INTEGRABILITY OF THE POISSON ALGEBRA ON A LOCALLY CONFORMAL SYMPLECTIC MANIFOLD

Stefan Haller   Tomasz Rybicki

Abstract. It is proven that the Poisson algebra of a locally conformal symplectic manifold is integrable by making use of a convenient setting in global analysis. It is also observed that, contrary to the symplectic case, a unified approach to the compact and non-compact case is possible.

1. Introduction

The aim of this note is to find the Lie group for the Poisson algebra of a locally conformal symplectic structure. The locally conformal symplectic manifolds constitute an intermediate notion between almost symplectic and symplectic manifolds.

An almost symplectic manifold (M, Ω) is called *locally conformal symplectic* (l.c.s.) if there exists an open covering $\{U_i\}_{i \in I}$ and a family of positive functions $\alpha_i \in C^\infty(U_i, \mathbb{R})$ such that $d(\alpha_i \Omega) = 0$ on U_i. It was first observed by H.C.Lee in [10] that then $d \ln \alpha_i$ glue up to a closed 1-form ω. So equivalently, and this will be our working definition, an l.c.s. manifold is a triple (M, Ω, ω) where ω is a closed 1-form and Ω is a non-degenerate 2-form satisfying

$$d^\omega \Omega := d\Omega + \omega \wedge \Omega = 0.$$

Since Ω is non-degenerate we get a canonical vector bundle isomorphism $\flat : TM \cong T^*M$ given by $X \mapsto i_X \Omega$. By $\sharp$ we denote the inverse of $\flat$.

The properties of l.c.s. structures have been studied in [4], [6], [7], [8], [14]. The l.c.s. manifolds are geometrically significant because they are exactly transitive Jacobi manifolds of even dimension, and any even dimensional leaf of the characteristic foliation of a Jacobi manifold carries an l.c.s. structure. But they also provide a physics motivation as they can serve as a more general model of the phase-space with all its good properties, cf. [14].

The third theorem of Lie asserts that any finite dimensional Lie algebra is actually the Lie algebra of a Lie group. This is no longer true in the infinite dimensional case, see, e.g., [5]. However there are several remarkable generalizations of this theorem, cf. [1], [3], [12], [13].

In the present note we extend some integrability results ([13],[1]) in the symplectic geometry to the l.c.s. case. It is surprising that, contrary to the symplectic case, the integrability of the l.c.s. Poisson algebra can be proven without discerning the compactness and non-compactness of a manifold (provided it is not globally conformal symplectic).

---

[0]This paper is in final form and no version of it will be submitted for publication elsewhere.

1991 *Mathematics Subject Classification.* 58B 53C 22E.

Key words and phrases. locally conformal symplectic structure, Poisson algebra, Lie group, Hamiltonian vector field, integrability, flux homomorphism.

The first autor is supported by the Fonds zur Förderung der Wissenschaftlichen Forschung, P-10037-PHY. The second author is supported by a grant in AGH

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

2. Preliminaries and the "weak" integrability

Let (M,Ω,ω) be an l.c.s. manifold. Alternatively it is defined by a couple (Λ,E) where Λ is a non-degenerate skew-symmetric $(2,0)$-tensor field, and E is a vector field which satisfy the relations

$$[\Lambda,\Lambda]=2E\wedge\Lambda,\qquad L_E\Lambda=[E,\Lambda]=0,$$

where $[\cdot,\cdot]$ denotes the Schouten-Nijenhuis bracket. The bracket on $C^\infty(M,\mathbb{R})$ is then given by:

$$\{u,v\}=\Lambda(du,dv)+udv(E)-vdu(E),$$

and $\mathcal{P}_c(M,\Omega,\omega)$ will stand for the Poisson algebra $(C^\infty(M,\mathbb{R}),\{\cdot,\cdot\})$.

Suppose a is a nowhere vanishing function on M. Then one can define a new bracket by $\{f,g\}_a:=\frac{1}{a}\{af,ag\}$. We then have $\Lambda_a=a\Lambda$, $E_a=aE+\Lambda(da)$ and $\Omega_a=\frac{1}{a}\Omega$, $\omega_a=\omega+\frac{da}{a}$. Moreover (M,Ω_a,ω_a) is again an l.c.s. manifold which is called conformally equivalent to the given one. We will denote this by $(M,\Omega,\omega)\sim(M,\Omega_a,\omega_a)$. An l.c.s. manifold is called globally conformal symplectic iff it is conformally equivalent to a symplectic manifold. This is the case if and only if ω is exact.

If g is a diffeomorphism of M then $(M,g^*\Omega,g^*\omega)$ is again an l.c.s. manifold. We write $\mathrm{Diff}^\infty_c(M,\Omega,\omega)$ for the group of all compactly supported diffeomorphisms that preserve the l.c.s. structure (Ω,ω) up to conformal equivalence, i.e.

$$\mathrm{Diff}^\infty_c(M,\Omega,\omega):=\big\{g\in\mathrm{Diff}^\infty_c(M):(M,g^*\Omega,g^*\omega)\sim(M,\Omega,\omega)\big\}.$$

More explicitly, $g\in\mathrm{Diff}^\infty_c(M,\Omega,\omega)$ iff there exists $a\in C^\infty(M,\mathbb{R}\setminus 0)$ such that $g^*\Omega=\frac{1}{a}\Omega$ and $g^*\omega=\omega+d(\ln|a|)$. If $\dim M>2$ then the first equation implies the second since ω is unique. Next we set

$$\mathfrak{X}_c(M,\Omega,\omega):=\big\{X\in\mathfrak{X}_c(M):\exists u\in C^\infty(M,\mathbb{R}):L_X\Omega=-u\Omega,L_X\omega=du\big\}.$$

Again, if $\dim M>2$ then the equation $L_X\Omega=-u\Omega$ implies the equation $L_X\omega=du$.

Notice that any Hamiltonian vector field $X_u=\sharp d^\omega u\in\mathfrak{X}_c(M,\Omega,\omega)$ for $u\in C^\infty_c(M,\mathbb{R})$, since $L_{X_u}\Omega=-\omega(X_u)\Omega$ and $L_{X_u}\omega=d(\omega(X_u))$. In terms of Λ and E the Hamiltonian is $X_u=\Lambda(du)+uE$. Then we have

$$\{u,v\}=-\Omega(X_u,X_v)=X_uv+v\omega(X_u)=-(X_vu+u\omega(X_v)).$$

In [8] it is shown that $\mathfrak{X}_c(M,\Omega,\omega)$ is the Lie algebra of $\mathrm{Diff}^\infty_c(M,\Omega,\omega)$ in the following sense:

Proposition 2.1. *Let $g\in C^\infty\big((\mathbb{R},0),(\mathrm{Diff}^\infty_c(M),\mathrm{id})\big)$. Then we have:*

$$\begin{aligned}g\in C^\infty\big(\mathbb{R},\mathrm{Diff}^\infty_c(M,\Omega,\omega)\big)&\Leftrightarrow\delta^rg\in\Omega^1\big(\mathbb{R};\mathfrak{X}_c(M,\Omega,\omega)\big)\\&\Leftrightarrow\dot g_t\in\mathfrak{X}_c(M,\Omega,\omega)\end{aligned}$$

(Here δ^rg is the right logarithmic derivative, cf. [9].) Especially $\mathrm{Fl}^X\in C^\infty(\mathbb{R},\mathrm{Diff}^\infty_c(M,\Omega,\omega))$ *iff $X\in\mathfrak{X}_c(M,\Omega,\omega)$.*

The mapping $\mathcal{H}:\mathcal{P}_c(M,\Omega,\omega)\ni u\mapsto X_u\in\mathfrak{X}_c(M,\Omega,\omega)$ is a Lie algebra homomorphism, i.e. $X_{\{u,v\}}=[X_u,X_v]$. We have that $\mathcal{P}_c(M,\Omega,\omega)$ is weakly integrable (in terms of [1]) and, moreover, in a more unified manner than in the symplectic case (cf. [1]).

Proposition 2.2 [14]. *Suppose (M,Ω,ω) is not globally conformal symplectic. Then $\mathcal{H}$ is an isomorphism from $\mathcal{P}_c(M,\Omega,\omega)$ onto its image $\mathcal{H}_c(M,\Omega,\omega)$.*

In fact, the mapping $u\mapsto d^\omega u$ is injective, since its kernel is $H^0_{\mathrm{d}^\omega_c}(M)=0$ for a non-exact ω. See the section below for this (Lemma 3.1).

3. d_c^ω-cohomology

Let ω be a closed 1-form on a manifold M and consider the differential:

$$d^\omega : \Omega^*(M) \to \Omega^{*+1}(M) \qquad d^\omega \alpha := d\alpha + \omega \wedge \alpha$$

Since ω is closed we have $d^\omega \circ d^\omega = 0$. We denote the corresponding cohomology by $H^*_{d^\omega}(M)$, and the cohomology with compact support by $H^*_{d^\omega_c}(M)$. If $[\omega] = [\omega'] \in H^1(M)$ we can choose $a \in C^\infty(M, \mathbb{R})$ such that $\omega' = \omega + \frac{da}{a} = \omega + d\ln|a|$ and obtain an isomorphism $\frac{1}{a} : H^*_{d^\omega}(M) \to H^*_{d^{\omega'}}(M)$ given by multiplication with $\frac{1}{a}$. So the d^ω-cohomology does only depend on the cohomology class of ω. Especially it is isomorphic to the ordinary de Rham cohomology if ω is exact.

Consider the sheaf $\mathcal{F}_\omega(U) := \{f \in C^\infty(U, \mathbb{R}) : d^\omega f = 0\}$. It is a locally constant sheaf and the following is a fine resolution:

$$0 \to \mathcal{F}_\omega \to \Omega^0 \xrightarrow{d^\omega} \Omega^1 \xrightarrow{d^\omega} \Omega^2 \to \cdots$$

So $H^*_{d^\omega}(M)$ is simply the sheaf cohomology of $\mathcal{F}_\omega$. Moreover one can easily construct a bundle of coefficients on M such that the sheaf of sections is precisely $\mathcal{F}_\omega$. Thus $H^*_{d^\omega}(M)$ is a kind of twisted de Rham cohomology. See [7] and [8] for more details.

Lemma 3.1. *Let M be connected and let ω be a closed 1-form which is not exact. Then $H^0_{d^\omega}(M)$ as well as $H^0_{d^\omega_c}(M)$ vanishes.*

Proof. Consider first the case $M = S^1$. We may assume $\omega = \lambda d\theta$ for some $\lambda \in \mathbb{R}$. We consider $f \in \Omega^0(S^1)$ as periodic function on the line. Being d^ω-closed translates to $f' + \lambda f = 0$, which has no nontrivial periodic solution. So $H^0_{d^\omega}(S^1) = 0$.

For arbitrary M we may choose a mapping $i : S^1 \to M$ such that $i^*\omega$ is non-exact. If $f \in \Omega^0(M)$ is d^ω-closed we get $d^{i^*\omega} i^* f = 0$ and thus $i^* f = 0$ by the previous paragraph. Since the zero set of a d^ω-closed function is open and closed, we get $f = 0$ from the connectedness of M. □

4. Some invariants

The following facts have been established in [8].

Proposition 4.1. *A compactly supported vector field $X \in \mathfrak{X}_c(M, \Omega, \omega)$ if and only if there exists a locally constant function $c_X \in C^\infty(M, \mathbb{R})$ with $d^\omega(\flat X) = c_X \Omega$. In this case $c_X = i_X \omega - u_X$, where $u_X \in C^\infty(M, \mathbb{R})$ satisfies $L_X \Omega = -u_X \Omega$ and $L_X \omega = du_X$. Moreover the mapping*

$$\varphi : \mathfrak{X}_c(M, \Omega, \omega) \to H^0_c(M), \qquad X \mapsto [c_X]$$

is a Lie algebra homomorphism. If M is compact it is surjective iff Ω is d^ω-exact.

If $(M, \Omega, \omega) \sim (M, \Omega', \omega')$ then $\mathfrak{X}_c(M, \Omega, \omega) = \mathfrak{X}_c(M, \Omega', \omega')$ and $\varphi = \varphi'$. If $g \in \mathrm{Diff}^\infty_c(M, \Omega, \omega)$ then $\mathfrak{X}_c(M, \Omega, \omega) = \mathfrak{X}_c(M, g^*\Omega, g^*\omega)$, $\varphi \circ g^* = g^* \circ \varphi$ and if $g \in \mathrm{Diff}^\infty_c(M, \Omega, \omega)_\circ$ have even get $\varphi \circ g^* = \varphi$. (The subscript $\circ$ indicates the identity component.)

That φ does not vanish in general, cf. [8] and the above examples.

Proposition 4.2. *The Lie algebra homomorphism φ integrates to a group homomorphism*

$$\tilde{\Phi} : \widetilde{\mathrm{Diff}}^\infty_c(M, \Omega, \omega)_\circ \to H^0_c(M),$$

i.e. $\tilde{\Phi} \circ \exp = \exp \circ \varphi$, *or* $\tilde{\Phi}(\mathrm{Fl}^X) = \varphi(X)$. $\tilde{\Phi}$ *is expressed as*

$$\tilde{\Phi}(g) = \textstyle\int_I \varphi_*(\delta^r g) = \int_0^1 \varphi(\dot{g}_t)dt = \big[\int_0^1 c_{\dot{g}_t} dt\big] = \big[\int_0^1 g_t^* c_{\dot{g}_t} dt\big].$$

If $(M, \Omega, \omega) \sim (M, \Omega', \omega')$ *then* $\widetilde{\mathrm{Diff}}_c^\infty(M, \Omega, \omega)_\circ = \widetilde{\mathrm{Diff}}_c^\infty(M, \Omega', \omega')_\circ$ *and* $\tilde{\Phi}' = \tilde{\Phi}$.

Now we define an analog of the flux homomorphism. This concept is essentially due to E.Calabi [2]. The following facts ensure us that this concept works on the ground of the l.c.s. theory.

Proposition 4.3. *We have a surjective Lie algebra homomorphism*

$$\psi : \ker \varphi \to H^1_{d_c^\omega}(M), \qquad X \mapsto [\flat X].$$

If $(M, \Omega, \omega) \sim (M, \Omega', \omega')$ *with* $\Omega' = \frac{1}{a}\Omega$, $\omega' = \omega + d(\ln|a|)$ *then* $\ker \varphi = \ker \varphi'$ *and* $\frac{1}{a}\psi = \psi'$.

If $g \in \mathrm{Diff}_c^\infty(M, \Omega, \omega)$ *and* $(M, \Omega'', \omega'') := (M, g^*\Omega, g^*\omega)$ *then* $\ker \varphi = \ker \varphi''$ *and* $\frac{1}{a}\psi \circ g^* = g^* \circ \psi$ *and if* $g \in \ker \Phi$ *we even get* $\psi \circ g^* = \psi$.

Proposition 4.4. *The Lie algebra homomorphism* ψ *integrates to a surjective group homomorphism*

$$\tilde{\Psi} : \widetilde{\ker \Phi} \to H^1_{d_c^\omega}(M),$$

i.e. $\tilde{\Psi} \circ \exp = \psi \circ \exp$, *or* $\tilde{\Psi}(\mathrm{Fl}^X) = \psi(X)$. $\tilde{\Psi}$ *is defined by*

$$\tilde{\Psi}(g) = \textstyle\int_I \psi_*(\delta^r g) = \int_0^1 \psi(\dot{g}_t)dt = \big[\int_0^1 i_{\dot{g}_t}\Omega dt\big] = \big[\int_0^1 a_t g_t^* i_{\dot{g}_t}\Omega dt\big],$$

where $g_t^*\Omega = \frac{1}{a_t}\Omega$. *If* $(M, \Omega, \omega) \sim (M, \Omega', \omega')$ *with* $\Omega' = \frac{1}{a}\Omega$ *and* $\omega' = \omega + d(\ln|a|)$ *then* $\widetilde{\ker \Phi'} = \widetilde{\ker \Phi}$ *and* $\frac{1}{a}\tilde{\Psi}' = \tilde{\Psi}$.

We let $\Gamma := \tilde{\Psi}(\pi_1(\ker \Phi))$. Then Γ is countable, cf. [8]. $\tilde{\Psi}$ descends to a surjective homomorphism

$$\Psi : \ker \Phi \to H^1_{d_c^\omega}(M)/\Gamma$$

Suppose now that (M, Ω, ω) is exact, i.e. $\Omega = d^\omega \alpha$. Then for $g \in \widetilde{\ker \Phi}$ we have

$$\tilde{\Psi}(g) = [a_1 g_1^* \alpha - \alpha] \in H^1_{d_c^\omega}(M),$$

where $g_t^*\Omega = \frac{1}{a_t}\Omega$ and $g_t^*\omega = \omega + d(\ln|a_t|)$. In particular, $\Gamma = 0$.

The following gives a characterization of Hamiltonian isotopies.

Proposition 4.5. *Let* $g \in C^\infty\big((\mathbb{R}, 0), (\mathrm{Diff}_c^\infty(M, \Omega, \omega), \mathrm{id})\big)$. *Then*

$$g \in C^\infty(\mathbb{R}, \ker \Psi) \quad \Leftrightarrow \quad \delta^r g \in \Omega^1(\mathbb{R}; \ker \psi) \quad \Leftrightarrow \quad \dot{g}_t \in \ker \psi.$$

Specifically, $\mathrm{Fl}^X \in C^\infty(\mathbb{R}, \ker \Psi)$ *iff* $X \in \ker \psi$. *Moreover,* $\ker \Psi$ *is connected by smooth arcs, and* $\widetilde{\ker \Psi} \cong \ker \tilde{\Psi}$.

Let us now give some examples of l.c.s. manifolds which motivate our interest.

As in the symplectic case there is also a 'canonical' l.c.s. structure on T^*M. Given a closed 1-form ω on M we let $\omega' := \pi^*\omega \in \Omega^1(T^*M)$, where $\pi : T^*M \to M$ is the projection. Let $\Theta_M \in \Omega^1(T^*M)$ be the canonical 1-form and set $\Omega' := d^{\omega'}\Theta_M$. Then (T^*M, Ω', ω') is an l.c.s. manifold. It is globally conformal symplectic iff ω is exact. Moreover the graph of $\alpha \in \Omega^1(M)$ is a Lagrangian submanifold iff α is d^ω-closed.

On S^3 there exists a global frame of 1-forms α, β, γ satisfying $d\alpha = \beta \wedge \gamma$, $d\beta = \gamma \wedge \alpha$ and $d\gamma = \alpha \wedge \beta$. Let $\omega := dt \in \Omega^1(S^1)$ be the angular form and $\Omega := d^\omega \alpha \in \Omega^2(S^1 \times S^3)$. Then $(S^1 \times S^3, \Omega, \omega)$ is an exact l.c.s. manifold with non-vanishing φ. Notice that there exists no symplectic structure on $S^1 \times S^3$.

Finally consider the 4-dimensional torus T^4 and let dx, dy, dx', dy' denote the invariant 1-forms. Let $\omega := dx$, $\alpha := \sin(y)dx' + \cos(y)dy'$ and $\Omega := d^\omega \alpha$. Then one can easily check that (M, Ω, ω) is an exact l.c.s. manifold with non-vanishing φ.

5. $\ker \tilde{\Psi}$ is a Lie group

In this section we will show that, under some assumption, $\ker \tilde{\Psi}$ is a Lie group and the Lie algebra of it agrees with $\ker \psi = \mathcal{H}_c(M, \Omega, \omega)$. Hence in view of Proposition 2.2 $\mathcal{P}_c(M, \Omega, \omega)$ is then integrable. We will assume that (M, Ω, ω) satisfies the condition:

$(*)$ the space $B^1_{\mathrm{d}^\omega_c}(M)$ of d^ω_c-exact forms is a convenient direct summand in the space $Z^1_{\mathrm{d}^\omega_c}(M)$ of all d^ω_c-closed forms.

This is of course satisfied if $H^1_{\mathrm{d}^\omega_c}(M)$ is finite dimensional, especially all compact manifolds satisfy $(*)$.

Theorem 5.1. $\mathrm{Diff}^\infty_c(M, \Omega, \omega)$ *is a Lie group in the sense of [9] modeled on the convenient vector space* $\mathfrak{X}_c(M, \Omega, \omega)$*. Furthermore, under the condition* $(*)$ *the group* $\ker \tilde{\Psi}$ *is a regular Lie group with Lie algebra* $\mathcal{H}_c(M, \Omega, \omega)$*.*

Proof. In [7] one of us has shown the first assertion. To prove the second it will be useful to give a sketch of the proof of the first.

Assume Ω is not d^ω_c-exact (i.e. $\varphi = 0$). The remaining case is a bit more complicated but similar. Let us consider the l.c.s. manifold (T^*M, Ω', ω') where $\omega' = \pi^*\omega$, $\Omega' = \mathrm{d}^{\omega'} \Theta_M$, Θ_M is the canonical 1-form living on T^*M, and $\pi : T^*M \to M$ is the projection, cf. Section 4. It is well known that for any 1-form α one has $\alpha^*\Theta_M = \alpha$ and, consequently, $\alpha^*\Omega' = \mathrm{d}^\omega \alpha$.

If $p_1, p_2 : M \times M \to M$ be the projections to the first and second factor then $p_1^*\omega - p_2^*\omega$ is closed and equal to zero on the diagonal Δ. Therefore one can choose a smooth function λ defined locally around Δ such that $p_2^*\omega - p_1^*\omega = \mathrm{d}\ln\lambda$ and $\lambda|\Delta = 1$.

On some neighborhood of Δ we consider the l.c.s. structure $(\tilde{\Omega}, \tilde{\omega})$ with $\tilde{\omega} := p_1^*\omega$ and $\tilde{\Omega} := p_1^*\Omega - \lambda p_2^*\Omega$. Then we have

$$g \in \mathrm{Diff}^\infty_c(M, \Omega, \omega) \quad \Leftrightarrow \quad (\mathrm{id}, g)^*\tilde{\Omega} = 0.$$

Let $\exp : TM \to M \times M$ be the exponential mapping of a Riemannian metric on M. We have a diffeomorphisms

$$\gamma := \exp \circ \sharp : T^*M \supset V \to W \subset M \times M.$$

Then one can arrange so that $(V, \gamma^*\tilde{\Omega}, \gamma^*\tilde{\omega})$ and (V, Ω', ω') are equivalent and the zero section is their common Lagrange submanifold.

There is an open neighborhood U of id in $\mathrm{Diff}^\infty_c(M)$ with a chart $u : U \to u(U) \subset \Omega^1_c(M)$ for $\mathrm{Diff}^\infty_c(M)$ at id given by

$$u(g) := \gamma^{-1} \circ (\mathrm{id}, g) \circ (\pi \circ \gamma^{-1} \circ (\mathrm{id}, g))^{-1}.$$

Now $g \in \mathrm{Diff}^\infty_c(M,\Omega,\omega) \Leftrightarrow \mathrm{d}^\omega_c(u(g))=0$ and, consequently

$$u(U\cap \mathrm{Diff}^\infty_c(M,\Omega,\omega)) = u(U)\cap Z^1_{\mathrm{d}^\omega_c}(M).$$

Observe that the 1-form $u(g)$ is d^ω_c-exact iff so is the pullback of Θ_M on $L=u(g)(M)$, or $\Theta_1=(\gamma^{-1})^*\Theta_M$ on W pulls back to a d^ω_c-exact form on the graph of g. Equivalently, $(\mathrm{id},g)^*\Theta_1$ is d^ω_c-exact on M.

The chart u may be also regarded as a chart on $\widetilde{\mathrm{Diff}^\infty_c(M,\Omega,\omega)}$. Let $g_t\in U$, $t\in[0,1]$, with $g_0=\mathrm{id}$. Then $i_{\dot g_t}\Omega$ is d^ω_c-exact if and only if $(\mathrm{id},g_t)^*\Theta_1$ is d^ω_c-exact for any t. We wish to show that, shrinking U if necessary, one has

$$g\in U\cap \ker\tilde\Psi \quad\Leftrightarrow\quad u(g)\in u(U)\cap B^1_{\mathrm{d}^\omega_c}(M).$$

This follows by a standard argument (cf. [9]) provided $(*)$ is fulfilled. Finally it is visible from Proposition 4.5 that the Lie algebra of $\ker\tilde\Psi$ is actually $\mathcal{H}_c(M,\Omega,\omega)$. □

That $\ker\tilde\Psi$ is a Lie group can be derived also from a complicated theory due to H.Omori. By using both Frobenius theorem and implicit function theorem in the case of ILH-Lie groups he proved essentially the following result.

Theorem 5.2 [11], IX,7.2. $\mathrm{Diff}^\infty_c(M,\Omega,\omega)$ *is a strong ILH-Lie group.*

Proposition 5.3. *Let G be the kernel of a continuous, surjective homomorphism from* $\mathrm{Diff}^\infty_c(M,\Omega,\omega)$ *onto a finite dimensional Lie group. Then G carries a structure of a Fréchet-Lie group. Specifically,* $\ker\tilde\Psi$ *possesses a Lie group structure.*

Indeed, $\mathrm{Diff}^\infty_c(M,\Omega,\omega)$ is a strong ILH-Lie group, and an argument from [11], p.226, ensures us that G is a Fréchet-Lie group (but not necessarily an ILH-Lie group).

References

[1] A.Banyaga, P.Donato, *Some remarks on the integration of the Poisson algebra*, J.Geom.Phys. **19**(1996), 368-378.

[2] E.Calabi, *On the group of automorphisms of a symplectic manifold*, Problems in Analysis (symposium in honour of S.Bochner), Princeton University Press (1970), 1-26

[3] A.Coste, P.Dazord, A.Weinstein, *Groupoides symplectiques*, Publ. Dpt. Mathématiques, Univ. C. Bernard - Lyon 1, **2/A**(1987), 1-62.

[4] P.Dazord, A.Lichnerowicz, C.M.Marle, *Structure locale des variétés de Jacobi*, J.Math. pures et appl. **70**(1991), 101-152.

[5] W.T. van Est, T.J.Korthagen, *Non enlargible Lie algebra*, Indag.Math. **26**(1964), 15-31.

[6] F.Guedira, A.Lichnerowicz, *Géométrie des algèbres de Lie locales de Kirillov*, J. Math. pures et appl. **63**(1984), 407-484.

[7] S.Haller, Perfectness and Simplicity of Certain Groups of Diffeomorphisms, Thesis, University of Vienna (1998).

[8] S.Haller, T.Rybicki, *On the group of diffeomorphisms preserving a locally conformal symplectic structure*, to appear in Ann. Global Anal. Geom. (preprint ESI No. 636)

[9] A.Kriegl, P.W.Michor, The Convenient Setting of Global Analysis, American Mathematical Society 1997.

[10] H.C.Lee, *A kind of even dimensional differential geometry and its application to exterior calculus*, Amer. J. Math. **65**(1943), 433-438.

[11] H.Omori, Infinite-Dimensional Lie Groups, Transl. of Math. Monographs, A.M.S., 1997.

[12] J.Pradines, *Troisième théorème de Lie sur les groupoides différentiables*, C.R.Acad. Sc. Paris **267**(1968), 21-23.

[13] T.Ratiu, R.Schmid, *The differentiable structure of three remarkable diffeomorphism groups*, Math.Z. **177**(1981), 81-100.

[14] I.Vaisman, *Locally conformal symplectic manifolds*, Internat. J. Math. & Math. Sci. **8**(1985), 521-536.

(St.H.)
Institute of Mathematics
University of Vienna
Strudlhofgasse 4
1090 Wien, AUSTRIA
e-mail: stefan@nelly.mat.univie.ac.at

(T.R.)
Dpt. of Appl. Mathematics AGH
al.Mickiewicza 30
30-59 Kraków, POLAND
e-mail: rybicki@im.uj.edu.pl