

Complex Ray–Singer torsion

Consider a closed connected Riemannian manifold (M, g) and a flat complex vector bundle (E, ∇) over M . Moreover, suppose E admits a fiber wise non-degenerate bilinear form. We obtain a non-degenerate bilinear form β on $\Omega^*(M; E)$, the space of differential forms with values in E . Let d denote the deRham differential on $\Omega^*(M; E)$ and let d^\sharp denote its formal transpose with respect to β . Consider the operator $\Delta := (d + d^\sharp)^2 = dd^\sharp + d^\sharp d$ acting on $\Omega^*(M; E)$. This is a generalized Laplacian, and depends holomorphically on ∇ . One can, in the spirit of Ray–Singer, define a complex valued analytic torsion with the help of zeta regularized products of eigen values of Δ . Up to rather easily computable terms this complex valued analytic torsion coincides with the combinatorial torsion defined with the help of a Morse–Smale vector field. Particularly, this complex Ray–Singer torsion essentially computes the phase of the combinatorial torsion. This is joint work with Dan Burghelea.