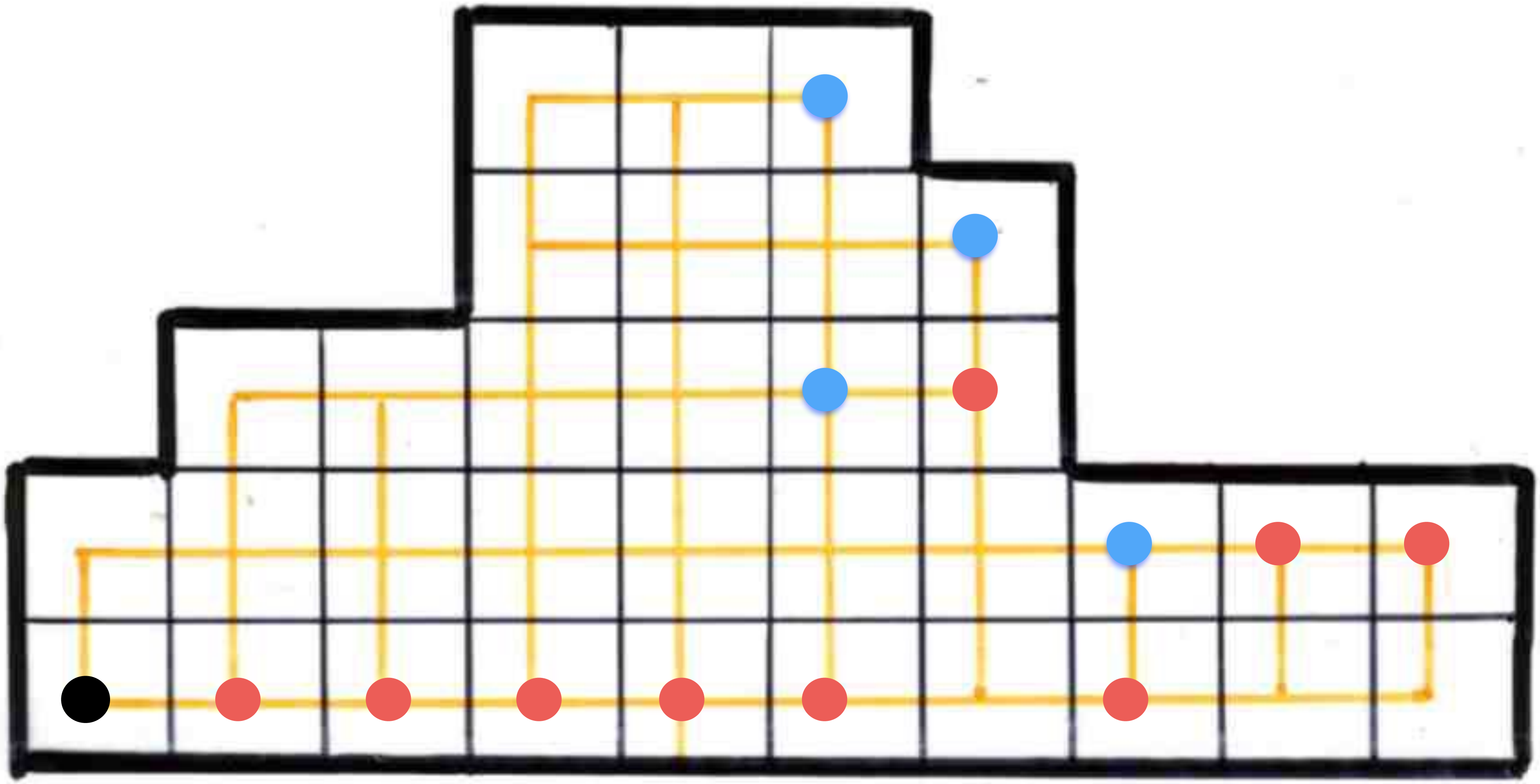


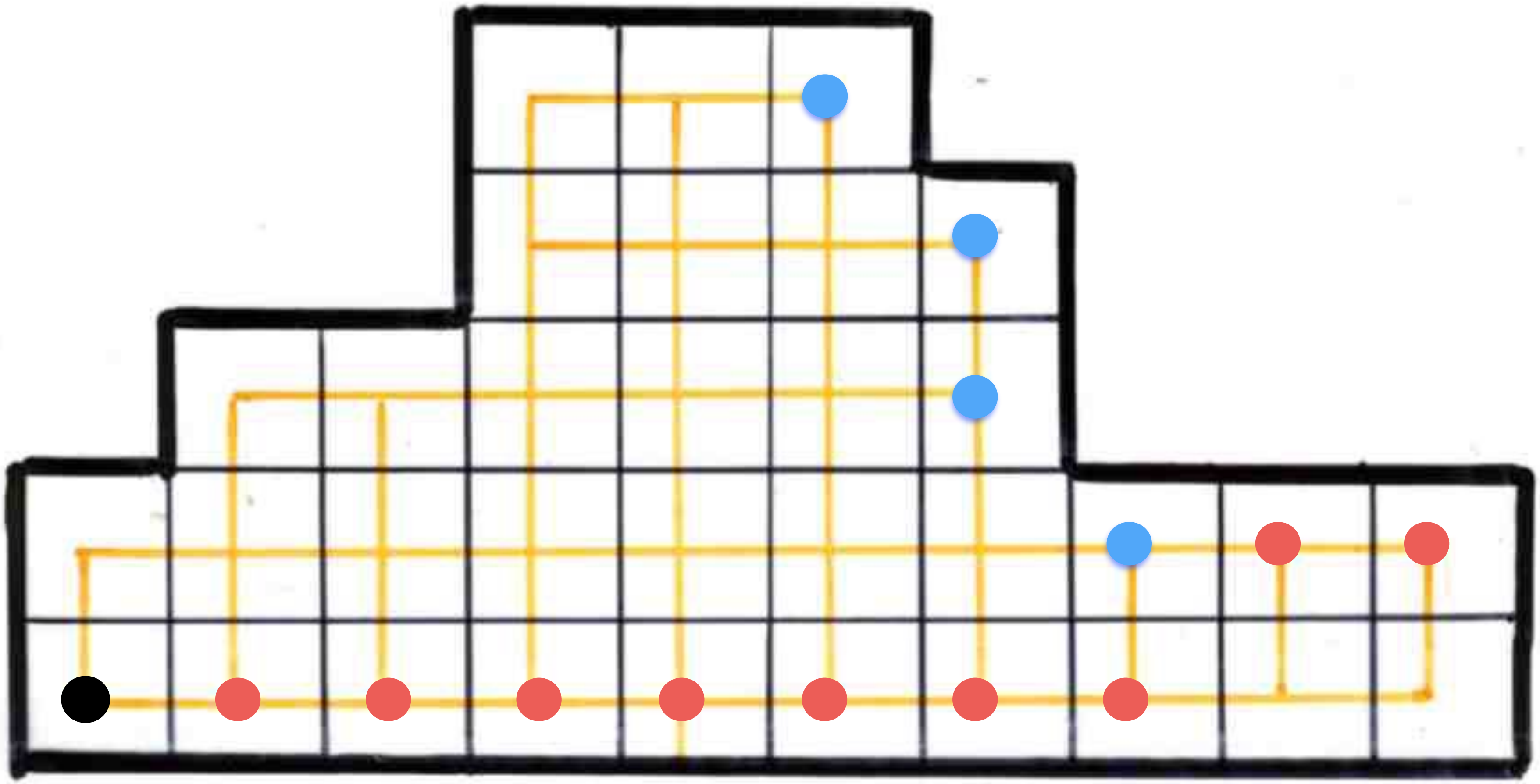


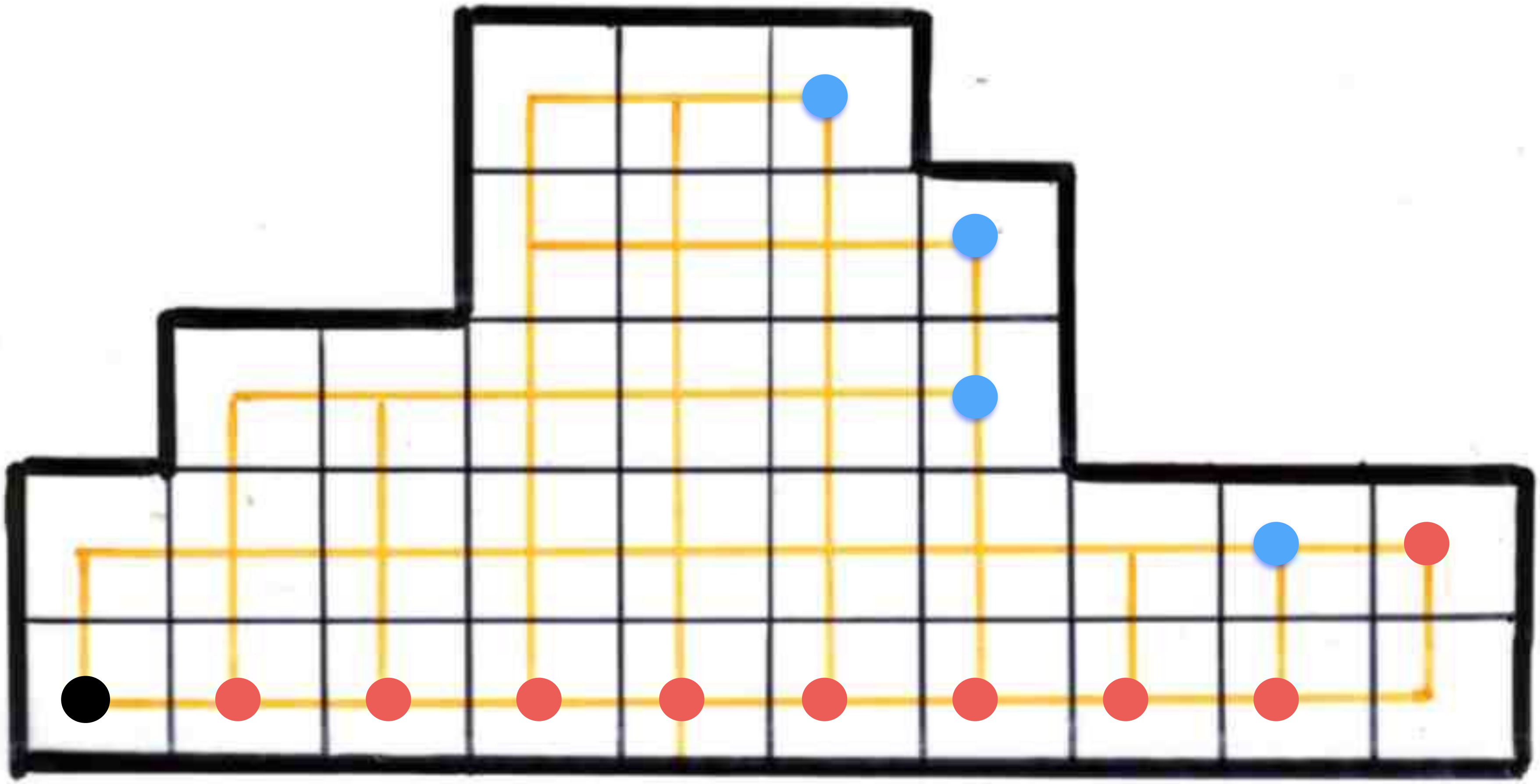


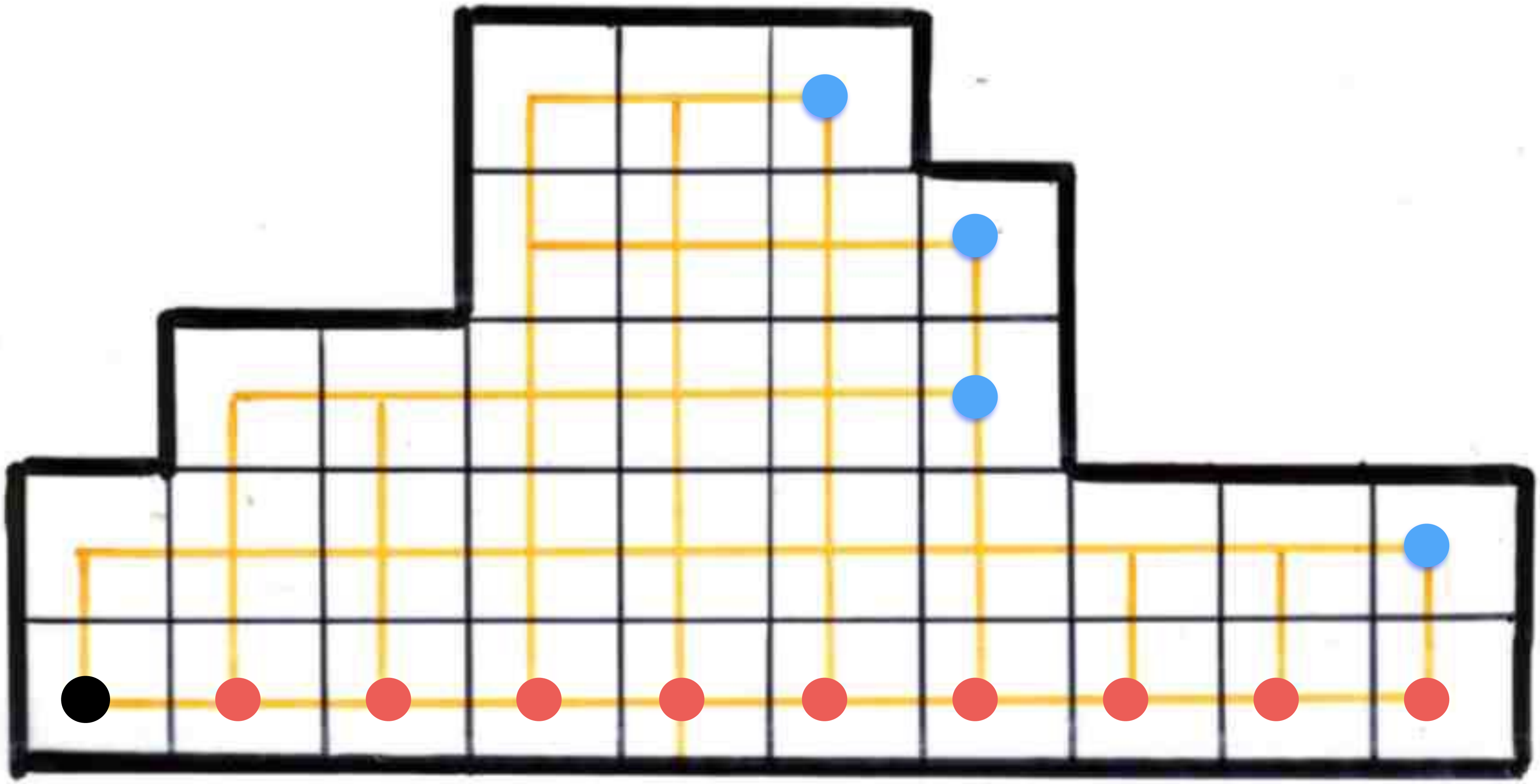










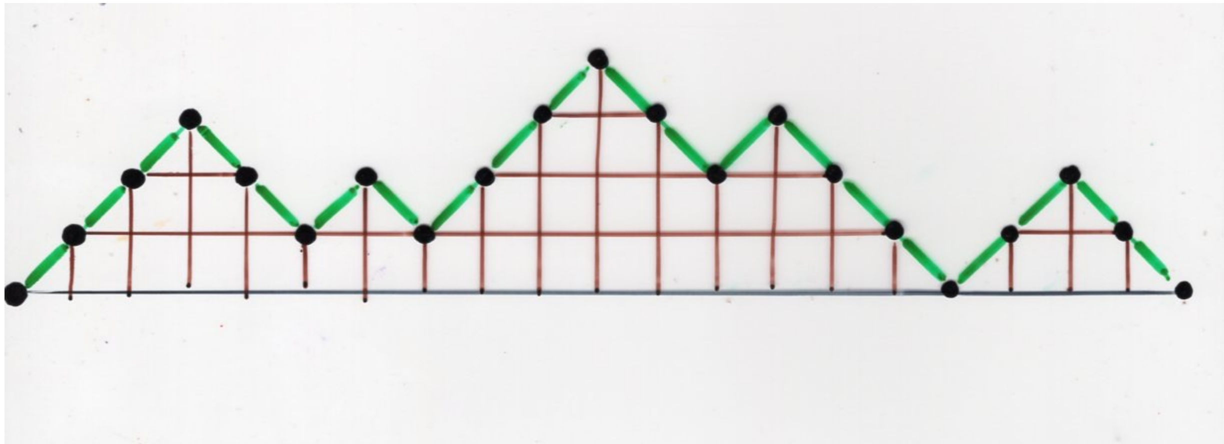


a festival of bijections

Pair of paths

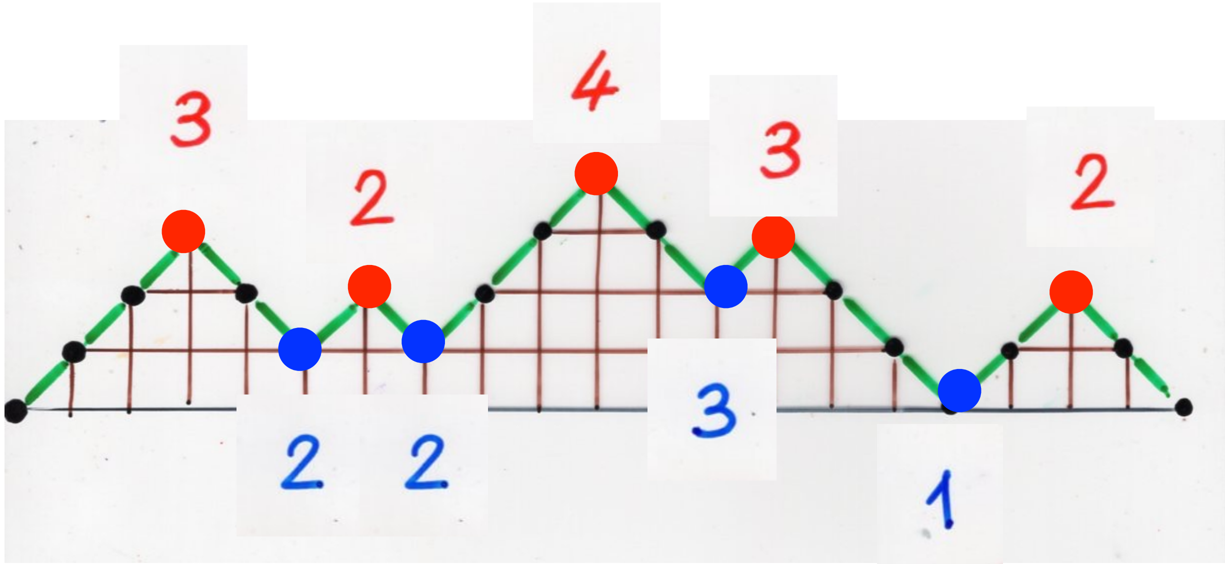
Dyck paths

Staircase polygons

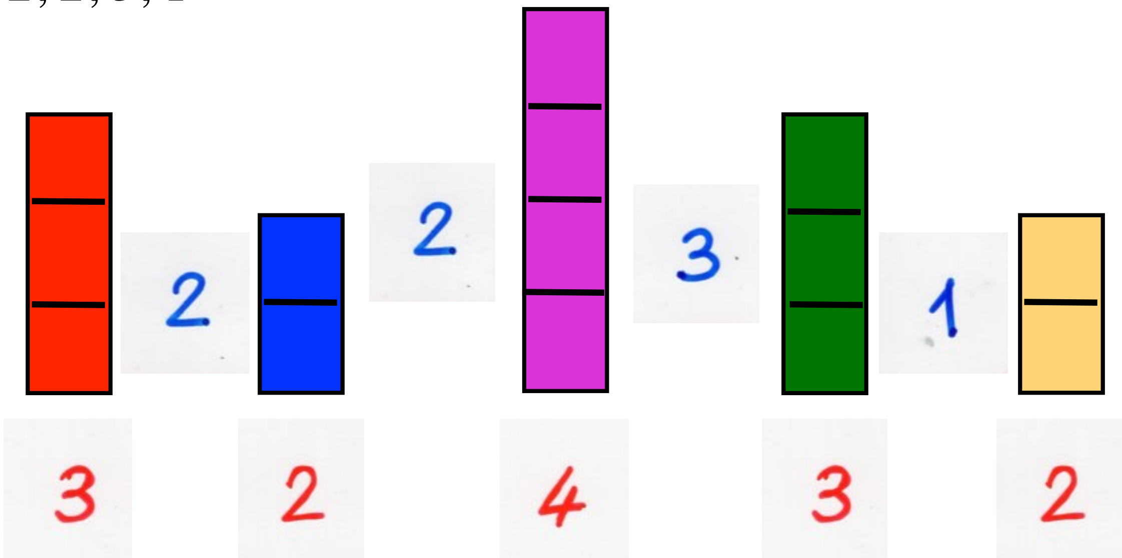


We described a bijection between Dyck paths and pairs (u,v) of paths, defined first by M. Delest and X.V., for the enumeration of convex polygons, with a formulation given by J.M. Fedou.

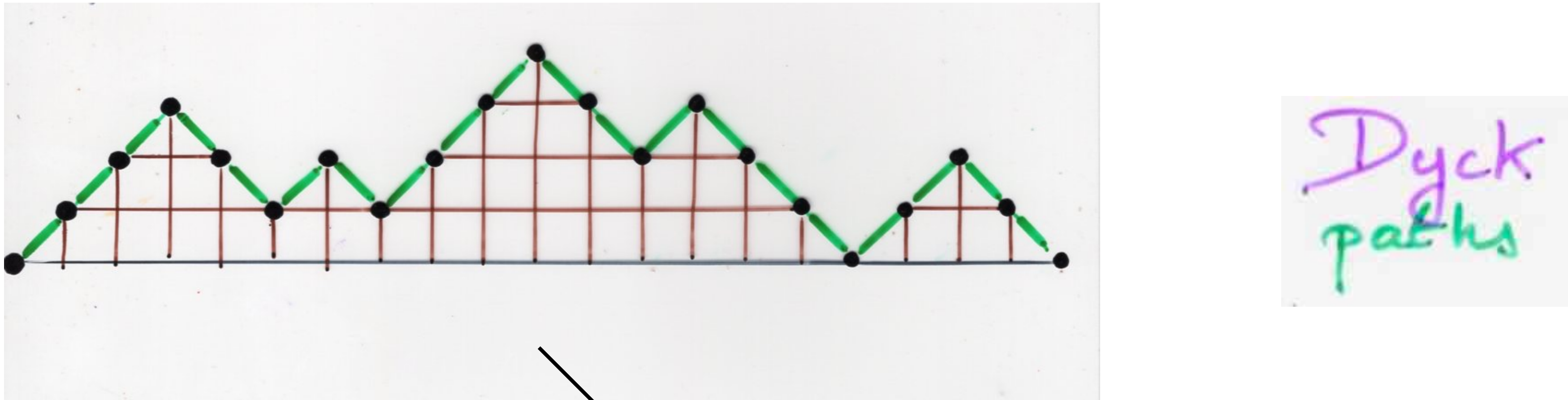
M. Delest and X.V., Algebraic languages and polyominoes enumeration, Theoretical Computer Science, 34 (1984) 169-206

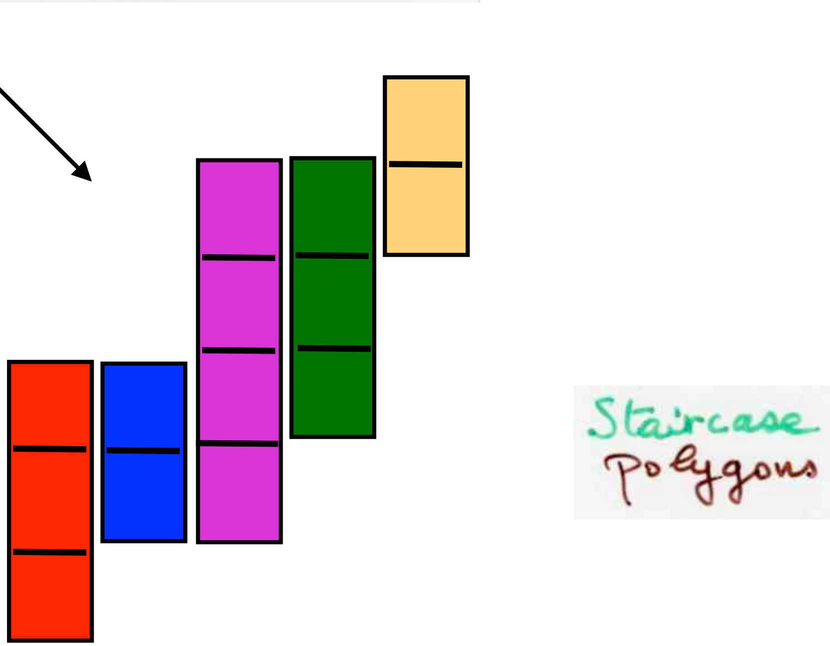


Height of the peaks: 3, 2, 4, 3, 2
1 + height of the valley: 2, 2, 3, 1

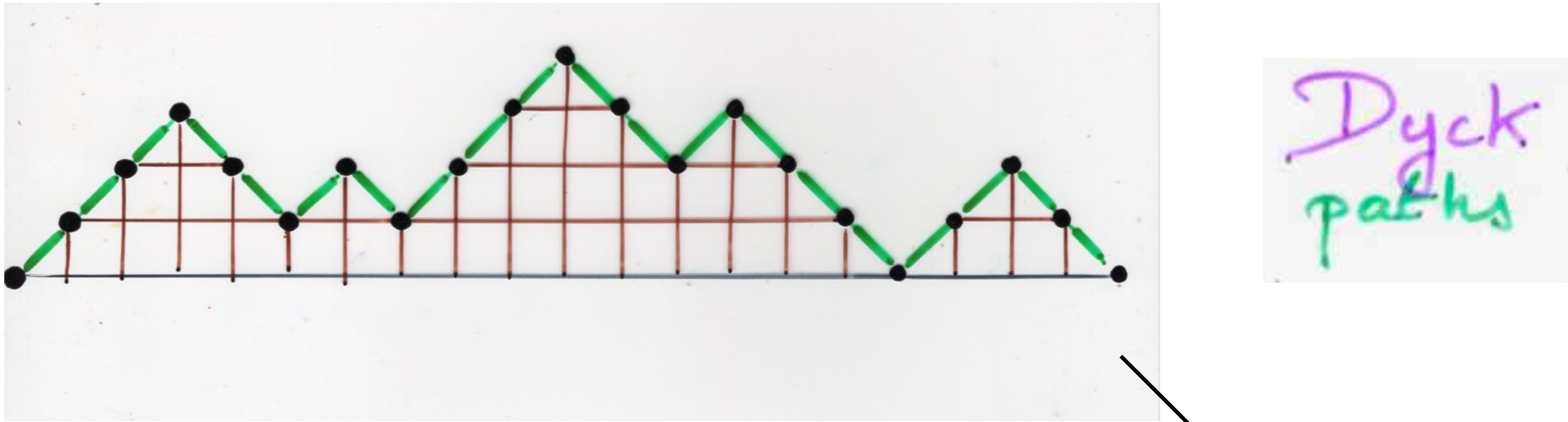


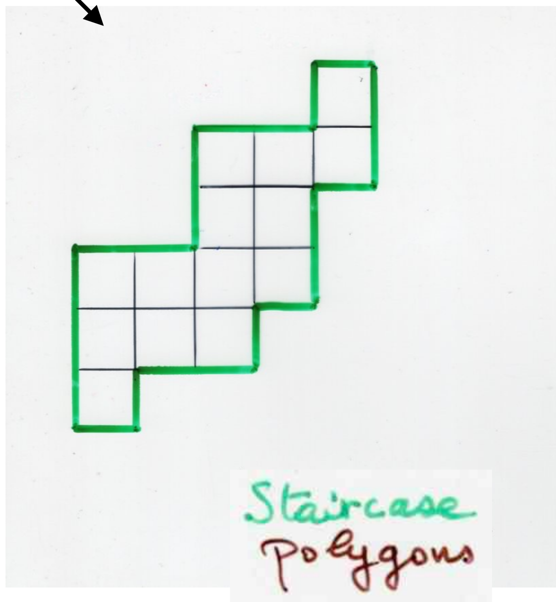
A sequence of columns from the red numbers

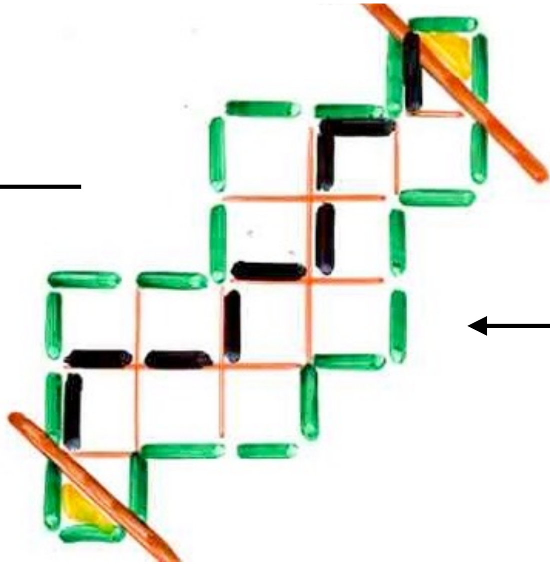




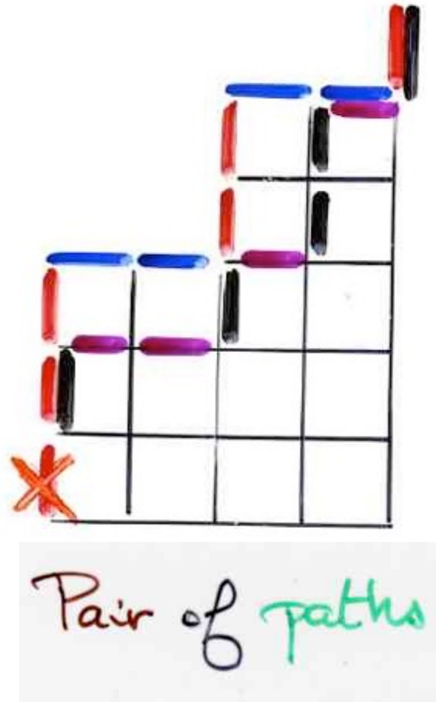
gluing the columns according to the blue numbers

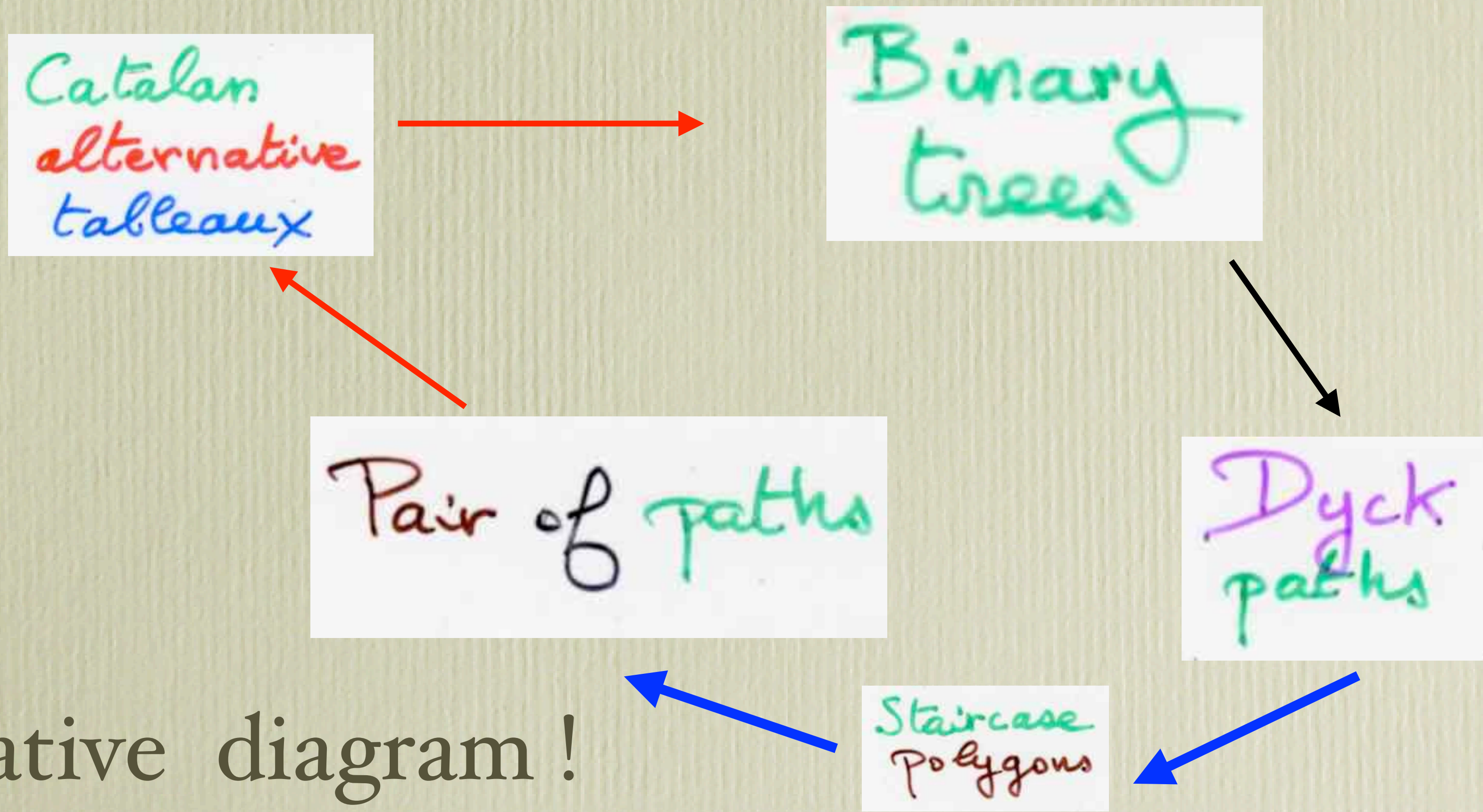




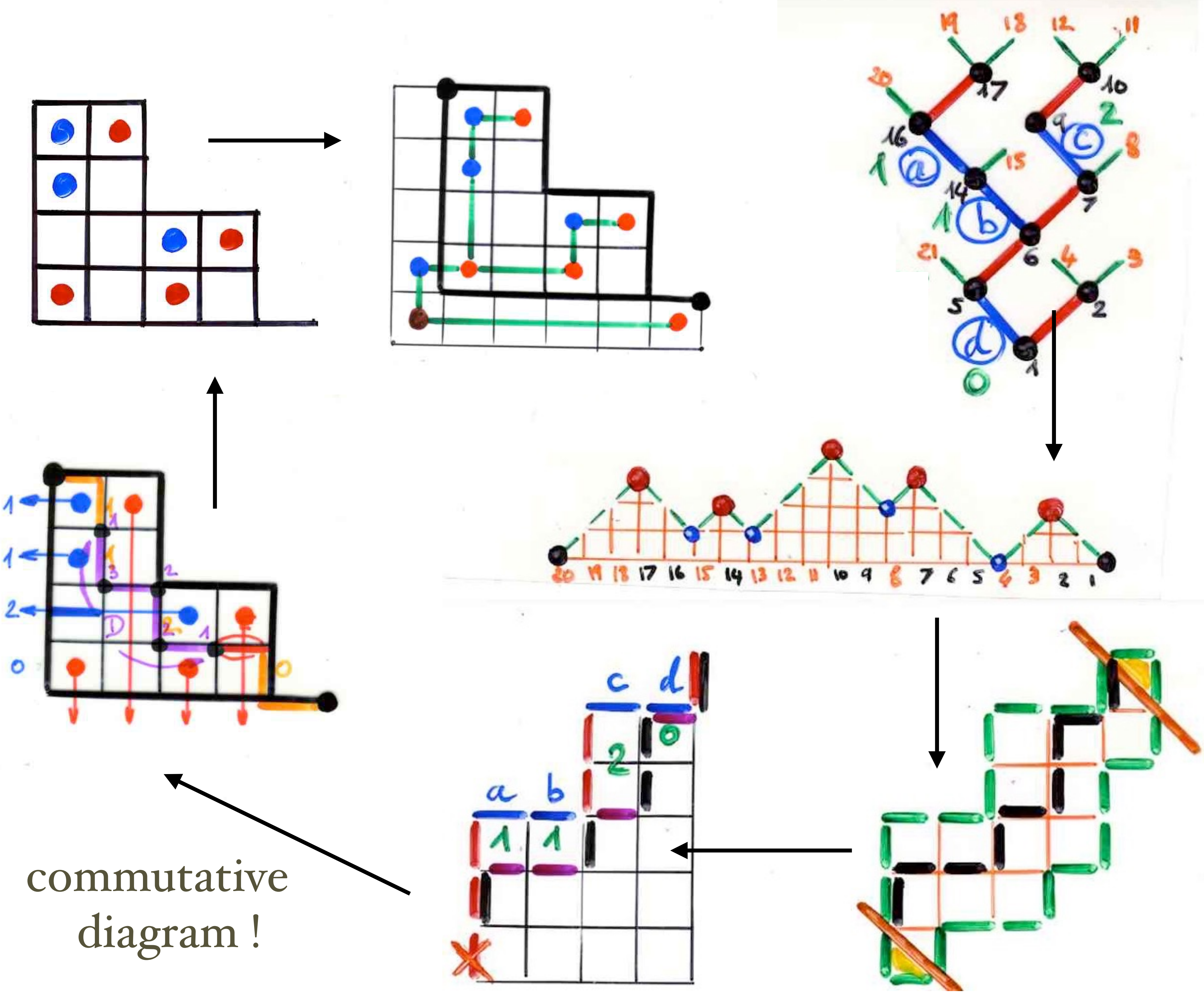


sliding the SE border up one step

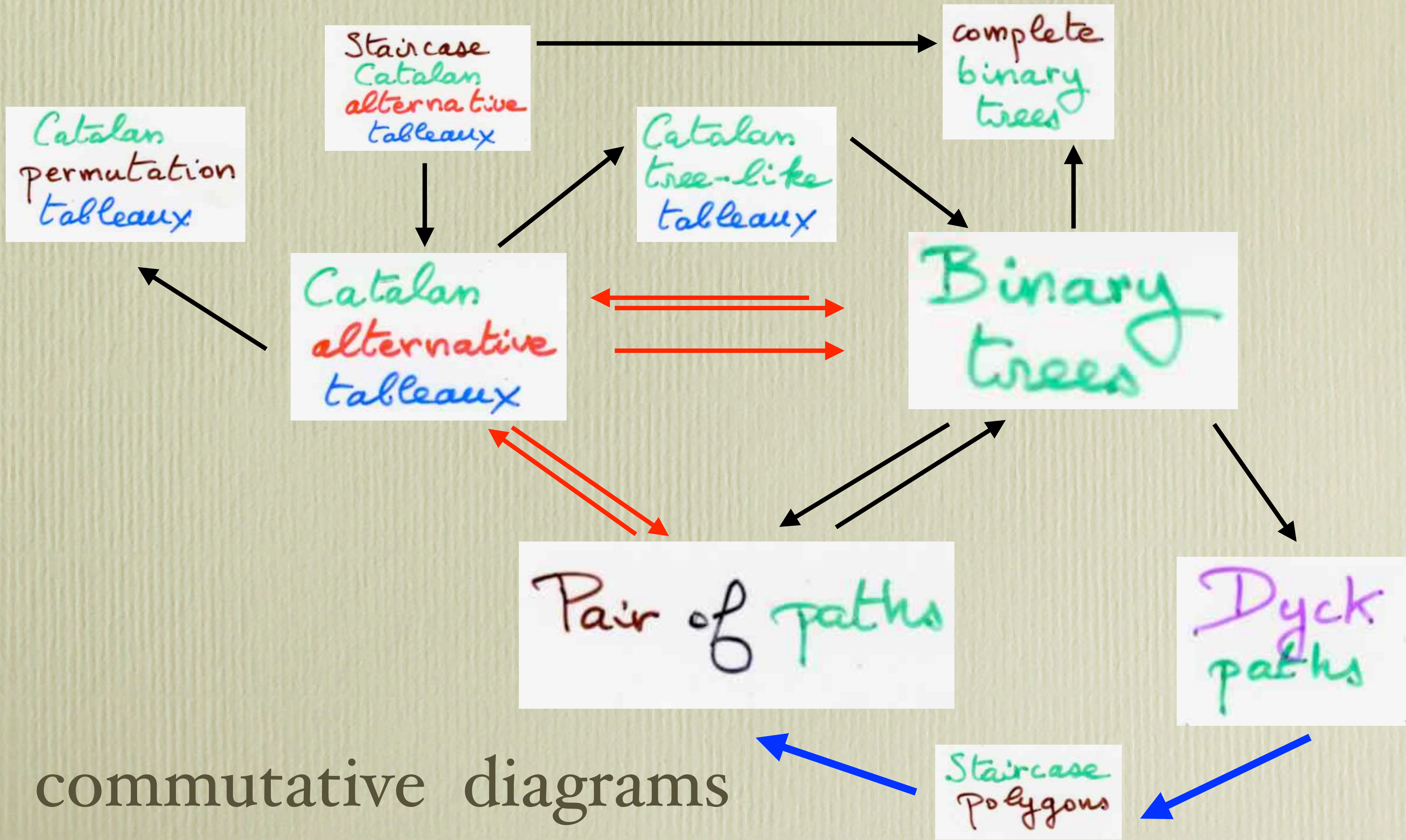




commutative diagram !



commutative
diagram !



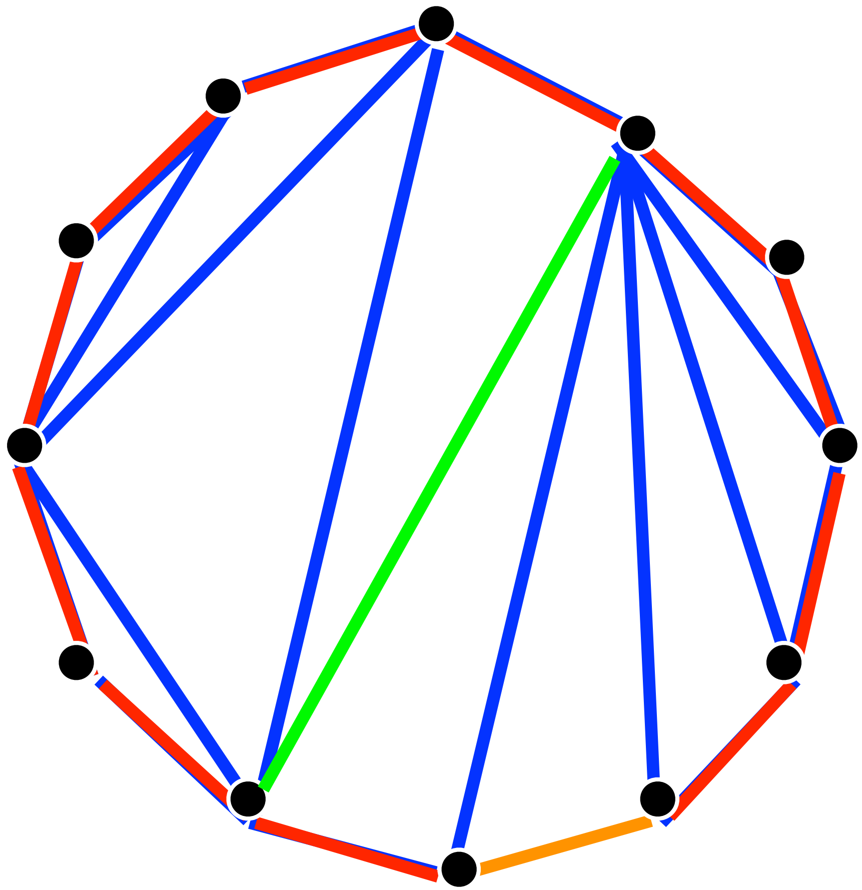
commutative diagrams

the work of

Ceballos, Padrol, Sarmiento
(2016) (2017)

Ceballos, Padrol, Sarmiento
(2016) (2017)

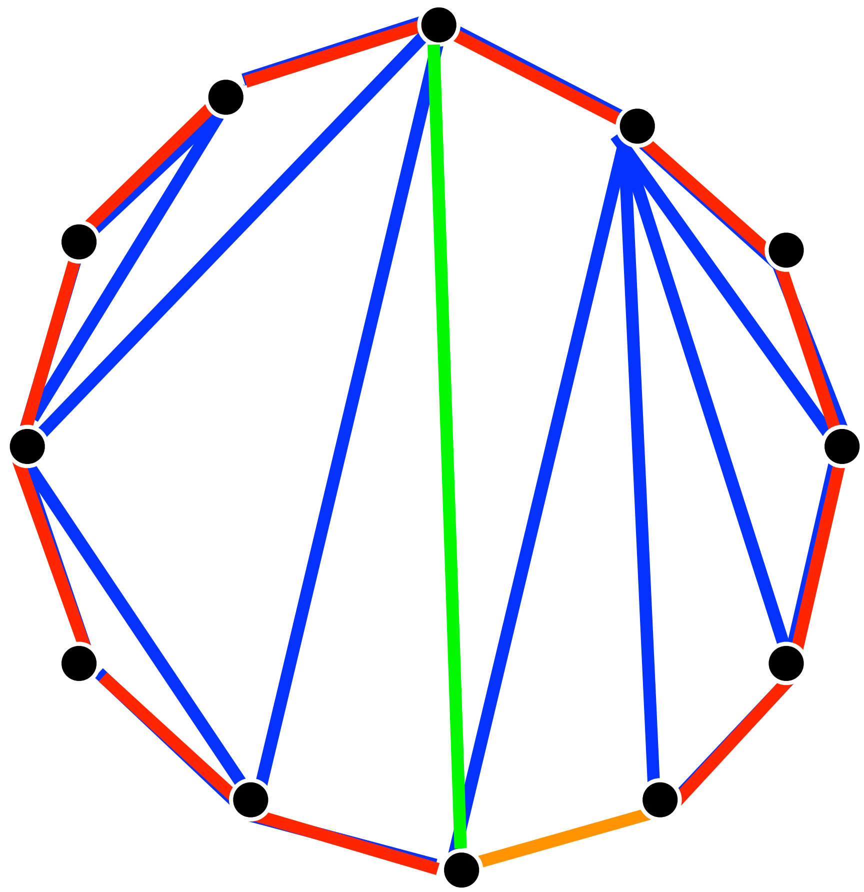
Triangulations
(of a regular polygon)



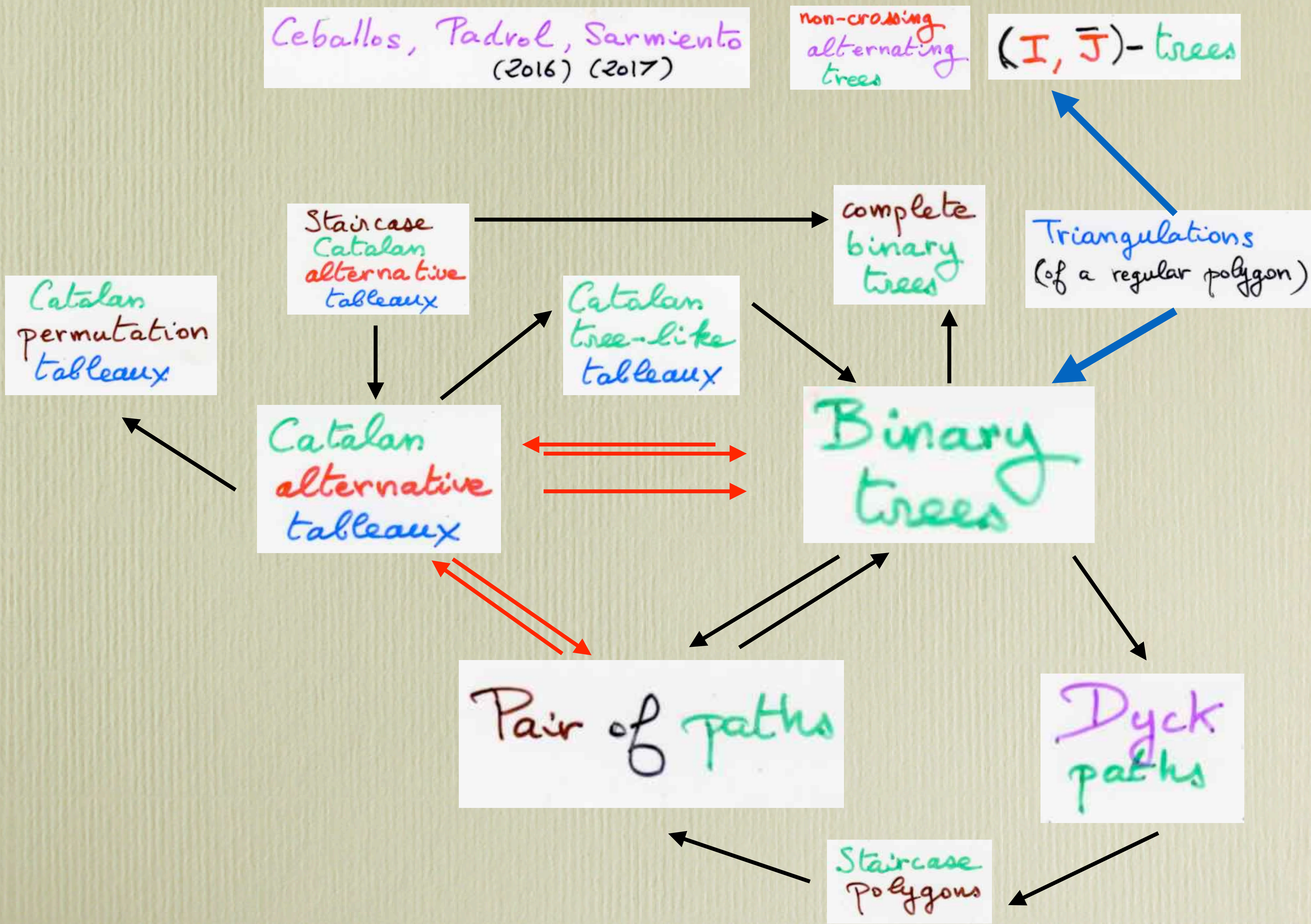
a flip in a triangulation defining the Tamari lattice

Ceballos, Padrol, Sarmiento
(2016) (2017)

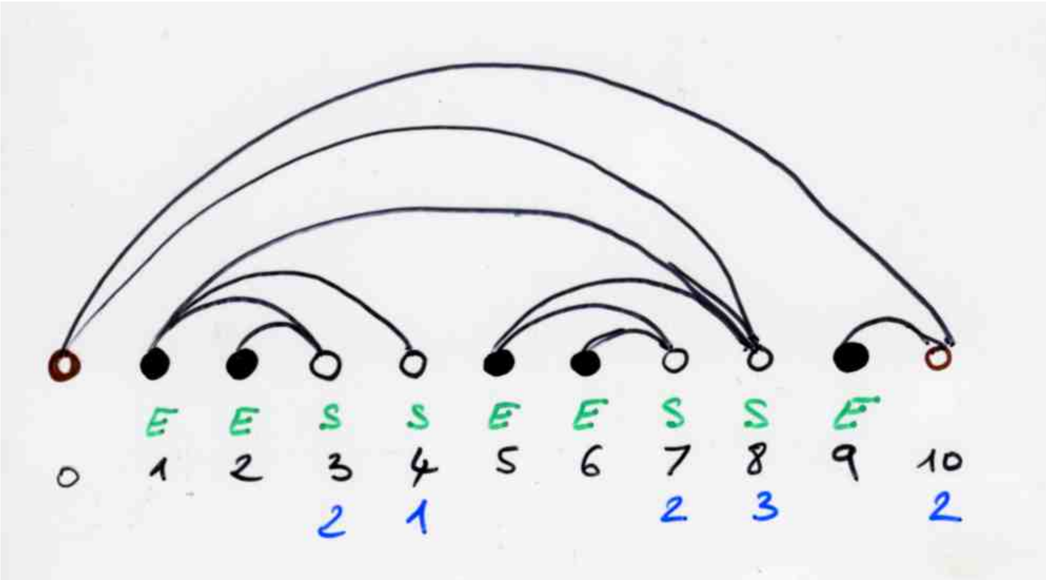
Triangulations
(of a regular polygon)



a flip in a triangulation defining the Tamari lattice


Ceballos, Padrol, Sarmiento
(2016) (2017)
non-crossing alternating trees
(I, J)- trees
Staircase Catalan alternative tableaux
complete binary trees
Triangulations (of a regular polygon)
Catalan permutation tableaux
Catalan tree-like tableaux
Catalan alternative tableaux
Binary trees
Pair of paths
Dyck paths
Staircase polygons

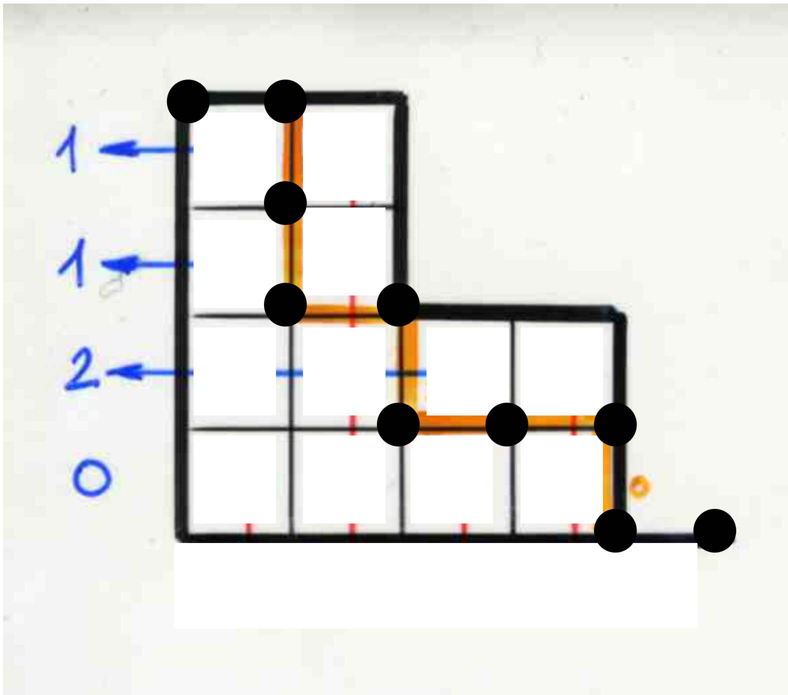
Ceballos, Padrol, Sarmiento
(2016) (2017)



Triangulations
(of a regular polygon)

$(I, \bar{J})$-trees

Pair of paths



Ceballos, Padrol, Sarmiento
(2016) (2017)

non-crossing
alternating
trees

$(I, \bar{J})$- trees

Triangulations
(of a regular polygon)

ν-trees

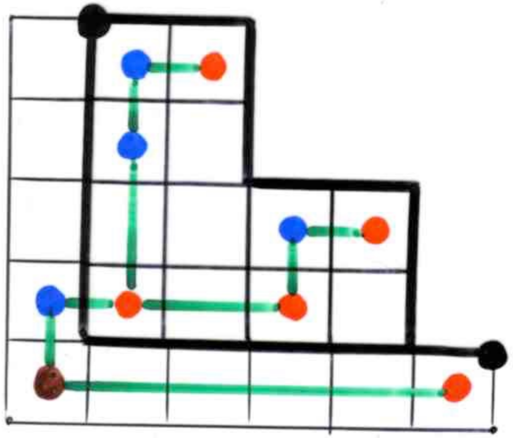
Catalan
tree-like
tableaux

Pair of paths

Ceballos, Padrol, Sarmiento
(2016) (2017)

v-tree introduced by the 3 authors are the same as the binary tree underlying an alternative tableau, or equivalently a tree-like tableau

v-trees

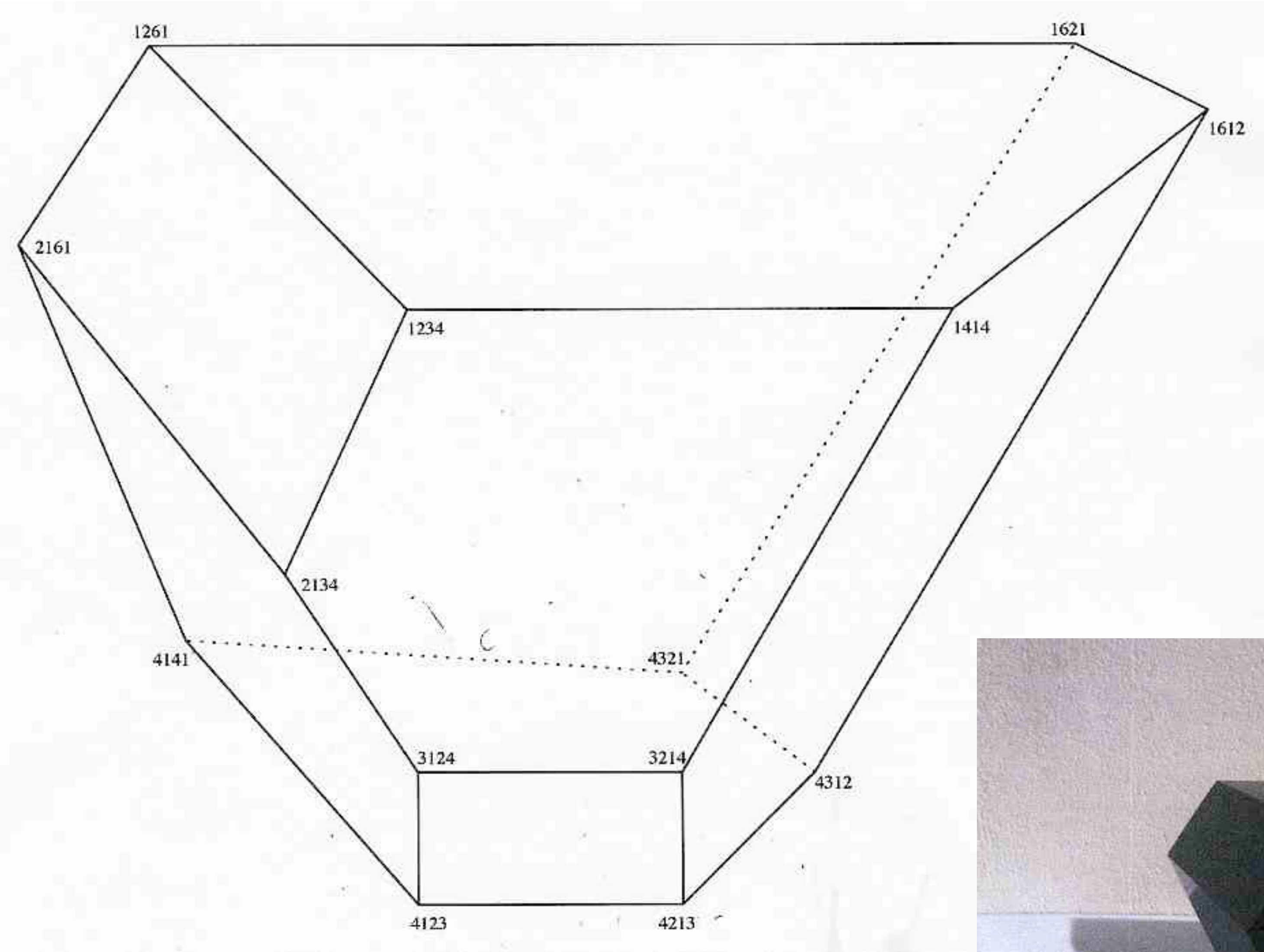


Catalan
tree-like
tableaux

geometry of v-Tamari lattice

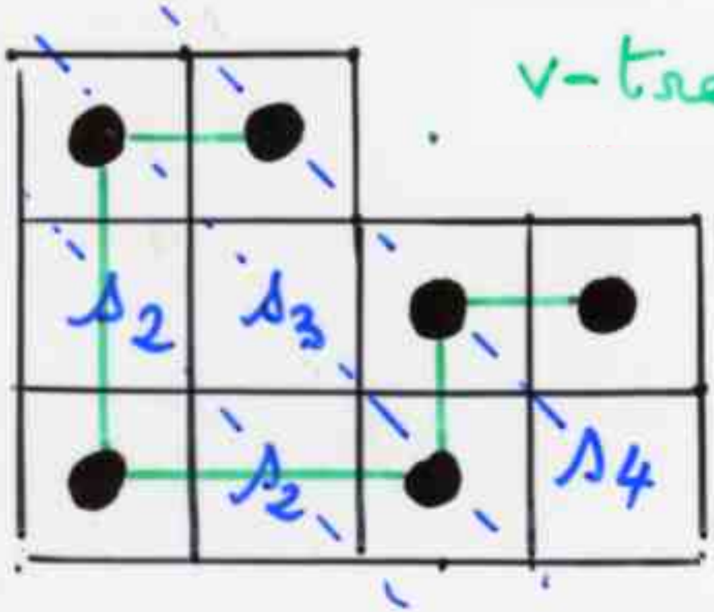
v-Tamari lattice :
dual of a well chosen

subword complex



associahedron

Ceballos, Padrol, Sarmiento
(2016) (2017)

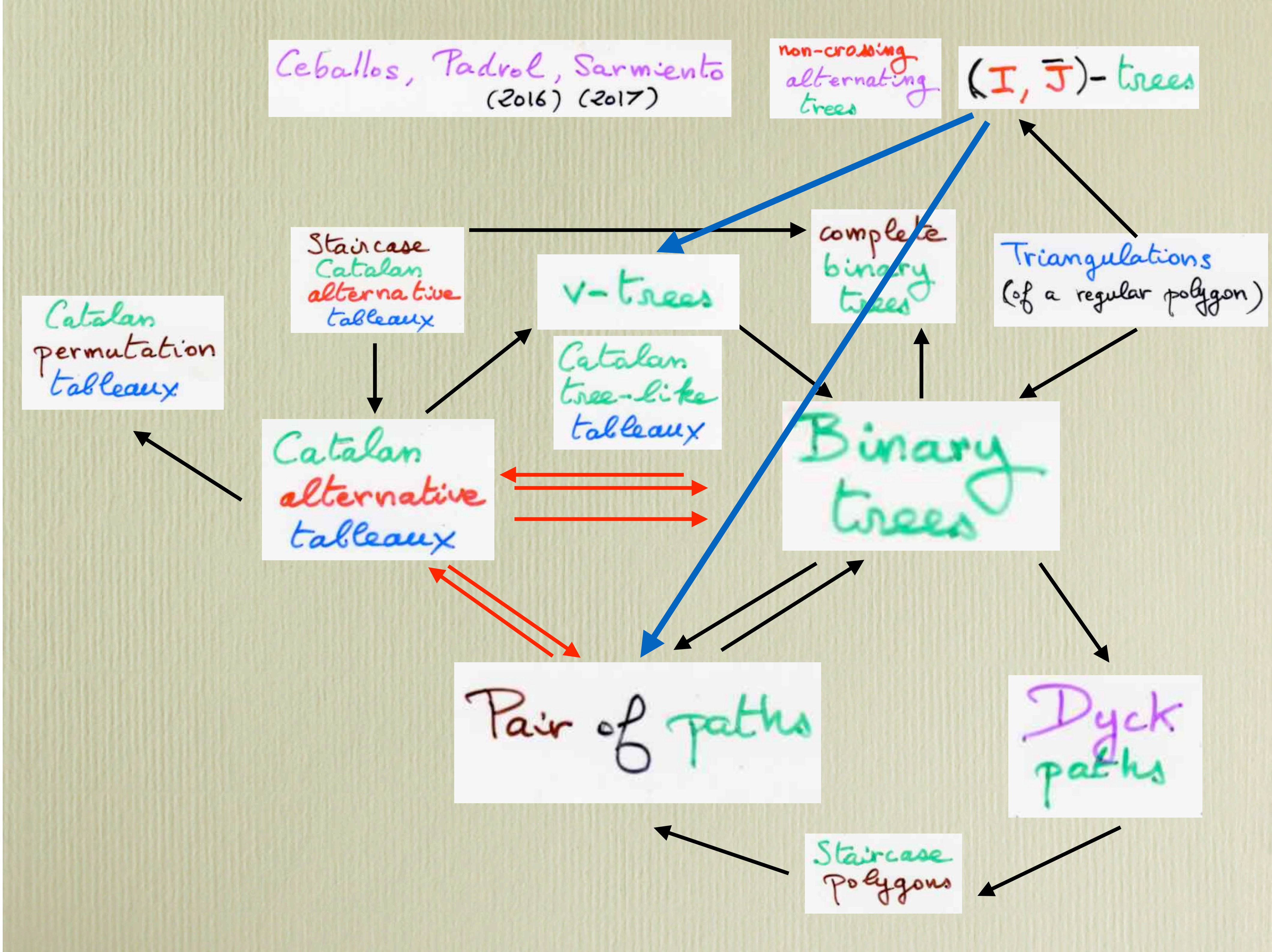


v-trees

subword complex

v-Tamari lattice :
dual of a well chosen

$s_2 s_3 s_2 s_4 = [1, 4, 3, 5, 2, 6]$



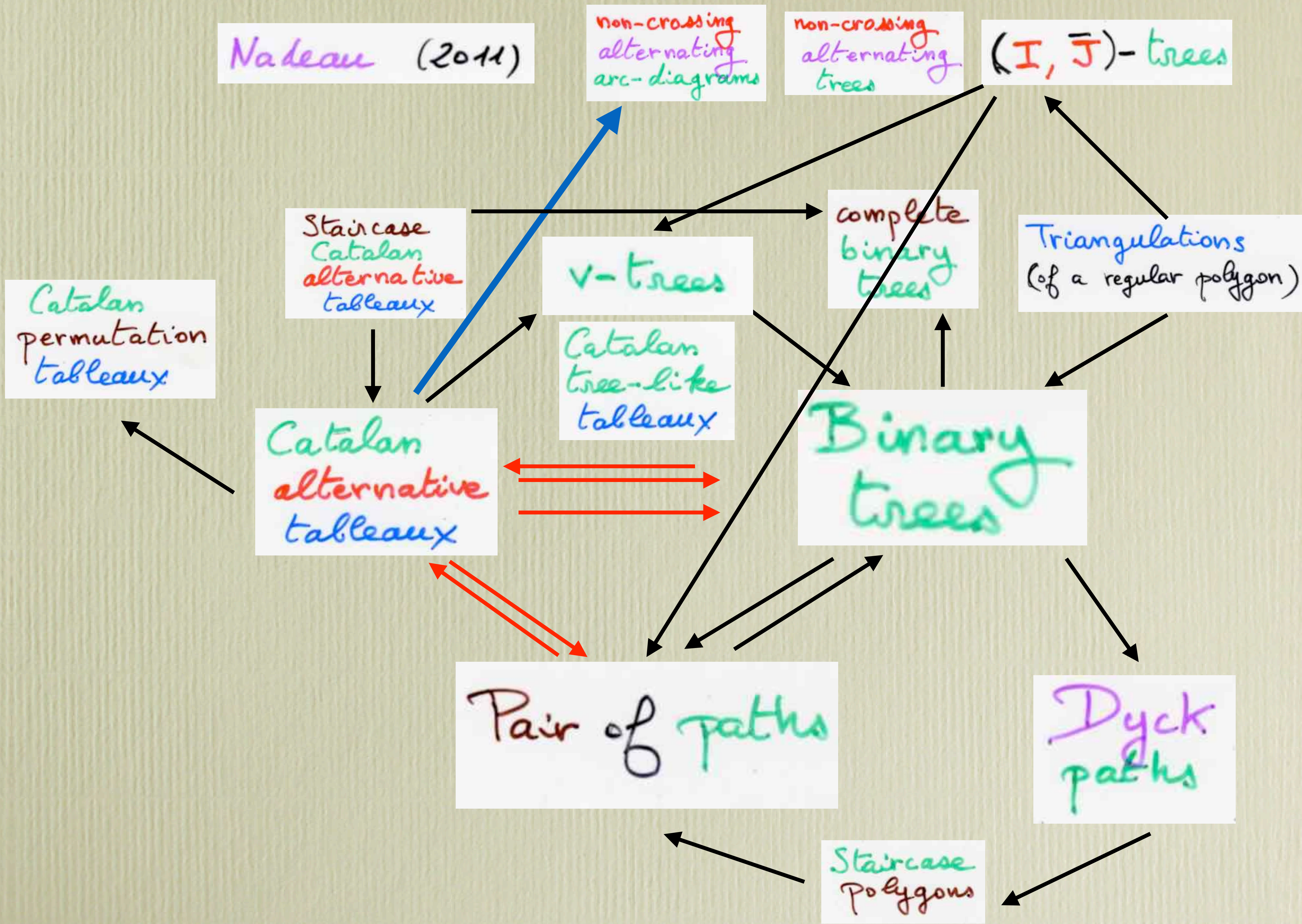
Ceballos, Padrol, Sarmiento
(2016) (2017)
non-crossing alternating trees
(I, J)- trees
Staircase Catalan alternative tableaux
v-trees
complete binary trees
Triangulations (of a regular polygon)
Catalan permutation tableaux
Catalan tree-like tableaux
Catalan alternative tableaux
Binary trees
Pair of paths
Dyck paths
Staircase polygons

a festival of commutative diagrams !

Nadeau (2011) non-crossing alternating arc-diagrams = $(I, \bar{J})$-trees

more with ...

Catalan alternative tableaux



Nadeau (2011)
non-crossing alternating arc-diagrams
non-crossing alternating trees
(I, J)-trees
Staircase Catalan alternative tableaux
v-trees
complete binary trees
Triangulations (of a regular polygon)
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Catalan alternative tableaux
Binary trees
Pair of paths
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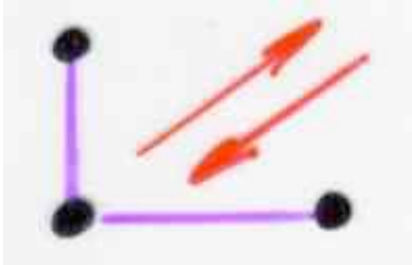
Staircase polygons

comments, remarks, references

Lam, Williams (2008)
total positivity for cominuscule Grassmannians



┘- move

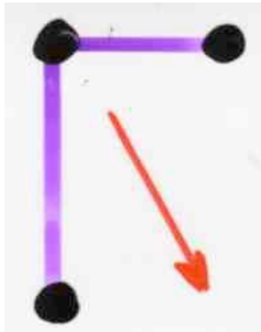
┐- move

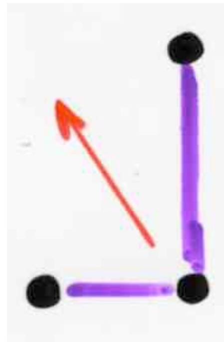
L- move

Karp, Williams, Zhang (2017)
decompositions of amplituhedra
m=4 scattering amplitudes in $\mathcal{N}=4$ supersymmetric Yang-Mills theory

┘- move

Γ- move

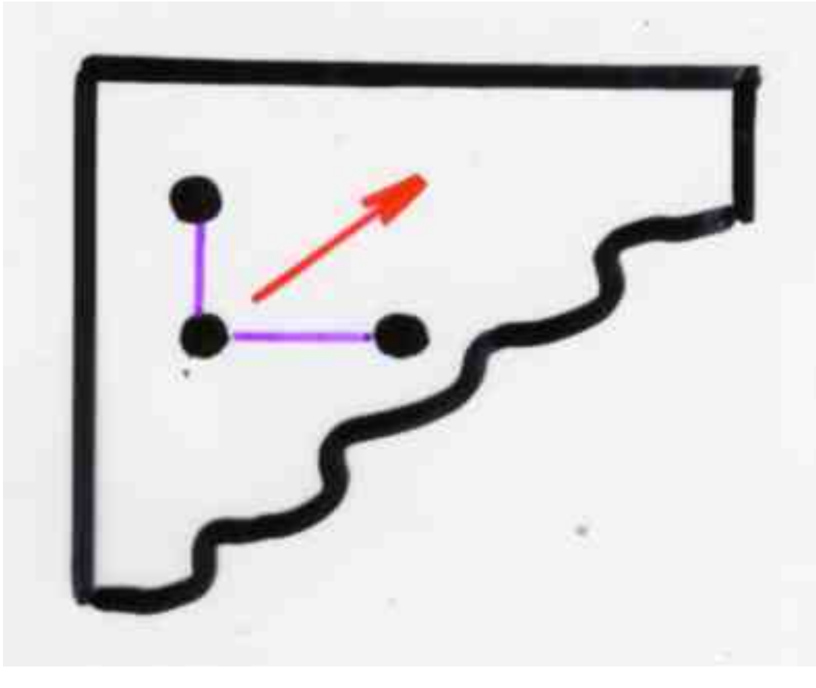


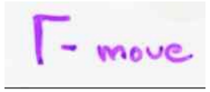


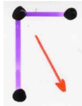
N. Bergeron, S. Billey (2010)
RC-graphs and Schubert polynomials

M. Rubey (2010)
Maximal 0-1 fillings of moon polyominoes
with restricted chain length and RC-graphs

chute move



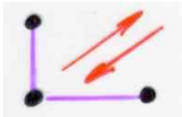
other references using what I call « Γ-move » 

 are:

N. Bergeron and S. Billey, RC-graphs and Schubert polynomials, Experiment Math. 2 (1993), n°4, 257-269 available from http://projecteuclid.org/getRecord?id=euclid.em/1048516036.
(Γ-moves in the case of rectangle with 2 rows)

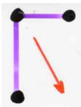
T. Lam and L. Williams, total positivity for cominuscule Grassmannians, New-York J. math., 14: 53-99, 2008, arXiv: 0710.2932 [math.CO]

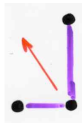
in here fact 

 Ferrers diagrams are in french notations

M. Rubey, Maximal 0-1-fillings of moon polyominoes with restricted chain lengths and RC-graphs, arXiv: 1009.3919v4 [math.CO] ((Γ-moves called « chutes »)

S. Karp, L. Williams, Y. Zhang, Decompositions of amplituhedra, ArXiv: 1708.09525 [math.CO]
here Γ-moves are

 and « Le-move » 



number of maximal chains in Tamari(n) ?

number of chains with maximum length

Nelson (2016) Ph.D.

Fishel, Nelson (2014)
bijection with standard shifted tableaux of staircase shape

This bijection is an immediate consequence of fact that the classical Tamari lattice is a maule: maximal chains with maximum length correspond to Γ-moves which are elementary, that is the corresponding rectangle is reduced to a cell of the square lattice. This property extends to Tamari(v) and the extension mixing Young and Tamari (slides 55-68, part II)

references:

S.Fishel and L.Nelson, Chains of maximum length in the Tamari lattice, Proc. Amer. math Soc. 142 (10):3343-3353, 2014

L.Nelson, Toward the enumeration of maximal chains in the Tamari lattices, Ph.D. Arizona sSate University, August 2016

L.Nelson, A recursion on maximal chains in the Tamari lattices, arXiv: 1709.02987 [math;CO]

alternative tableaux
and avatars

algebraic structures

Hopf algebra

dim 2^{n-1} C_n $n!$

Catalan

Boolean lattice inclusion $\longleftrightarrow$ Tamari order $\longleftrightarrow$ weak Bruhat order

J.-L.Loday, M. Ronco (1998, 2012)

Some references for alternative tableaux and its avatars (enumerated by n!):

permutations tableaux: A. Postnikov, Total positivity, Grassmannians and networks, arXiv math/0609764, 2006

alternative tableaux, X.V. ("video-preprint") talk at Newton Institute, 23 April 2008, slides and video at https://sms.cam.ac.uk/media/1004

P. Nadeau, "On the structure of alternative tableaux", JCTA, Volume 118, Issue 5, July 2011, p1638-1660 or ArXiv 0908.4050,

P. Nadeau introduced a class of "alternative trees" in bijection with alternative tableaux, and a subclass of "non-crossing alternative trees" in bijection with Catalan alternative tableaux, objects which are the same as "(I,J_) trees ".

staircase tableaux: S. Corteel and L. Williams, Duke Math J. 159 (2011), 385--415 , arXiv math/0910.1858, 2009

tree-like tableaux, J.C. Aval, A. Boussicault and P. Nadeau (FPSAC2011, Reikjavik) and Electronic Journal of Combinatorics, Volume 20, Issue 4 (2013), P34

more with permutations tableaux:

S. Corteel, A simple bijection between permutations tableaux and permutations, arXiv: math/0609700

S. Corteel and P. Nadeau, Bijections for permutation tableaux, Europ. J. of Combinatorics, 2007

S. Corteel and L.K. Williams, Tableaux combinatorics for the asymmetric exclusion process, Adv in Apl Maths, to appear, arXiv:math/0602109

E. Steingrimsson and L. Williams Permutation tableaux and permutation patterns, J. Combinatorial Th. A., 114 (2007) 211-234. arXiv:math.CO/0507149

about the cellular ansatz: (mentioned in slide 115-119 about the Adela bijection)

X.V., Alternative tableaux, permutations, a Robinson-Schensted like bijection and the asymmetric exclusion process in physics, (dedicated to to the memory of P. Leroux), talk presented at the 61th SLC, Curia, Portugal, slides available at http://www.mat.univie.ac.at/~slc/

For the four subclasses enumerated by Catalan numbers see:

X.V., FPSAC 2007, Tianjiin : Chine (2007) or arXiv math/ 0905.3081 (bijection Catalan permutation tableaux -- pair of paths (u,v))

J.C. Aval and X.V., (about Catalan alternative tableaux and Loday-Ronco Hopf algebra of trees) SLC, 63 (2010) B63h or arXiv math 0912.0798

here we have rewritten the above bijection Catalan permutation tableaux -- pair (u,v) as a bijection Catalan alternative tableaux -- pair of paths (u,v).

the bijection Catalan alternative tableaux -- Catalan tree-like tableaux can be easily found as a special case of the bijection between alternative tableaux -- tree-like tableaux, see for example: tree-like tableaux, J.C. Aval, A.Boussicault and P.Nadeau (FPSAC2011, Reikjavik) and Electronic Journal of Combinatorics, Volume 20, Issue 4 (2013), P34

more material in:

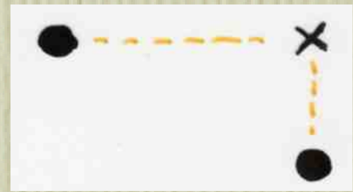
the slides of a "petite école" I gave in Bordeaux:
http://cours.xavierviennot.org/Petite_Ecole_2011_12.html Chapter 2, Slides PEC6 of 4 Nov 2011
see also the course given at IIT Bombay in 2013:
http://cours.xavierviennot.org/IIT_Bombay_2013.html , chapter 4 about the TASEP and Catalan tableaux

the paper introducing the lattice Tamari(v) is:
P.-L. Préville-Ratelle and X.V., « An extension of Tamari lattices », Transactions AMS, 369 (2017) 5219-5239
note: curiously the title in the Transactions « The enumeration of generalised Tamara intervals » is wrong (!). This is the title of the paper [13] quoted in our paper.
An extended abstract of the paper can be found in the Proceeding of the FPSAC'2015, Daejon, South Korea, DMTCS proc. FPSAC'15, 2015, 133-144

The work of C.Ceballos, A.Padrol and C.Sarmiento we very briefly mentioned in slides 79-90 (part II) can be found in:
C.Ceballos, A.Padrol and C.Sarmiento, Geometry of v-Tamari in types A and B, ArXiv: 1611.09794 [math.CO] (47 pages)
and in the slides of a talk at the 78th SLC devoted to the 60th birthday of Jean-Yves Thibon
http://www.mat.univie.ac.at/~slc/ see « preface » with the talk of Cesar Ceballos « v-Tamari lattices via subwords complex »

v-trees introduced by the 3 authors are the same as the binary tree underlying an alternative tableau, or equivalently a tree-like tableau n!

permutations



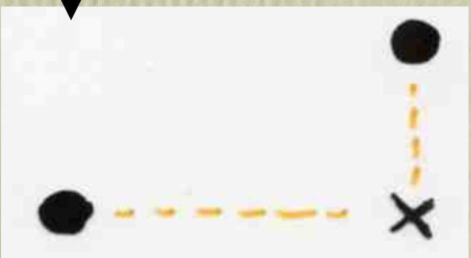
permutation tableaux

alternative tableaux

X.V. (2008)

tree-like tableaux

Aval, Boussicault, Nadeau (2013)



⅃-diagrams

decorated permutations

Steingrímsson, Williams (2007)

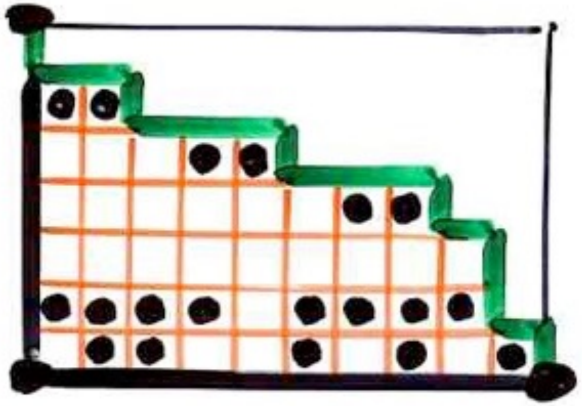
Postnikov (2006)

totally non-negative part of the type A Grassmannian

Permutation Tableau

Ferrers diagram $F \subseteq k \times (n-k)$

rectangle



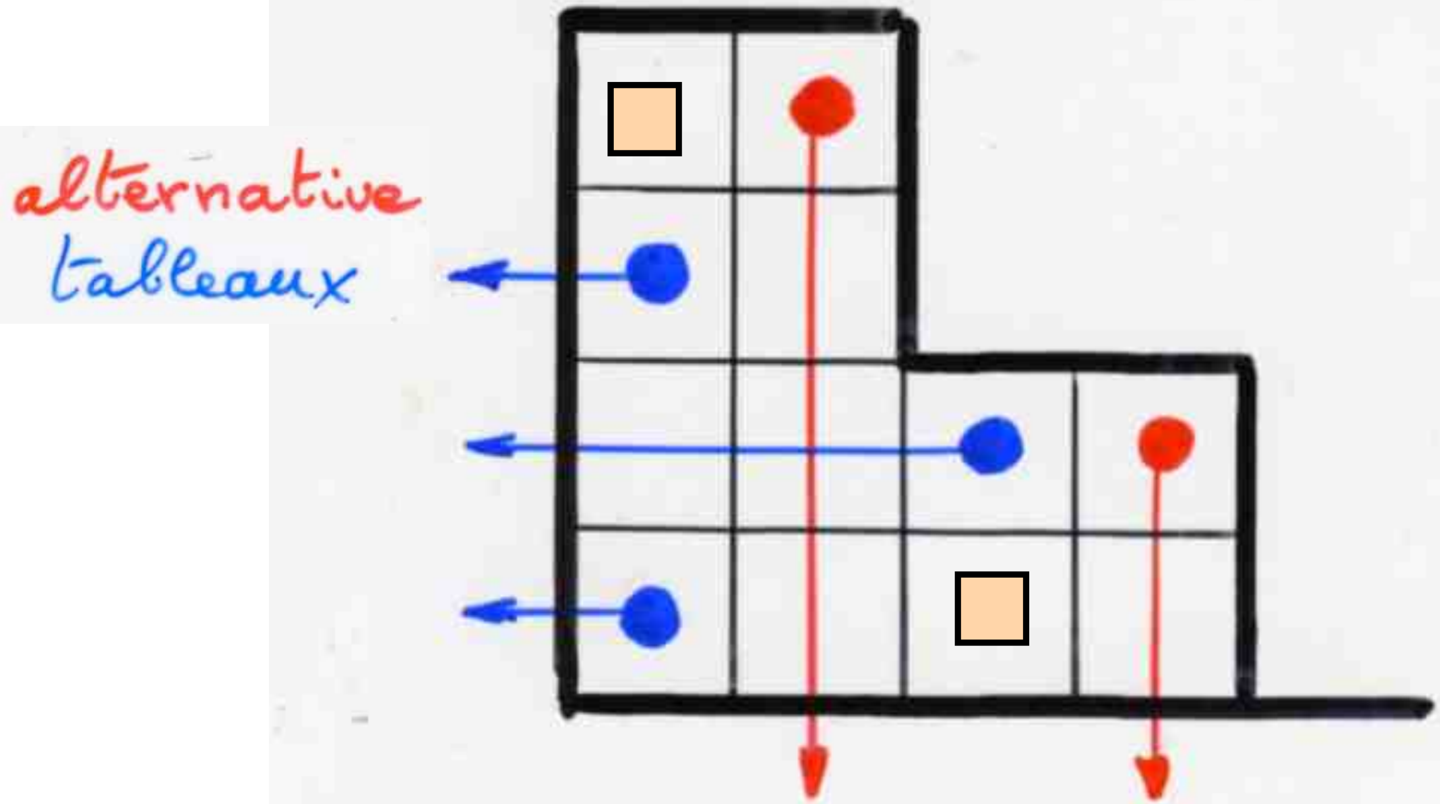
□ = 0   ⊡ = 1

filling of the cells with 0 and 1

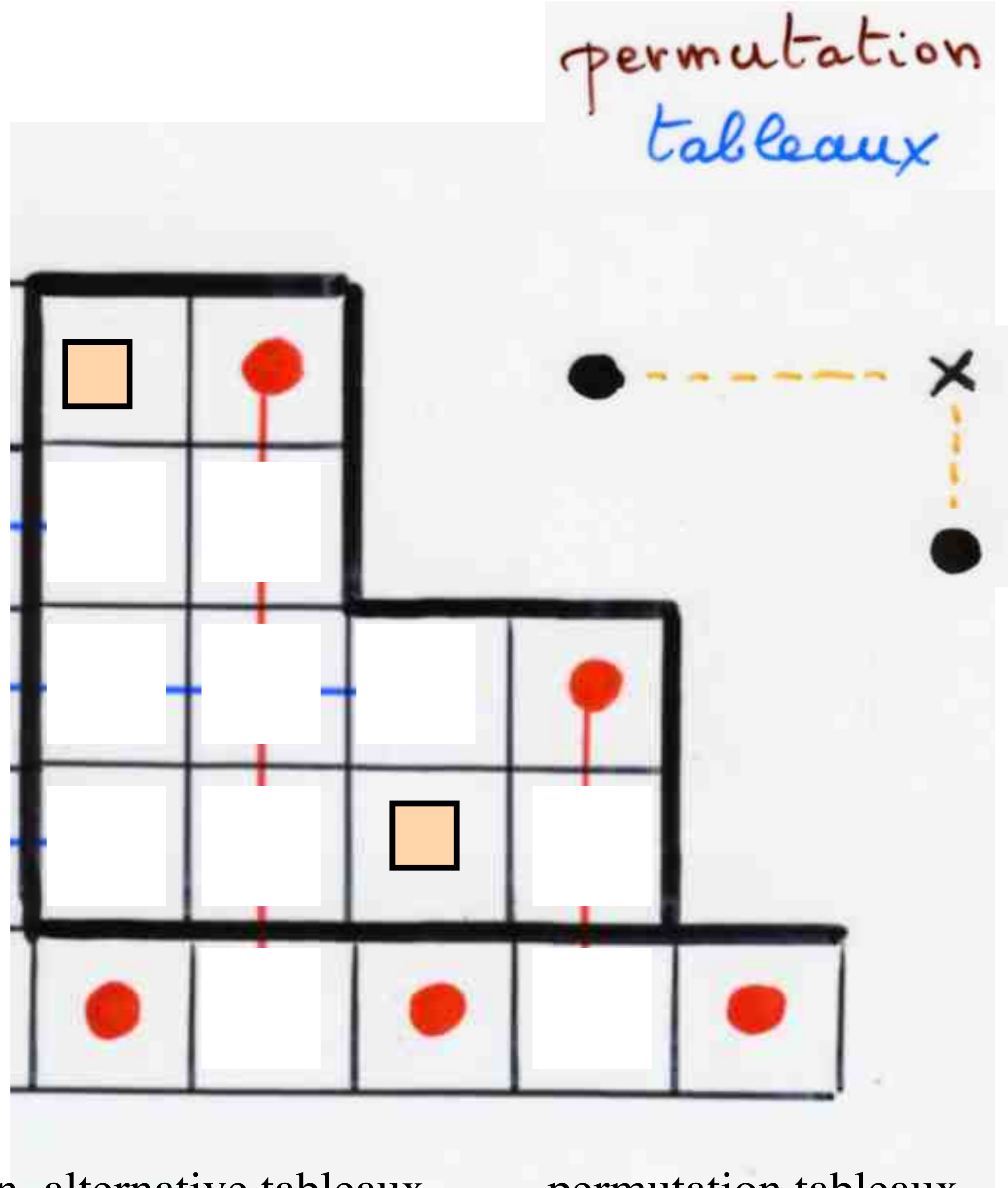
(i) in each column : at least one 1

(ii) 1 - - - - 0 forbidden

$\quad\quad\quad\quad\ \ $ 1



Prop. The number of alternative tableaux of size n is $(n+1)!$

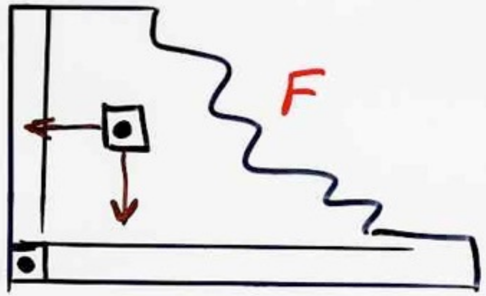


The bijection alternative tableaux —— permutation tableaux

In the Catalan case, we get back the bijection described slides 77-78 part I

Def- Tree-like tableaux

(Aval, Boussicault, Nadeau) 2011



Ferrers diagram

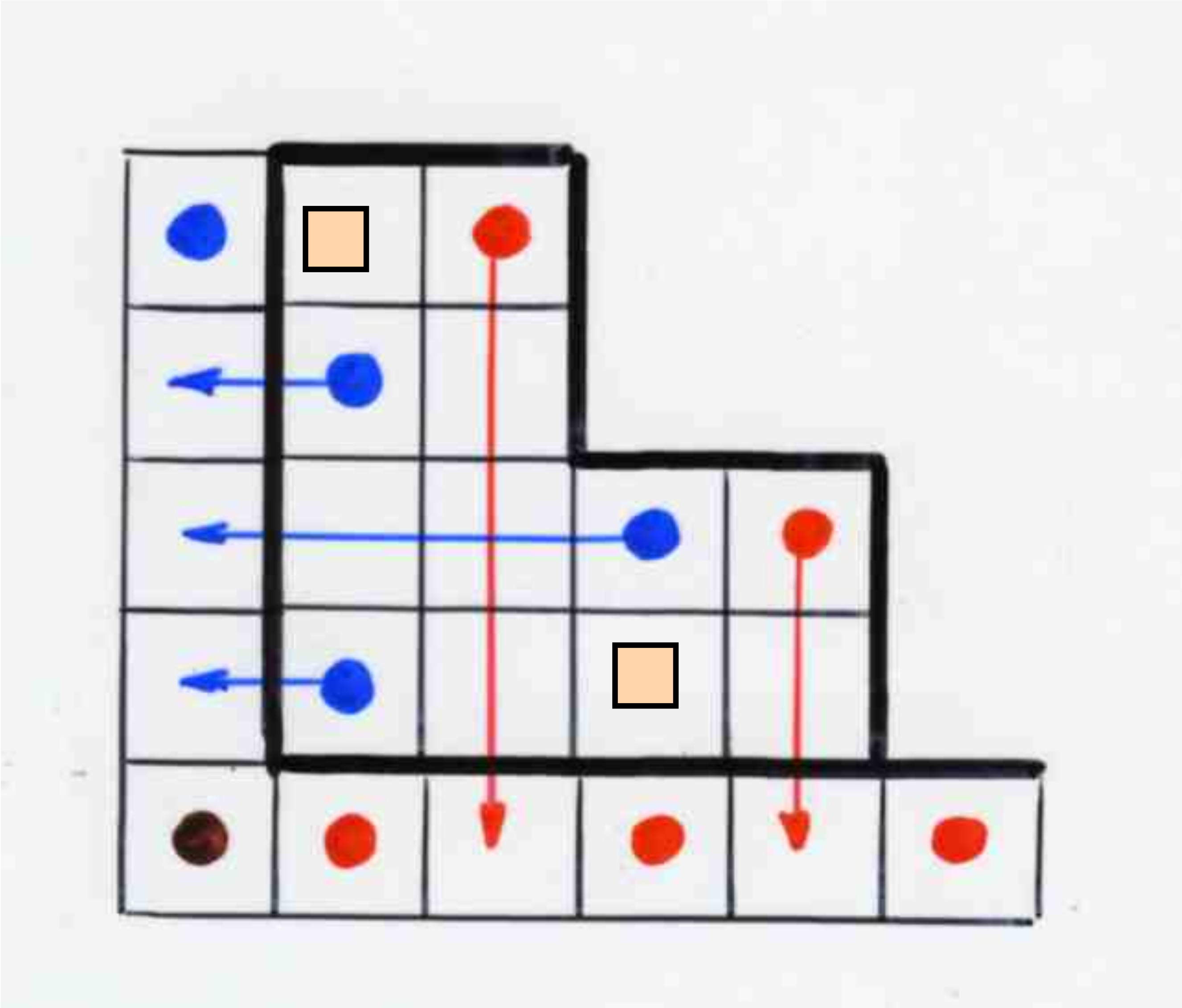
empty cell

pointed cell

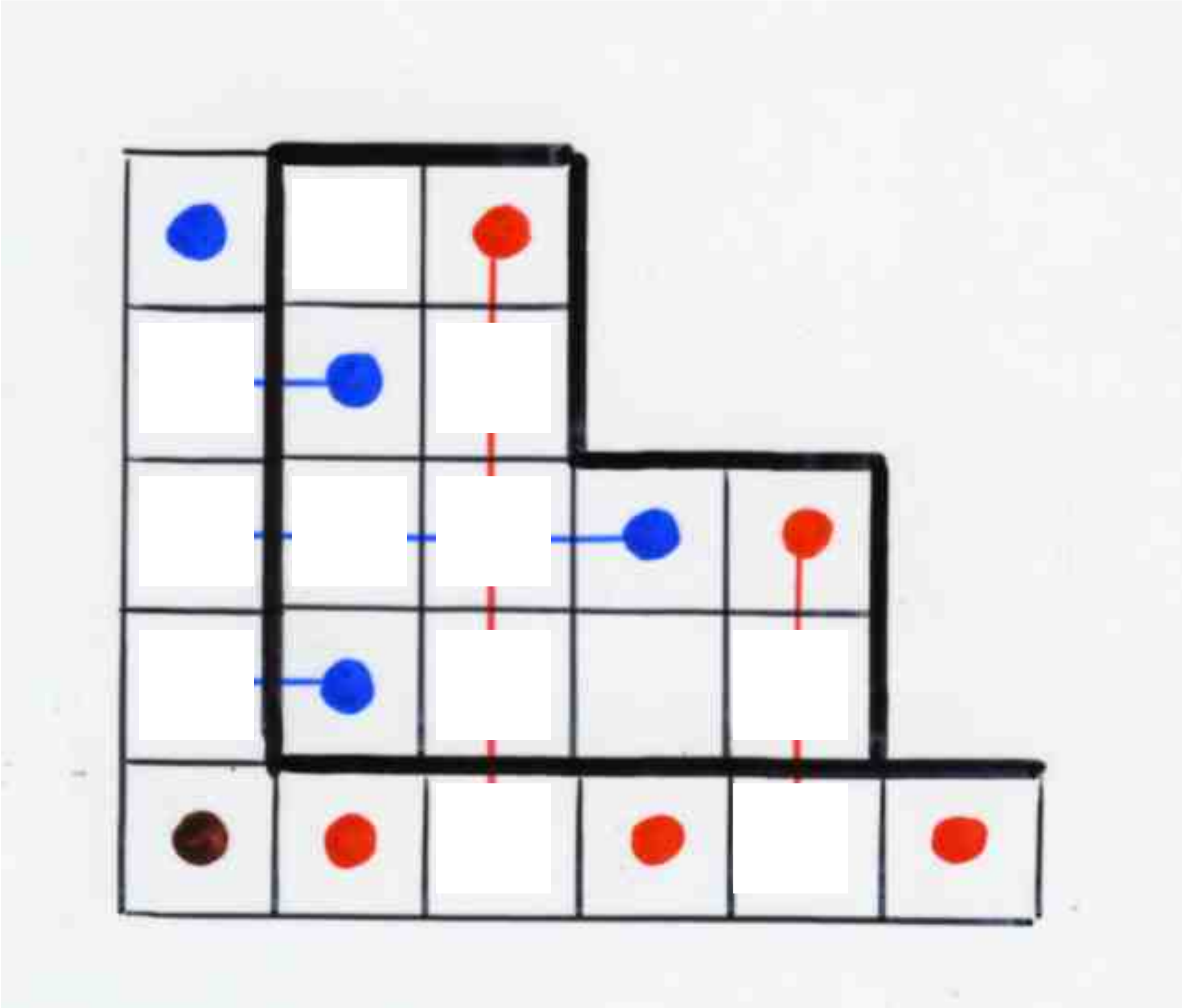
(i) botom left cell possess a point called root point

(ii) for every non-root pointed cell c, ∃ either a pointed cell below c in same column or a pointed cell to its left in same row but no both

(iii) every column and every row possess at least one pointed cell

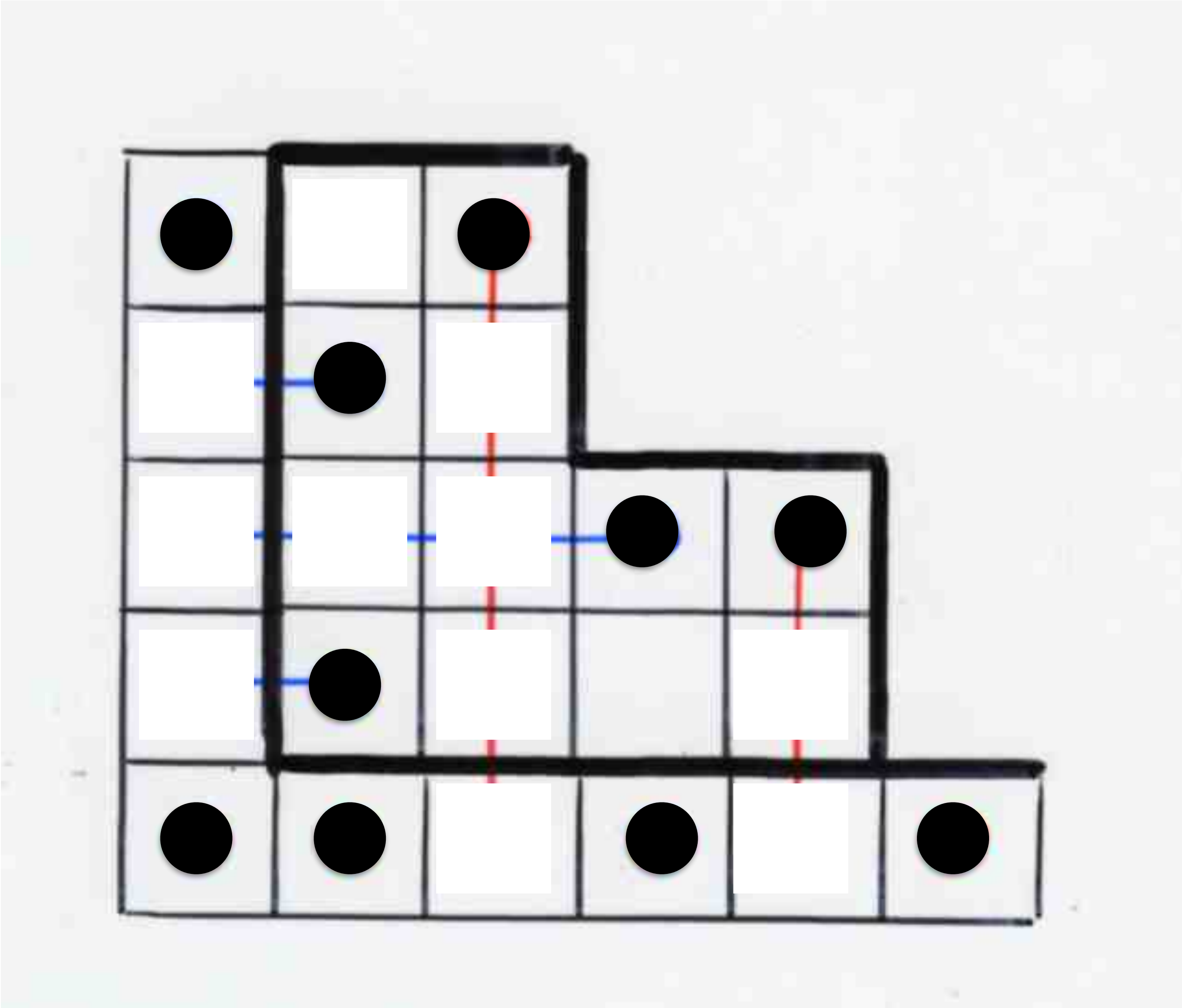


augmented tableau (as in slides 77-82 part I, for Catalan alternative tableau) tree-like tableaux

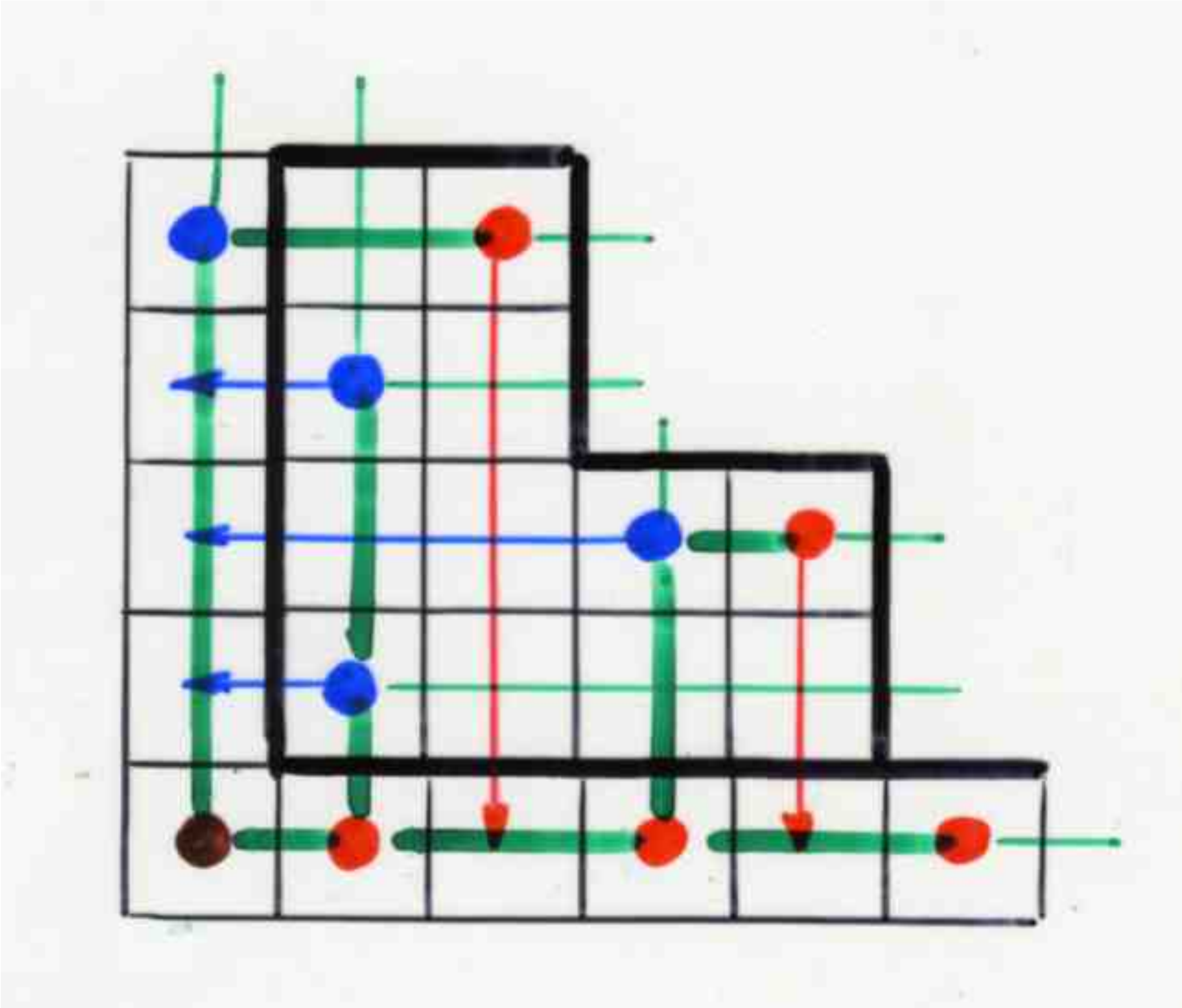


The bijection alternative tableaux —— tree-like tableaux

tree-like
tableaux

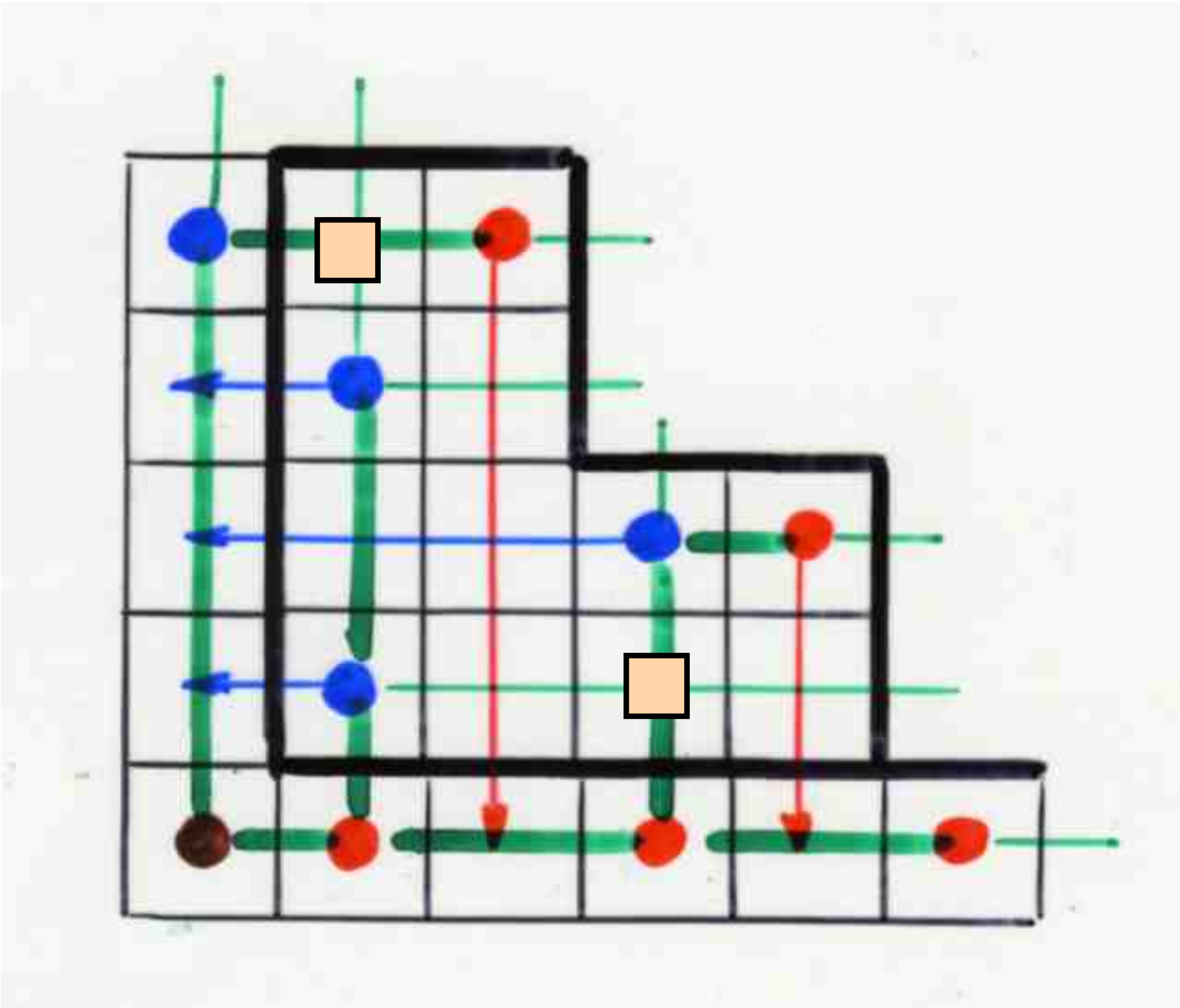


The bijection alternative tableaux — — tree-like tableaux



« non-ambiguous tree » associated to an alternative tableau

analog to the case of Catalan alternative tableau, slide 98, part I.



yellow cells correspond to crossings in the « non-ambiguous tree »

the Adela bijection

This a bijection between alternative tableaux T and a pair (P,Q) of vectors of integers

$$\mathrm{Adela}(T) = (P, Q)$$

The row vector P is obtained by associating to each row:

- 0 if there are no blue point in the row
- 1 + the number of cells in the row which are of the type
  (i.e. there is a blue point at its right, but no red point above)

as in the Catalan case, see slide 39, this part II.

The column vector Q is obtained by associating to each column:

- 0 if there are no red point in the column
- 1 + the number of cells in the column which are of the type
  (i.e. there is a red point above, but no blue point on its right) .

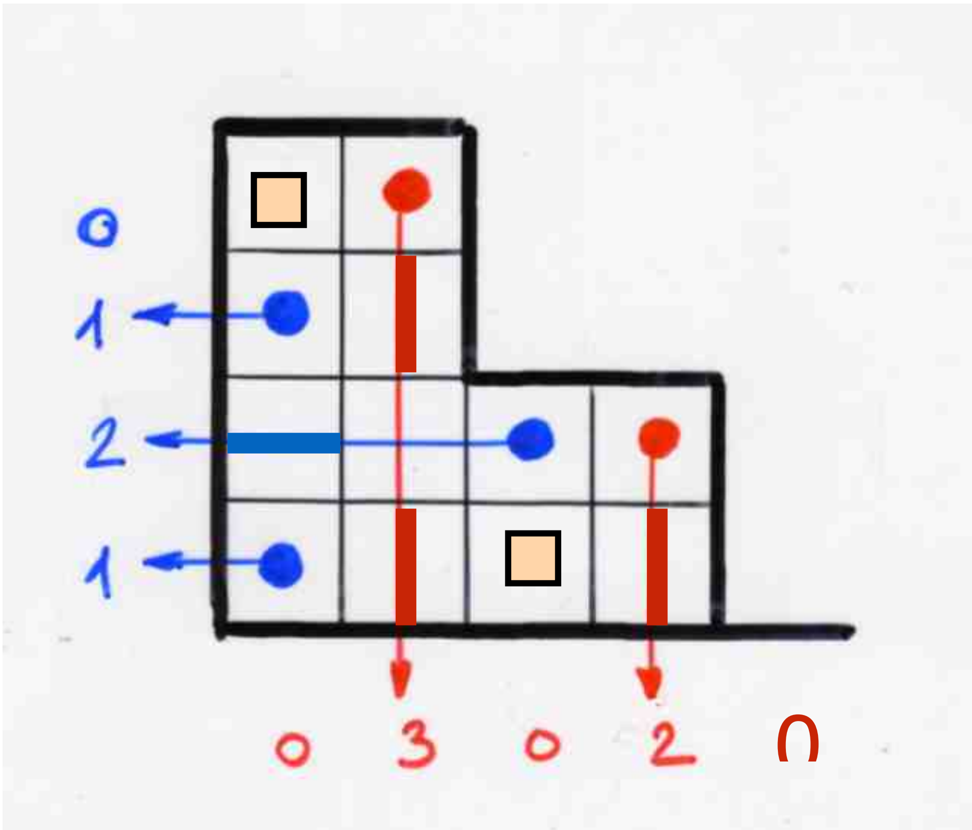
the Adela bijection

an example

Adela$(T) = (P, Q)$

$P = (0, 1, 2, 1)$

$Q = (0, 3, 0, 2, 0)$



the Adela bijection

$$\text{Adela}(T) = (P, Q)$$

The map $T \longrightarrow (P, Q)$ is a bijection between alternative tableaux and some pairs (P, Q) of integers.

This fact can be proved using the « **cellular ansatz** » methodology described in the series of lecture given at Bordeaux in 2011/12 or at IIT Bombay in 2013, see:
http://cours.xavierviennot.org/Petite_Ecole_2011_12.html
http://cours.xavierviennot.org/IIT_Bombay_2013.html

The cellular ansatz methodology associate certain combinatorial objects to some quadratic algebra, together with a systematic way to construct some bijections analogue to the RSK bijection between permutations and pair of Young tableaux. In the case of the so-called PASEP algebra defined by generators E, D and the relation DE = ED+E+D, we get the alternative tableaux enumerated by n!.
In the case of the Weyl-Heisenberg algebra defined by UD = DU+Id, we get the permutations.

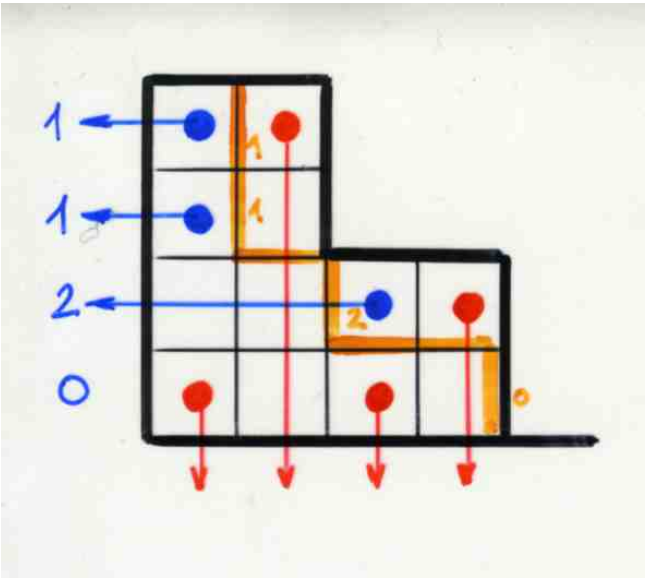
Then we define a methodology called « demultiplication » of equations (see Chapter 5, slides PEC15) of the « petite école » or Chapter 7 of the course at IIT Bombay), which gives the RSK bijection in the case of the algebra UD = DU+Id, and the above Adela bijection in the case of the PASEP algebra.

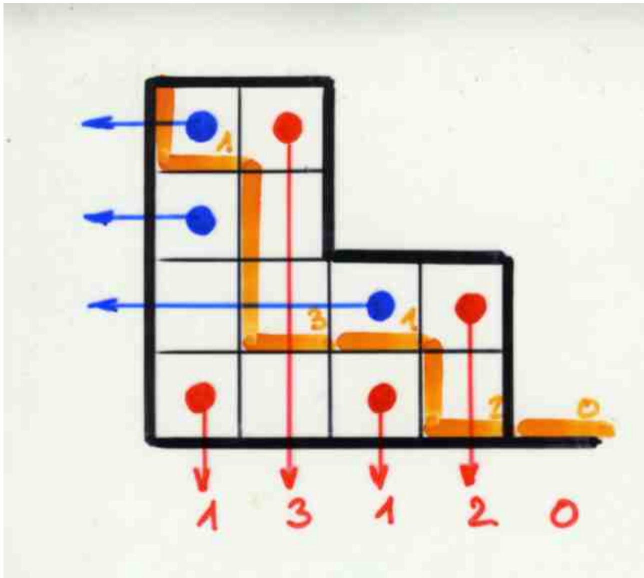
the Adela duality

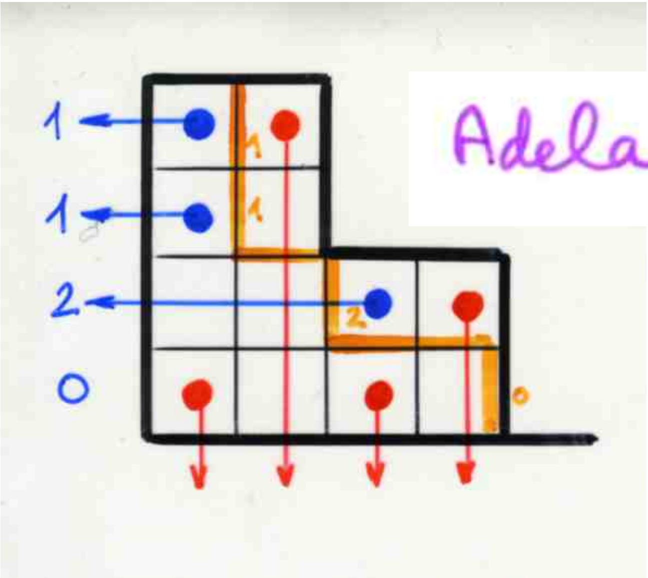
$$\text{Adela}(T) = (P, Q)$$

In the case of Catalan alternative tableaux, the column vector Q is determined by the row vector P and in that case the Adela bijection is reduced to the bijection T $\longrightarrow$ P described in this talk (slide 39 of this part II).

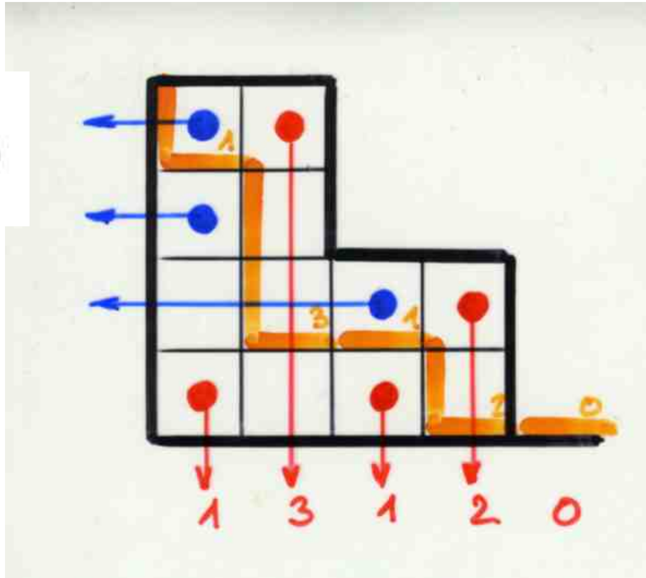
In that case I call the map exchanging P $\longrightarrow$ Q « the Adela duality » (see next slide). This is equivalent to the duality described on slides 21 and slide 22 (theorem 2), part II.





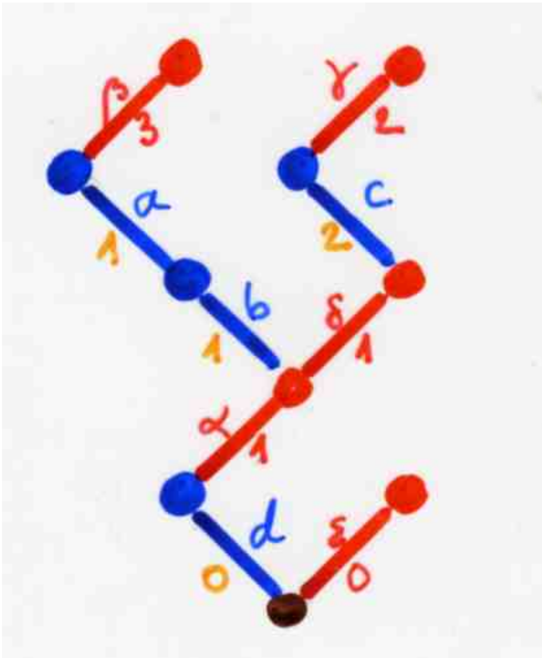


$\text{Adela}(T) = (P, Q)$



the Catalan case

Adela duality



a	b	c	d
1	1	2	0

α	β	γ	δ	ε
1	3	1	2	0

right height
of
left edges

inorder
(= symmetric order)

left height
of
right edges

Isla Negra
Pablo Neruda

The names «Adela bijection» and «Adela duality» is in honour of my friend Adela where part of this research was done in her house in Isla Negra, Chile, inspiring place where Pablo Neruda spent many years in his house in front of the Pacific Ocean.

Thank you !

new website (in construction): www.viennot.org

old website: www.xavierviennot.org/xavier